# Exercises - Lecture 2

February 2, 2011

**1. Interpolation**

**(a)** Create a program that compares the exact value and an interpolation of the function $sin(x^2)$ from 0 to 3.

**(b)** Find the error for $x_1 = \sqrt{3} - 1$ and $x_2 = \sqrt{3} + 1$, for $n = 15$, $n = 30$ and $n = 60$.

**2. Differentiation**

**(a)** Modify the program from the foil "Differences on a Mesh" to calculate $\frac{d^4}{dx^4} sin(x)$. (Add more for loops.)

**(b)** Plot $sin(x)$ and the numerical approximation of $\frac{d^4}{dx^4} sin(x)$ for $n = 20$ and $n = 40$.

**3. Differential Equations**

Consider the initial value problem

$$u'(t) = 1 + 4u(t), \quad (1)$$

$$u(0) = 0. \quad (2)$$

**(a)** Verify that the analytical solution is

$$u(t) = \frac{e^{4t} - 1}{4} \quad (3)$$

**(b)** Write a program that finds an approximate solution for $u(t_k)$ for a time $t_k$. (Try to implement it in *C* and save data in a file.)

**(c)** Plot the approximate and the analytical solution for $u(t)$, $t \in [0, 1]$, with $n$ time steps. Use $n = 5$, $n = 10$, $n = 20$ and $n = 100$.

**4. Finite Difference Scheme**

**(a)** Adjust the function heat so that it also takes the boundary values as inputs and run the following program:

```
from scitools.std import *
import time
f = 1 if x < 0.3 else 3
f = StringFunction(f)
M = 20
N = 200
T = 0.1
b0 = 1
b1 = 3
heat(T, N, M, f, b0, b1)
```

Plot the result for t = 0.01 and t = 0.10.

**(b)** Adjust heat so that it can also handle diffusion reaction equation $u_t = u_{xx} - 5u^3$ with initial value $f(x) = 5x(x-1)$ and boundary conditions $u(0,t) = u(1,t) = 0$. Find the maximum value of the function $u$ at time $t = 0.1$ (You may use $M = 100, N = 5000$).

Please make both *Python* and *C* implementations and compare run speed. For the *C* code, please implement main function and heat function in different files and use the 2-step compiling approach mentioned in the previous lecture. Try different optimization flags (-O, -O1, ..., -O3) of *gcc* and observe how it affects the code performance.