UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in INF3470/4470 — Digital signal processing

Day of examination: December 12, 2007

Examination hours: 14.30 – 17.30

This problem set consists of 5 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Sampling and reconstruction

a) For each of the four signals, use Shannons samplings theorem to find all sampling frequencies that will not give aliasing: 1 p.

1. $x(t) = \cos(600\pi t + \pi/2)$.
2. $x(t) = \cos(2\pi 300 t - \pi/4)$.
3. $x(t) = \cos(150\pi t) + \sin(151\pi t)$.
4. $x(t) = \begin{cases} 1 & \text{for} \quad |t| \leq T \\ 0 & \text{for} \quad |t| > T. \end{cases}$

b) Given the following continuous time signal

$$x(t) = \cos(300\pi t + \pi/2) + \cos(600\pi t + \pi/3).$$

This signal is to be sampled 400 times per second to form the discrete-time-signal $x[n] = x(nT_s)$, where T_s is the time between two samples. Calculate $x[n]$ and sketch the amplitude spectrum of $x[n]$. 1 p.

Problem 2 LTI-systems

a) We usually seek stable LTI-systems. Formulate one requirement for a system with impulse response $h[n] = T\{\delta[n]\}$ to be stable. .5 p.

c) For a system to be realizable, it must be both stable and causal. Given an LTI-system with impulse response $h[n] = T\{\delta[n]\}$, formulate a requirement for this system to be causal. .5 p.

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c) Formulate the necessary requirements for a et system $y[n] = T\{x[n]\}$ to be *linear* and *time invariant*. 1 p.

d) By calculation, determine if the following systems are linear and time invariant or not: 1 p.

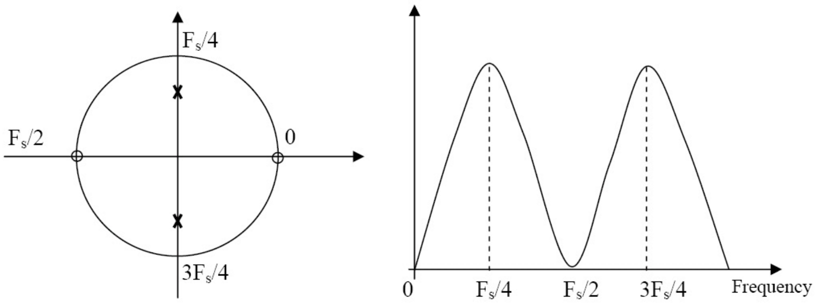
1. $y[n] = x[n] - 3x[n-1] + x[n-2]$.
2. $y[n] = x[n+1] - x[n] + x[n-1] + 5x[2]$.

Problem 3 Design av IIR filtre

A bandpass digital filter is required to meet the following specifications;

1. complete signal rejection at dc and at 500 Hz;
2. a narrow passband centered at 250 Hz;
3. a 3 dB bandwidth of 20 Hz.

The sampling frequency of 1000 Hz is used. The filter is realized by a second-order IIR filter, and a sketch of the pole-zero diagram and magnitude response is given in the following figure.



Sketch of pole-zero diagram and frequency response.

The radius r of the poles is determined by the desired bandwidth. An approximate relationship between r, for $r > 0.9$, and the bandwidth, bw, is given by

$$r \approx 1 - (\text{bw}/F_s)\pi,$$

where F_s is the sampling frequency.

a) Obtain the transfer function, $H(z)$, of the filter. Use $\pi = 3.14$ in your calculations. 1 p.

b) Find the difference equation of the filter. 1 p.

(Continued on page 3.)

Problem 4 Design of FIR-filters

You are given the task of designing a FIR-filter with impulse response $h[n]$ which meets the following requirements:

$$T\{\cos(0.5\pi n + \phi)\} = 0, \phi \in [0..2\pi]$$

$$T\{\cos(0.75\pi n + \phi)\} = 0, \phi \in [0..2\pi].$$

Additional requirements are

1. The filter is to be causal.
2. The filters is to have real coefficients.

How the filter is to react to other signals are not specified, and we can, therefore, disregard this point.

a) What is the minimum number of coefficients you need to make a filter which obey the given requirements? State your reasons. 1 p.

b) Find the zeros and poles of the system and draw a pole-zero plot. 1 p.

c) Find the system function, $H(z)$, and the impulse response, $h[n]$, of the system. 1 p.

d) You are given the following additional requirement; The DC-signal (constant signals) are to pass through the system without changing:

$$T\{\alpha\} = \alpha, \alpha \in \mathbf{R}$$

Find the new impulse response which obey this demand 1 p.
(**Hint:** The additional requirement means that $H(e^0) = 1$).

Problem 5 Filters og convolution

An LTI-system with impulse response $h[n]$ and input signal $x[n]$ has as output the signal $y[n]$ given as:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k],$$

where $*$ denotes the convolution operator as defined in the above equation.

a) What is the output, $y[n]$, expressed by the input, $x[n]$, when the impulse response is given as .5 p.

$$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2].$$

(Continued on page 4.)

b) Calculate the output, $y[n]$, when the input is: 1 p.

$$x[n] = \delta[n] + 3\delta[n-2] + 2\delta[n-4].$$

c) Prove the general relationship that convolution in time domain equals multiplication in frequency domain, i.e. that: 1 p.

$$Y(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}})X(e^{j\hat{\omega}}).$$

A causal running-average filter (RA-filter) of length M is given by the following impulse response and system function:

$$h[n] = \frac{1}{M}\sum_{k=0}^{M-1}\delta[n-k], \quad \text{and} \quad H(z) = \frac{1}{M}\frac{z^M - 1}{z^{M-1}(z-1)}.$$

d) Calculate the frequency response $H(e^{j\hat{\omega}})$ of the filter and explain the effect the RA-filter has on the input signal. (At most 3 sentences). 1 p.

e) We can construct a new filter by cascading K running-average filters in a series, i.e. .5 p.

$$h_K[n] = h[n] * h[n] * \cdots * h[n].$$

What is the spectrum, $H_K(e^{j\hat{\omega}})$ of $h_K[n]$?

Formelas

Some basic relations:

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= 2\cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}) \\
\sin\alpha &= \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 &\Leftrightarrow x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

(Continued on page 5.)

Convolution:

$$y(n) = x(n)*h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} x(n-k)h(k) = h(n)*x(n)$$

Discrete time Fourier transform (DTFT):

$$\begin{aligned} \text{Analyse:} \quad X(\hat{\omega}) &= X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\hat{\omega}n} \\ \text{Syntese:} \quad x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\hat{\omega}})e^{j\hat{\omega}n}d\hat{\omega} \end{aligned}$$

Discrete Fourier transform (DFT):

$$\begin{aligned} \text{Analyse:} \quad X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad 0 \le k \le N-1 \\ \text{Syntese:} \quad x(n) &= \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, \quad 0 \le k \le N-1 \end{aligned}$$

z-transform:

$$\text{Analyse:} \quad X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Expectation and variance

$$\begin{aligned} \text{Forventning:} \quad E\{x(\zeta)\} &\equiv \mu_x = \begin{cases} \sum_k x_k p_k & x(\zeta) \text{ discrete} \\ \int_{-\infty}^{\infty} x f_x(x)dx & x(\zeta) \text{ continuous} \end{cases} \\ \text{Varians:} \quad \text{var}[x(\zeta)] &= \sigma_x^2 = \gamma_x^{(2)} = E\{[x(\zeta)-\mu_x]^2\} \end{aligned}$$

Good luck!!!