UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in INF3470/4470 — Digital signal processing

Day of examination: December 11th, 2015

Examination hours: 09.00 – 13.00

This problem set consists of 7 pages.

Appendices: None

Permitted aids: None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

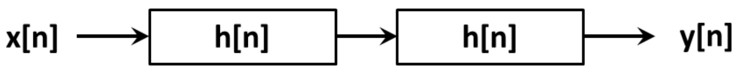
Note 1: All numbers and figure axes should have units.
Note 2: Read through the whole exercise set before you start!

Problem 1 FIR filters

We study a realizable low-pass FIR filter of order M with impulse response $h[n]$, and pass and stopband frequencies $\omega_p = 0.2$ rad/sample and $\omega_s = 0.3$ rad/sample. Its pass and stopband ripple levels are $\delta_p = 0.125$ and $\delta_s = 0.1$.

a)
- What is the width of the filter transition band? 0.5 p.
- Schematically draw the magnitude of the filter frequency response and indicate δ_p and δ_s on the vertical axis. 0.5 p.

b) The output $y[n]$ is obtained from the input $x[n]$ by cascading two filters with impulse response $h[n]$ as shown below.



Determine the order of the effective filter with impulse response $h_{tot}[n]$ used to get $y[n]$ from $x[n]$. Justify your answer. 1 p.

c) The absolute specifications for $h[n]$ corresponding to δ_p and δ_s are $A_p = 2.2$ dB and $A_s = 21$ dB. What are the absolute specifications for the passband and stopband ripple levels for the effective filter described in b)? 1 p.

d) The frequency response corresponding to $h[n]$ and $h_{tot}[n]$ are noted $H(e^{j\omega})$ and $H_{tot}(e^{j\omega})$.

- Give an expression for the phase of $H_{tot}(e^{j\omega})$ as a function of the phase of $H(e^{j\omega})$. 0.5 p.
- If $H(e^{j\omega})$ is linear phase, what can you say about the phase of $H_{tot}(e^{j\omega})$? Justify your answer. 0.5 p.

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Problem 2 Sampling and aliasing

Let us consider a signal $x(t)$ with frequency spectrum

$$X(\Omega) = 1 \quad \text{for } |\Omega| \leq 200\pi \text{ rad/s}$$
$$X(\Omega) = 0 \quad \text{for } |\Omega| > 200\pi \text{ rad/s}$$

a) Sketch the frequency spectrum of $x[n]$ the ideally sampled version of $x(t)$ at the sampling frequency $F_s = 300$ Hz. 1 p.

b)
- If $x[n]$ was a filter impulse response, what sort of filter would it correspond to? 0.5 p.
- Would $x[n]$ be realizable and why? 0.5 p.

c) Sketch the frequency spectrum of $x[n]$ the ideally sampled version of $x(t)$ at the sampling frequency $F_s = 150$ Hz. 1 p.

Problem 3 Transform analysis of LTI systems

You have recently been hired to work with a research project about heart rate monitoring of anxious exam candidates at UiO. You have developed a combined sensor and data logging system powered from mains (240 V @ 50 Hz), but you were sloppy with your hardware design and have problems with the measurements data being corrupted with 50 Hz noise. Your boss has discovered that you have taken a course in digital signal processing and has asked you to design a filter to remove this noise. You vaguely remember something about a notch filter. Since you unfortunately sold your old signal processing book, some desperate googling lead you to the following transfer function for a notch filter:

$$H(z) = b_0[1 - (2\cos(\phi))z^{-1} + z^{-2}] \tag{1}$$

a) Find the zeros of $H(z)$ as a function of ϕ. Provided that the sampling frequency in the system is $F_s = 1$ kHz, find ϕ to suppress the 50 Hz noise from the mains system. Draw a pole/zero plot for the filter. 1 p.

b) Compute the magnitude response $|H(e^{j\omega})|$ of the filter and sketch it. You may use that $b_0 = 1$ and $\cos(\phi) = 0.95$. Is the filter a minimum phase filter? Justify your answer. 1 p.

c) Find the impulse response and the differential equation of the system. Assume an input signal $x[n]$. 1 p.

d) Compute the system response $y[n]$ for the input signal $x[n] = s[n] + v[n]$ where $v[n] = A\cos(\pi/10n), n >= 0$. Comment on your findings. 1 p.

e) You struggle a bit with the filter removing too much of the original signal in the frequency range around 50 Hz. How can you modify your filter to improve this? Use a pole/zero plot and explain qualitatively how your change(s) will affect the frequency response of the system. 1 p.

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Problem 4 Match the systems!

Equations 2 to 5 describe 4 systems. Figures 1, 2 and 3 show corresponding plots. In the pole/zero plots, zeros have been added at the origin to get the same polynomial degree in the numerator and denominator. Match the systems and the plots. You may assume that all the systems are causal. Additionally, when possible, label each system with: 5 p.

- stability properties: (stable/unstable/marginally stable).
- phase response (minimum phase/mixed phase/maximum phase).
- impulse response length (FIR/IIR).
- type of filter (notch/ comb/ digital resonator/ allpass filter/ low pass/ high pass/band pass/ band stop)

Note 1! All answers are to be justified. Random combinations without justification are not rewarded.
Note 2! It is actually possible to solve this exercise almost completely by reasoning and without calculation!
Note 3! Each correct equation-figure combination is scored 0.33 points. Each correct additional information is scored 0.10 points. Maximum score is 5 points.

$$y[n] = 0.9\sqrt{2}y[n-1] - 0.81y[n-2] + x[n] \tag{2}$$

$$H(z) = \frac{1}{2}\sum_{k=0}^{M-1}(W_M^{-mk} + W_M^{mk})z^{-k}, M = 10, m = 1, W_m = e^{j2\pi/M} \tag{3}$$

$$H(z) = \frac{(3/4)(3/4 - \sqrt{2}z^{-1}) + z^{-2}}{1 - (3\sqrt{2}/4)z^{-1} + (3/4)^2 z^{-2}} \tag{4}$$

$$H(z) = \frac{z^{-1}}{1 + (6/5)z^{-1} + (6/5)^2 z^{-2}} \tag{5}$$

Fill in a table using the following format (Justifications and calculations to be provided outside the table):

Filter → Characteristics ↓	Equation 2	Equation 3	Equation 4	Equation 5
Pole/Zero (figure)				
Magnitude response (figure)				
Impulse response (figure)				
Stability properties				
Phase response				
Impulse response length				
Type				

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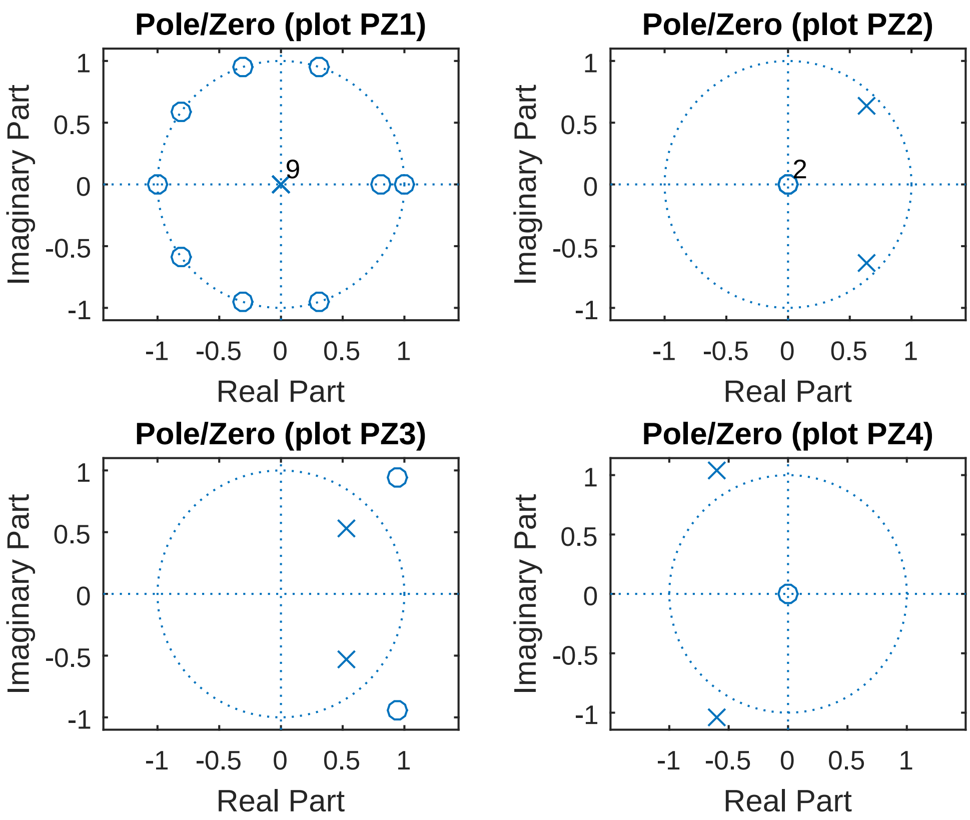

Figure 1: Plots a-c

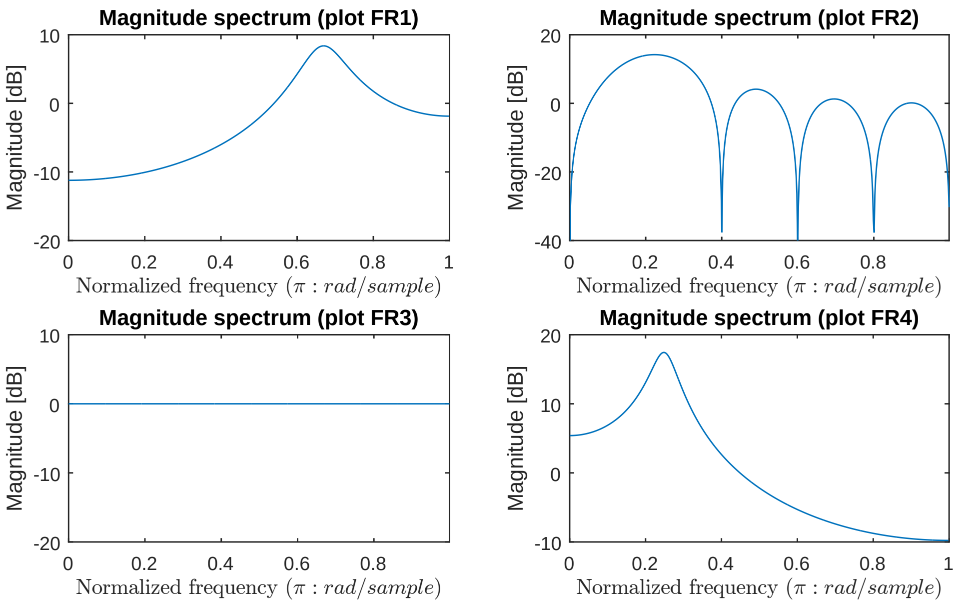

Figure 2: Plots d-f

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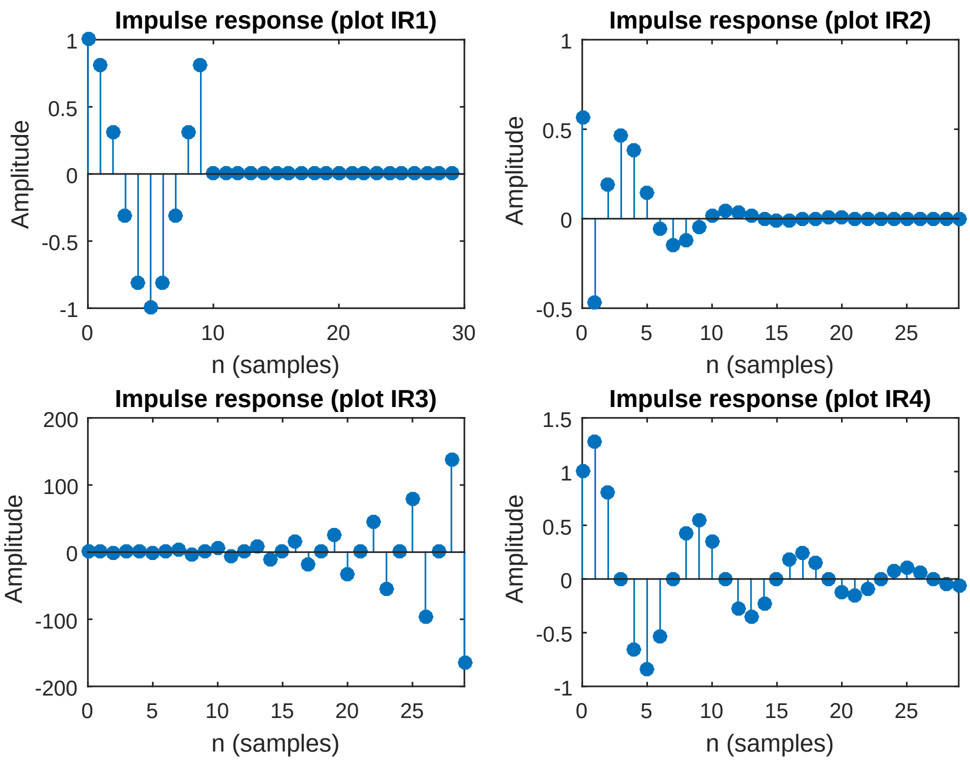

Figure 3: Plots g-j

Problem 5 Linear vs. circular convolution

Consider the two finite length sequences

$$x_1[n] = \{\underset{\uparrow}{1}, -2, 1, -3\}$$

$$x_2[n] = \{\underset{\uparrow}{0}, 2, -1, 0, 0, 4\}$$

a) Determine the linear convolution $x_1[n] * x_2[n]$. 1 p.

b) Determine the 6-point cicular convolution $x_1[n] \textcircled{6} x_2[n]$. 1 p.

c) What should be the smallest value of N so that the N-point circular convolution is equal to the linear convolution? 1 p.

d)
- When a filtering operation is done on computers it is usually done using multiplications in the frequency domain. What kind of convolution does it correspond to? 0.5 p.
- Explain why zero-padding is a useful technique in this case. 0.5 p.

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Formula sheet

Basic relations:

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}) \\
\sin\alpha &= \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 &\Leftrightarrow x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Linear convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = h[n] * x[n]$$

Circular convolution:

$$y[n] = x[n]\,\text{Ⓝ}\,h[n] = \sum_{k=0}^{N-1} x[k]h[< n-k >_N] = \sum_{k=0}^{N-1} x[< n-k >_N]h[k] = h[n]\,\text{Ⓝ}\,x[n]$$

Discrete Time Fourier Transform (DTFT):

$$\begin{aligned}
\text{Analysis:} \quad X(\Omega) &= = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n} \\
\text{Synthesis:} \quad x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}d\Omega
\end{aligned}$$

Discrete Fourier Transform (DFT):

$$\begin{aligned}
\text{Analysis:} \quad X[k] &= \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad 0 \le k \le N-1 \\
\text{Synthesis:} \quad x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N}, \quad 0 \le k \le N-1
\end{aligned}$$

(Continued on page 7.)

z-transform:

$$\text{Analysis:} \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$