UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	INF3470/4470 — Digital signal processing
Day of examination:	December 10th, 2014
Examination hours:	14.30 – 18.30
This problem set consists of 7 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 Frequency domain analysis

a) Find the DC gain of a 3-point filter whose impulse response is given as $h[n] = \left\{\alpha, \overset{\Downarrow}{\beta}, \alpha\right\}$ such that it completely blocks the normalized frequency $F = \frac{1}{3}$ and passes the normalized frequency $F = 0.125$ with unity gain, where $F = 1$ is the Nyquist frequency. 1 p.

b) A system is described by $y[n] = x[n] + 2x[n-1] + 3x[n-2]$.

1. Find $H[F]$ of the system. Obtain the DC gain and high frequency gain of the system. 1 p.
2. Find its impulse response $h[n]$. 1 p.

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Problem 2 Filter Concepts

a) Given in Figure 1 is the pole-zero plot of the transfer function $H[z]$ of four filters A, B, C and D respectively. All filters are causal and the radius of the circle in complex plane is $|z| = 1$.

Identify the characteristics of each filter from the respective options and reproduce and complete the table below. 2 p.

Characteristics:

1. *Stability*: Unstable, Marginally stable, Stable
2. *Phase response:* Minimum phase, Mixed phase, Maximum phase
3. *Magnitude response:* Low pass filter, High pass filter, Bandpass filter, Notch filter, Comb filter, All pass filter, Digital resonator.
4. *Type of filter:* FIR, IIR.

Reproduce and complete the table below.

Filters → Characteristics ↓	Filter A	Filter B	Filter C	Filter D
Stability				
Phase response				
Magnitude response				
Type of filter				

(Continued on page 3.)

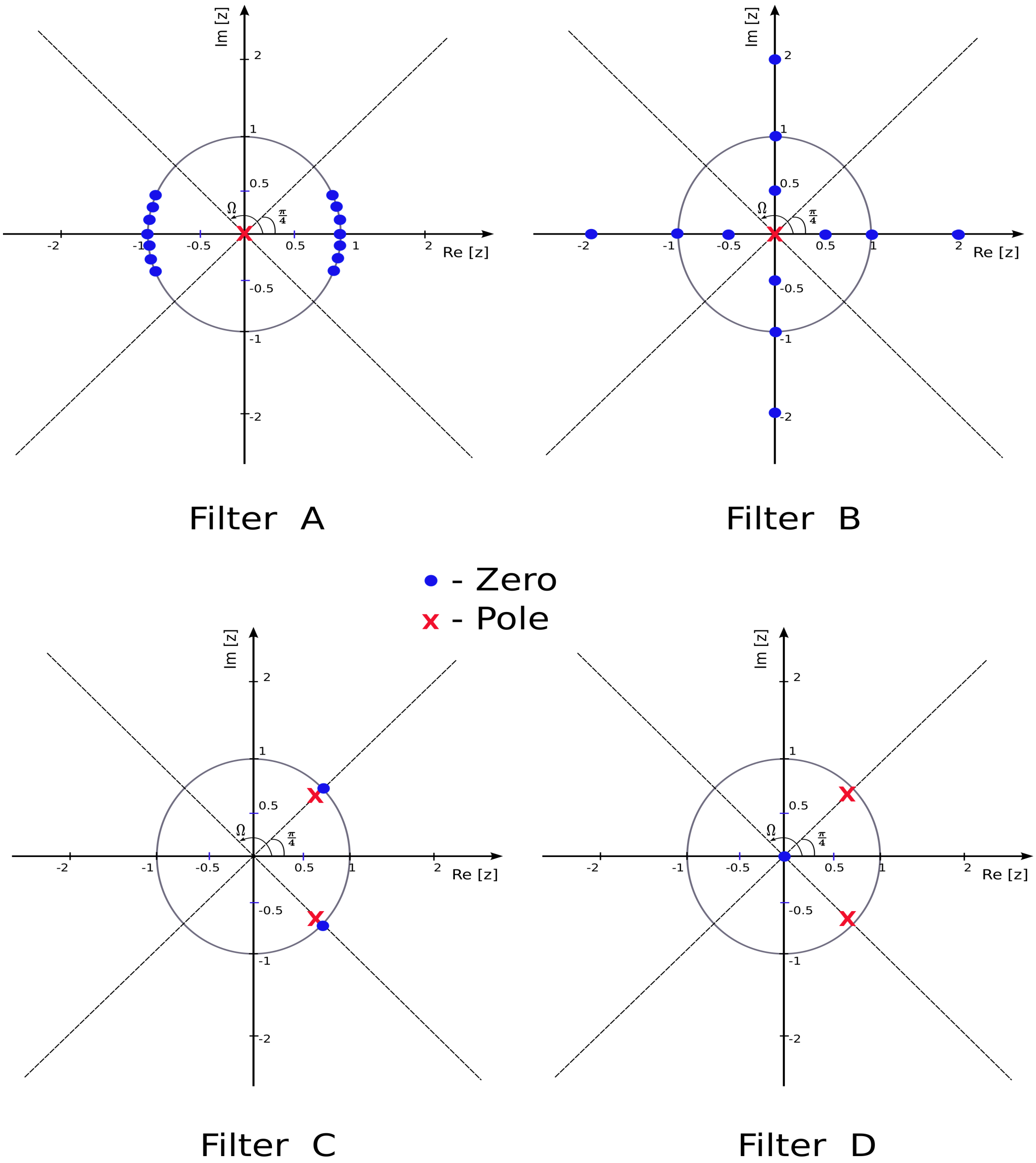

Figure 1: Pole-zero plots

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Problem 3 Z - Transform

a) Show that the Z transform does not exist for the discrete signal $x[n] = 1$, for $-\infty \leq n \leq \infty$. 1 p.

b) Prove the following *derivation* property of the two-sided Z transform:

$$\frac{d}{dz}X(z) = -\frac{\mathcal{Z}\{n\,x(n)\}}{z},$$

where $X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x[n]\,z^{-n}$. Here $\mathcal{Z}$ denotes the Z transformation operator. 1 p.

c) The Z transform $X[z]$ of a causal discrete signal $x[n]$ is

$$X[z] = \frac{10z^3 + 40z}{z(z^2 - 2z - 3)}$$

1. Find the poles and zeros of $X[z]$ and give their order. 1 p.
2. Use a suitable method (residue method or partial fraction decomposition) to obtain the complete expression for $x[n]$. Evaluate $x[0]$, $x[1]$ and $x[2]$. 1 p.
3. Use synthetic long division method (power series expansion) to obtain the values of $x[0]$, $x[1]$ and $x[2]$ and confirm the result you have obtained in 2. 1 p.

Problem 4 The DFT

a) The four-point DFT of a sequence $x[n]$ is given as

$$X[k] = \sum_{n=0}^{3} x[n]W_4^{nk} = 1 + W_4^k + 2W_4^{2k} - W_4^{3k}.$$

Find x[n]. (Show your reasoning). 1 p.

b) A four-point sequence $z[n]$ is given as the circular flipped version of $x[n]$ as in a), i.e.

$$z[0] = x[0],\ z[1] = x[3],\ z[2] = x[2],\ \text{ and } z[3] = x[1].$$

The sequence $z[n]$ is then zero-interpolated by a factor 3 to form the 12-point sequence $w[n] = z^{\uparrow}[n/3]$.
Find the complex values of the first six DFT-coefficients of this sequence, i.e.

$$W[k] = \text{DFT}\{w[n]\}, \quad k = 0 \ldots 5.$$

(Show you reasoning). 2 p.

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Problem 5 IIR-filter

A causal IIR filter with one real pole $z_p \in \Re$ has the impuls response:

$$h[n] = \alpha^n u[n], \quad 0 < \alpha < 1. \tag{1}$$

a) Given the system with the impulse response $h[n]$, find the system function $H(z)$ together with the ROC. Indicate the pole location in a pole-zero plot. 1 p.

b) Find the system function $G(z)$ together with the ROC for the system with (complex) impulse response

$$g[n] = h[n]e^{j\frac{\pi}{2}n}$$

where $h[n]$ is given in (1).
Indicate the pole location in a pole-zero plot. 1 p.

c) Find the system function $H'(z)$ together with the ROC to the system with impulse response given as

$$h'[n] = h[n]\cos(\frac{\pi}{2}n),$$

where $h[n]$ is given in (1).
Indicate the pole locations in a pole-zero plot.
Explain the functioning of the systems $h[n]$ og $h'[n]$ interpreted in the frequency domain. 1 p.

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Problem 6 Sampling and Filter Design

a)

1. Plot the magnitude response of an ideal low pass filter. In the same plot, sketch the response of a practical low pass filter and label the different frequency bands. .5 p.
2. Find the impulse response $h[n]$ of an ideal low pass filter. What is the difficulty in implementing it practically? What is a reasonable practical implementation of this filter? .5 p.
3. What is Gibbs effect? Draw a suitable sketch to demonstrate it. As a filter designer, what necessary steps would you take to minimize Gibbs effect? Draw suitable sketches to illustrate how the steps taken by you affect the magnitude response of the filter? 1 p.
4. What is the gain in dB of a filter if its output voltage is 10 times the input voltage? .5 p.

b) A given sinusoid signal $x(t)$ in time t is given as $x(t) = \sin(10\pi t + \pi/4)$.

Make suitable sketches (as applicable) to demonstrate aliasing, undersampling, oversampling, Nyquist frequency and folding when the signal is sampled at frequencies:

1. $f_1 = 2.5$ Hz .5 p.
2. $f_2 = 50$ Hz .5 p.
3. $f_3 = 10$ Hz. .5 p.

Formulas

Cosine values of angles in radians

Angle θ in radians	$\cos(\theta)$
0	1
$\frac{\pi}{6}$	0.866
$\frac{\pi}{4}$	0.7071
$\frac{\pi}{3}$	0.5
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	-0.5
$\frac{5\pi}{6}$	-0.866
π	-1

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Basic relations:

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}) \\
\sin\alpha &= \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 &\Leftrightarrow x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = h[n] * x[n]$$

Discrete-time Fourier transformation (DTFT):

$$\begin{aligned}
\text{Analysis:} \qquad X(\Omega) &= = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n} \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n} d\Omega
\end{aligned}$$

Discrete Fourier transformation (DFT):

$$\begin{aligned}
\text{Analysis:} \qquad X[k] &= \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad 0 \le k \le N-1 \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N}, \quad 0 \le k \le N-1
\end{aligned}$$

z-transformation:

$$\text{Analysis:} \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$