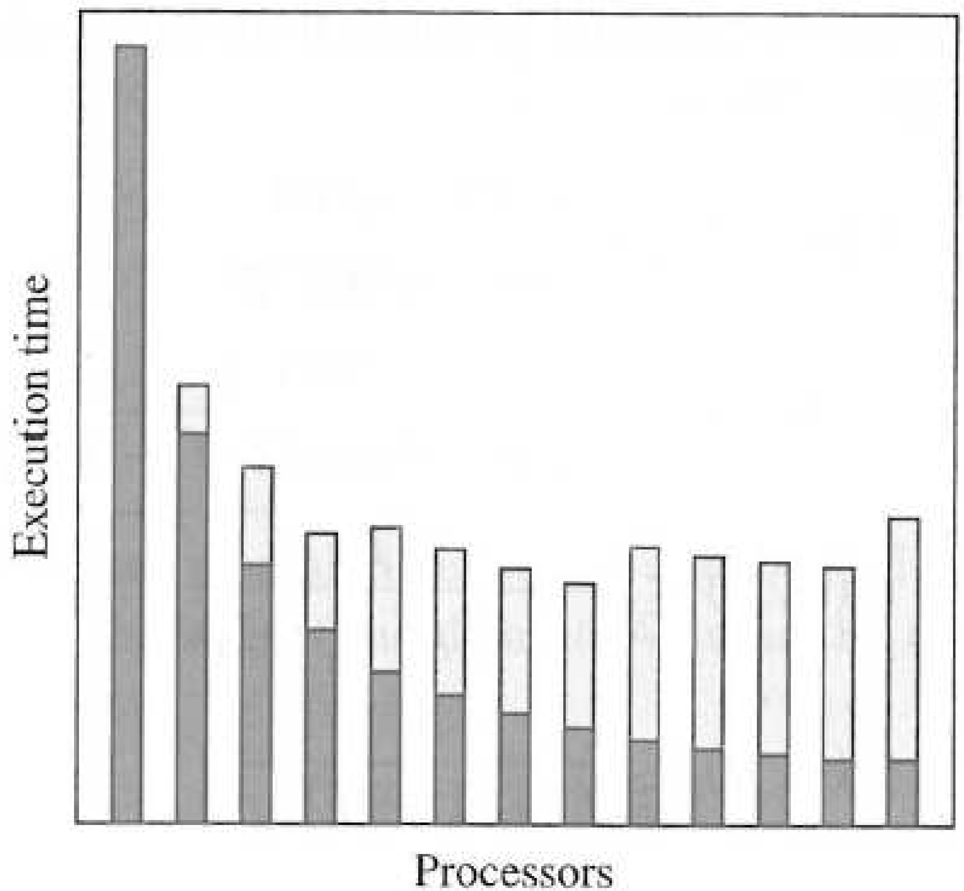
Lecture 8: Performance analysis

An example of time measurements

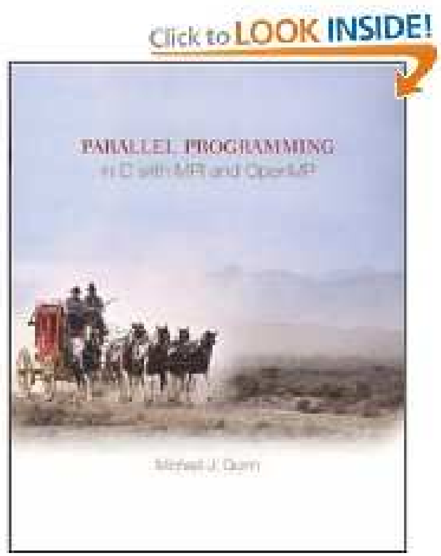


Dark grey: time spent on computation, decreasing with p
White: time spent on communication, increasing with p

Objectives of the lecture

- How to roughly predict computing time as function of p?
- How to analyze parallel execution times?
- Understand the limit of using more processors

Chapters 7 from *Michael J. Quinn*, **Parallel Programming in C with MPI and OpenMP**



Notation

n	problem size
p	number of processors
$\sigma(n)$	inherently sequential computaiton
$\varphi(n)$	parallelizable computation
$\kappa(n,p)$	parallelization overhead

$$\text{Speedup } \Psi(n,p) = \frac{\text{Sequential execution time}}{\text{Parallel execution time}}$$

$$\text{Efficiency } \varepsilon(n,p) = \frac{\text{Sequential execution time}}{\text{Processors used} \times \text{Parallel execution time}}$$

Simple observations

- Sequential execution time $= \sigma(n) + \varphi(n)$
- Parallel execution time $\geq \sigma(n) + \varphi(n)/p + \kappa(n,p)$

$$\text{Speedup } \Psi(n,p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p + \kappa(n,p)}$$

$$\text{Efficiency } \varepsilon(n,p) \leq \frac{\sigma(n) + \varphi(n)}{p\sigma(n) + \varphi(n) + p\kappa(n,p)}$$

Amdahl's Law

If the parallel overhead $\kappa(n,p)$ is neglected, then

$$\text{Speedup } \Psi(n,p) \leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

Suppose we know the inherently sequential portion of the computation,

$$f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)}$$

Can we predict the speedup $\Psi(n,p)$?

Amdahl's Law

Note that $f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)}$ means

$$1 - f = \frac{\varphi(n)}{\sigma(n) + \varphi(n)}$$

Therefore

$$\begin{aligned}
\Psi(n,p) &\leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p} \\
&= \frac{1}{\frac{\sigma(n)}{\sigma(n)+\varphi(n)} + \frac{\varphi(n)}{\sigma(n)+\varphi(n)} \frac{1}{p}} \\
&= \frac{1}{f + (1-f)/p}
\end{aligned}$$

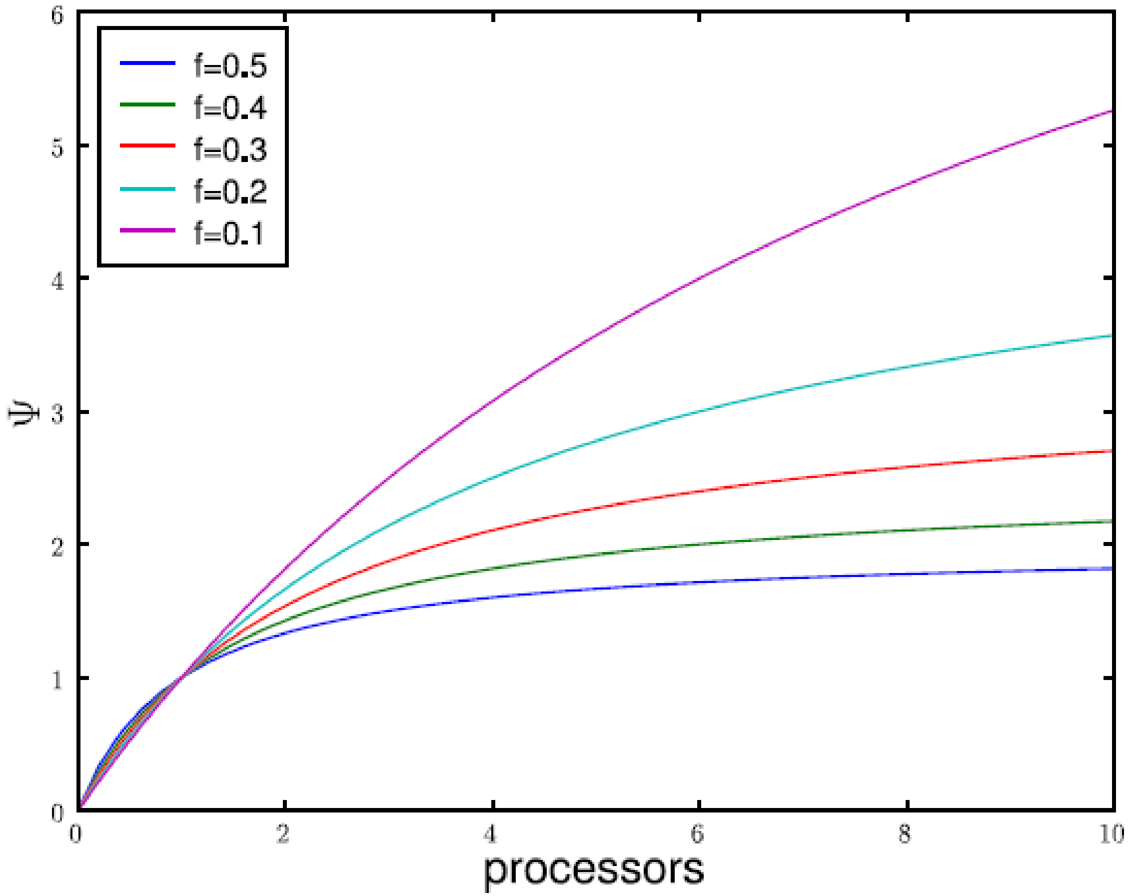
Amdahl's Law

Amdahl's Law: if $f = \sigma/(\sigma + \varphi)$ is known, then the best achievable speedup can be estimated as

$$\Psi \leq \frac{1}{f + (1-f)/p}$$

Upper limit (when p goes to infinity): $\Psi \leq \frac{1}{f+(1-f)/p} < \frac{1}{f}$

Amdahl's Law



Example 1

If we know that 90% of the computation can be parallelized, what is the maximum speedup we can expect from using 8 processors?

Solution

Since f=10%, Amdahl's Law tells us for $p = 8$

$$\Psi \leq \frac{1}{0.1 + \frac{(1-0.1)}{8}} \approx 4.7$$

Example 2

If 25% of the operations in a parallel program must be performed sequentially, what is the maximum speedup achievable?

Solution

The maximum speedup is

$$\lim_{p\to\infty} \frac{1}{0.25 + \frac{(1-0.25)}{p}} = \frac{1}{0.25} = 4$$

Example 3

Suppose

$$\begin{aligned}\sigma(n) &= 18000 + n \\ \varphi(n) &= \frac{n^2}{100}\end{aligned}$$

What is the maximum speedup achievable on a problem of size $n = 10000$?

Example 3 (cont'd)

Solution

Since

$$\Psi(n,p) \leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p}$$

and we know

$$\begin{aligned} \sigma(10000) &= 28,000 \\ \varphi(10000) &= 1,000,000 \end{aligned}$$

Therefore

$$\Psi(10000,p) \leq \frac{28,000+1,000,000}{28,000+1,000,000/p}$$

Comments about Amdahl's Law

- Parallelization overhead $\kappa(n, p)$ is ignored by Amdahl's Law
  - Amdahl's Law gives a too optimistic estimate of Ψ
- The problem size n is constant for $p = 1$ and increasing p values
  - Amdahl's Law doesn't consider solving larger problems with more processors
- The inherently sequential portion f may greatly decrease when n is increased
  - Amdahl's Law ($\Psi < \frac{1}{f}$) can be unnecessarily pessimistic for large problems

Gustafson–Barsis's Law

What if we want to solve larger problems when the number of processors p is increased?

That is, we may not know the computing time needed by a single processor, because the problem size is too big for one processor.

However, suppose we know the fraction of time spent by a parallel program (using p processors) on performing inherently sequential operations, can we estimate the speedup Ψ?

Gustafson–Barsis's Law

Definition: s is the fraction of time spent by a parallel computation using p processors on performing inherently sequential operations.
More specifically,

$$s = \frac{\sigma(n)}{\sigma(n) + \varphi(n)/p}$$

and

$$1 - s = \frac{\varphi(n)/p}{\sigma(n) + \varphi(n)/p}$$

Gustafson–Barsis's Law

We note

$$\begin{aligned}\sigma(n) &= (\sigma(n) + \varphi(n)/p)s \\ \varphi(n) &= (\sigma(n) + \varphi(n)/p)(1-s)p\end{aligned}$$

Therefore

$$\begin{aligned}\Psi(n,p) &\leq \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p} \\ &= \frac{(s+(1-s)p)(\sigma(n)+\varphi(n)/p)}{\sigma(n)+\varphi(n)/p} \\ &= s+(1-s)p \\ &= p+(1-p)s\end{aligned}$$

Gustafson–Barsis's Law

Given a parallel program solving a problem of size n using p processors, let s denote the fraction of total execution time spent in serial code. The maximum speedup Ψ achievable is

$$\Psi \leq p + (1 - p)s$$

Comments about Gustafson–Barsis's Law

- Gustafson–Barsis's Law encourages solving larger problems using more processors. The speedup obtained is thus also called scaled speedup.
- If n is large enough for p processors, n is probably too large (with respect to memory) for a single processor. However, this doesn't prevent Gustafson–Barsis's Law from predicting the best achievable speedup Ψ, when s is known.
- Since parallelization overhead $\kappa(n,p)$ is ignored, Gustafson–Barsis's Law may also overestimate the speedup.
- Since $\Psi \leq p + (1-p)s = p - (p-1)s$, so the best achievable speedup is $\Psi \leq p$. The smaller s the better Ψ.
- If $s = 1$, then there is no speedup at all, because $\Psi \leq p + (1-p) = 1$.

Example 1

An application executing on 64 processors uses 5% of the total time on non-parallelizable computations. What is the scaled speedup?

Solution

Since $s = 0.05$, the scaled speedup on 64 processors is

$$\Psi \leq p + (1-p)s = 64 + (1-64)(0.05) = 64 - 3.15 = 60.85$$

Example 2

If we want to achieve speedup $\Psi = 15000$ using $p = 16384$ processors, what can the maximum allowable value of the serial fraction s be?

Solution

Since

$$\Psi \leq p + (1-p)s = p - (p-1)s$$

then

$$s \leq \frac{p - \Psi}{p - 1} = \frac{16384 - 15000}{16384 - 1} \approx 0.084$$

Karp–Flatt Metric

Both Amdahl's Law and Gustafson–Barsis's Law ignore the parallelization overhead $\kappa(n,p)$, they may therefore overestimate the achievable speedup.

We recall

$$\begin{aligned}
\text{Parallel execution time } T(n,p) &= \sigma(n) + \varphi(n)/p + \kappa(n,p) \\
\text{Sequential exectuion time } T(n,1) &= \sigma(n) + \varphi(n)
\end{aligned}$$

Karp–Flatt Metric

If we consider the parallelization overhead as another kind of "inherently sequential work", then we can use Amdahl's law to experimentally determine a "combined" serial fraction e, which is defined as

$$e(n,p) = \frac{\sigma(n) + \kappa(n,p)}{\sigma(n) + \varphi(n)}$$

This experimentally determined serial fraction $e(n,p)$ may either stay constant with respect to p (meaning that the parallelization overhead is negliable) or increase with respect to p (meaning that parallelization overhead deteriorates the speedup).

Karp–Flatt Metric

If we know the actually achieved speedup $\Psi(n, p)$ using p processors, how can we determine the serial fraction $e(n, p)$?

Since

$$T(n, p) = T(n, 1)e + \frac{T(n, 1)(1 - e)}{p}$$

and we know the value of $\Psi(n, p)$, which is defined as

$$\Psi(n, p) = \frac{T(n, 1)}{T(n, p)} = \frac{T(n, 1)}{T(n, 1)e + \frac{T(n,1)(1-e)}{p}} = \frac{1}{e + \frac{1-e}{p}}$$

Therefore

$$\frac{1}{\Psi} = e + \frac{1 - e}{p} \quad \Rightarrow \quad e = \frac{1/\Psi - 1/p}{1 - 1/p}$$

Example 1

Benchmarking a parallel program on $1, 2, \ldots, 8$ processors produces the following speedup results:

p	2	3	4	5	6	7	8
Ψ	1.82	2.50	3.08	3.57	4.00	4.38	4.71

What is the primary reason for the parallel program achieving a speedup of only 4.71 on eight processors?

Example 1 (cont'd)

Solution

We can use Karp–Flatt Metric to experimentally determine the values of $e(n, p)$ as

p	2	3	4	5	6	7	8
Ψ	1.82	2.50	3.08	3.57	4.00	4.38	4.71
e	0.10	0.10	0.10	0.10	0.10	0.10	0.10

Since the experimentally determined serial fraction e is not increasing with p, the primary reason for the poor speedup is the large fraction (10%) of the computaiton that is inherently sequential. In other words, parallel overhead is not the reason for the poor speedup.

Example 2

Benchmarking a parallel program on $1, 2, \ldots, 8$ processors produces the following speedup results:

p	2	3	4	5	6	7	8
Ψ	1.87	2.61	3.23	3.73	4.14	4.46	4.71

What is the primary reason for the parallel program achieving a speedup of only 4.71 on eight processors?

Example 2 (cont'd)

Solution

We can use Karp–Flatt Metric to experimentally determine the values of $e(n, p)$ as

p	2	3	4	5	6	7	8
Ψ	1.87	2.61	3.23	3.73	4.14	4.46	4.71
e	0.070	0.075	0.080	0.085	0.090	0.095	0.1

Since the experimentally determined serial fraction e is steadily increasing with p, parallel overhead is also a contributing factor to the poor speedup.