



ifj

# Filterkonsepter kapittel 6

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# 6 Filterkonsepter

## 6.1 Frekvensrespons og filterkarakteristikker

- gain, forsinkelse, fase, lineær- og minimum-fase, grafisk betraktning

## 6.2 FIR filtre og lineær fase

- pol-nullpunkt, type 1-4, midlingsfiltre, kam-filtre

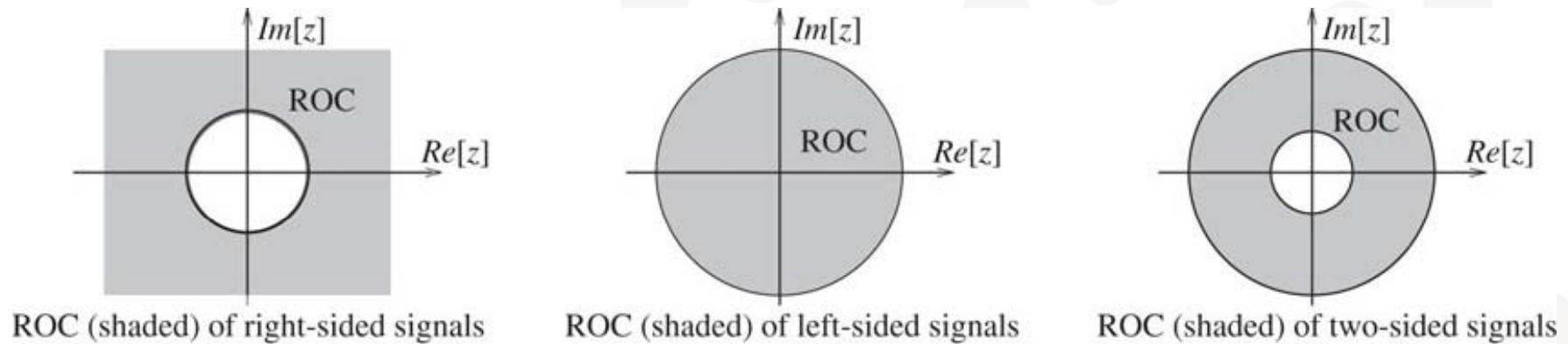
## 6.3 IIR filtre

- 1./2. ordens, pol/null plassering, resonator, notch

## 6.4 Allpass-filtre

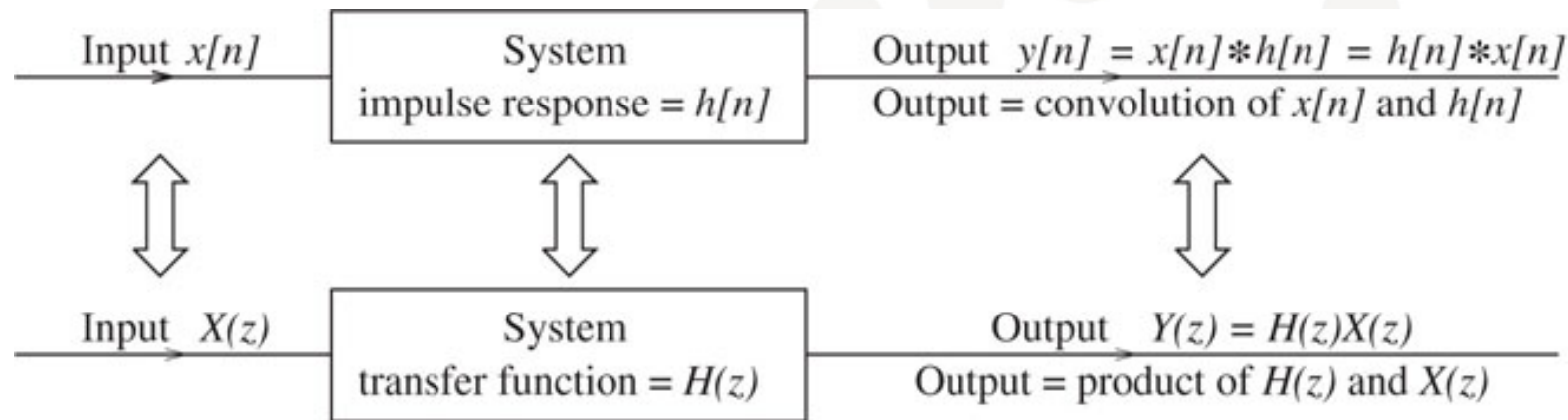
# Z-transform

$$H(z) = \sum_{m=-\infty}^{\infty} h[m]z^{-m}$$



**FIGURE 4.1** The ROC (shown shaded) of the z-transform for various sequences. For a right-sided signal or causal signal, the ROC lies outside a circle of finite radius. For a left-sided signal or anti-causal signal, the ROC lies inside a circle of finite radius. For a two-sided signal, the ROC is an annular region

# Konvolusjon $\Leftrightarrow$ produkt av z-transformer

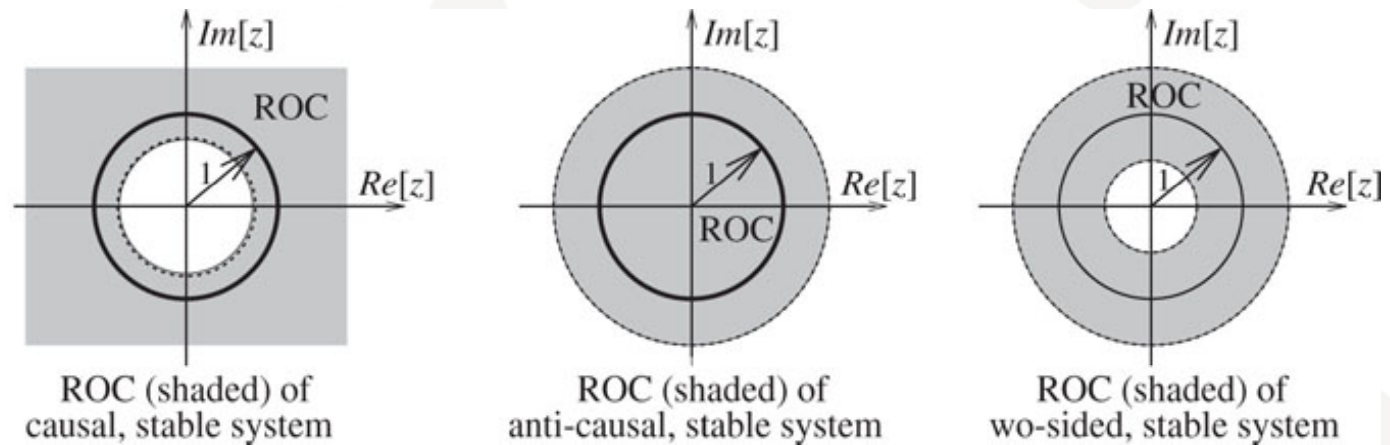


**FIGURE 4.3** System description in the time domain and z-domain. In the time domain, the system output is found by the convolution of  $x[n]$  and  $h[n]$ . In the z-domain, the transformed output  $Y(z)$  is found by the product of  $X(z)$  and  $H(z)$ . Convolution in one domain transforms to multiplication in the other

# Stabilitet og kausalitet

$$H(z) = \sum_{m=-\infty}^{\infty} h[m]z^{-m}$$

**FIGURE 4.8** The ROC of stable systems (shown shaded) always includes the unit circle

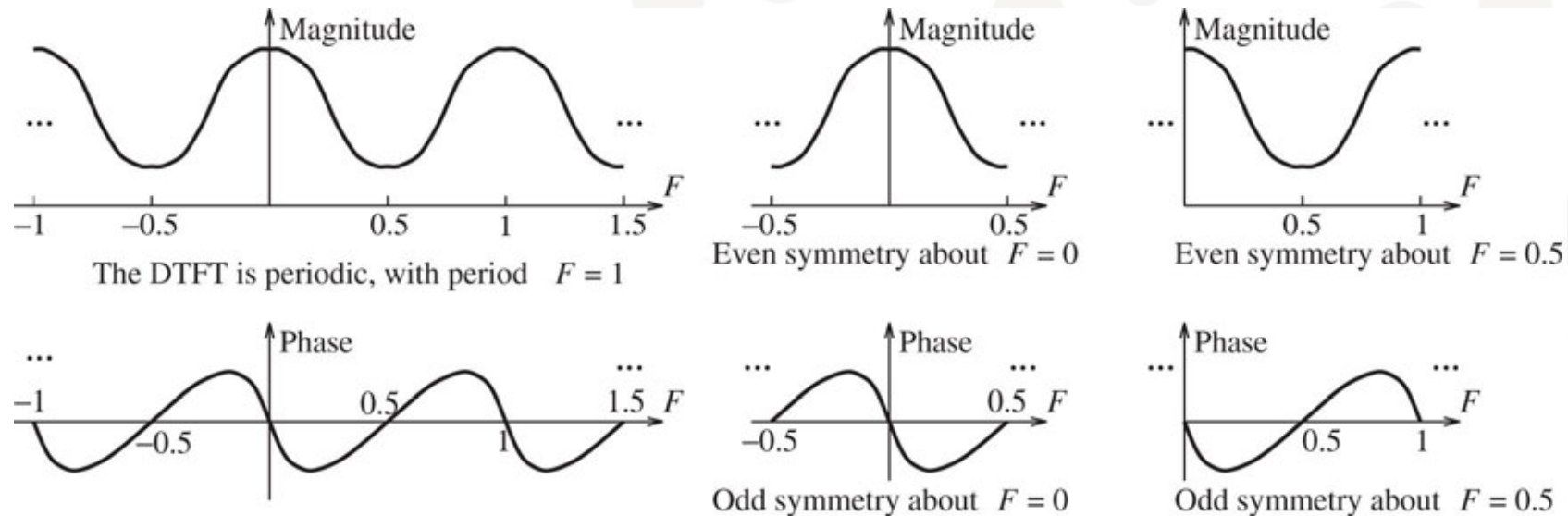




# Fourier transform, filter med reelle koeff

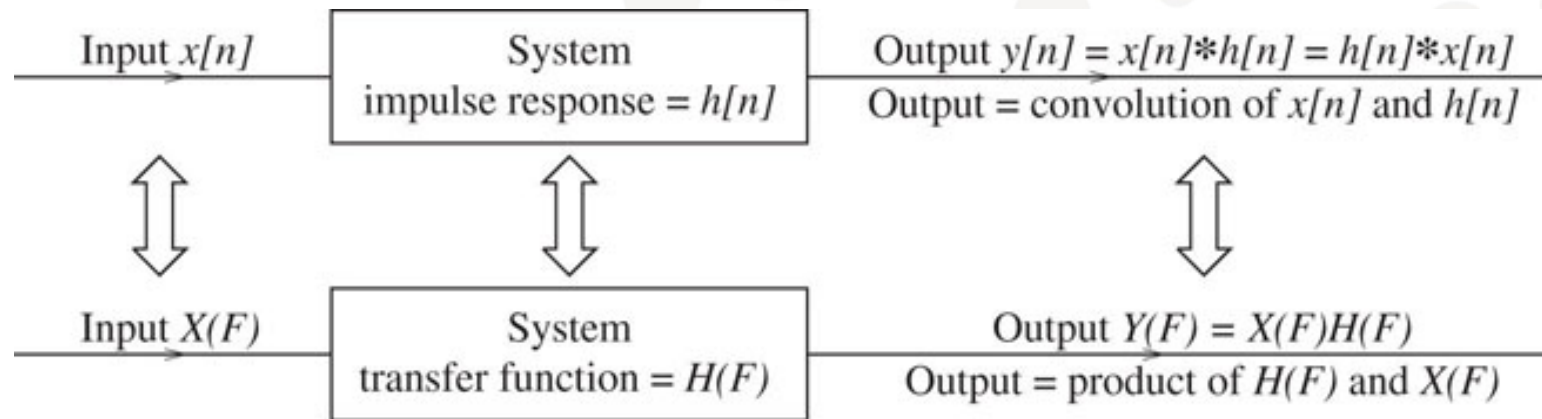
$$H(F) = H(z) \Big|_{z=e^{j2\pi F}} = \sum_{m=-\infty}^{\infty} h[m] e^{-j2\pi m F}$$

- Reell  $h[m] \Leftrightarrow H(F) = H^*(-F)$ :



**FIGURE 5.1** Illustrating the symmetry in the DTFT spectrum of real signals. The magnitude shows even symmetry about  $F = 0$  and about  $F = 0.5$ . The phase shows odd symmetry about  $F = 0$  and about  $F = 0.5$ . Both are periodic, with unit period (in  $F$ )

# Systemrespons: frekvensdomene

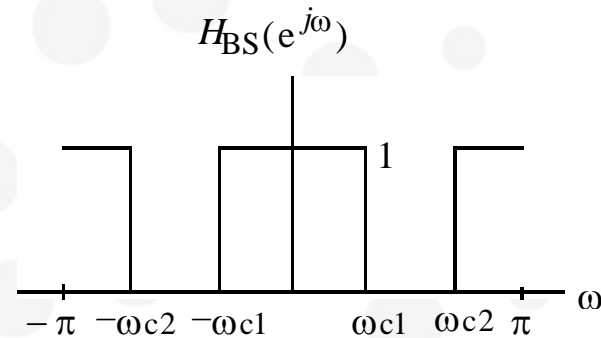
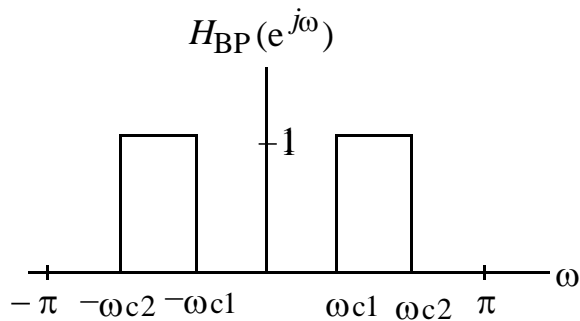
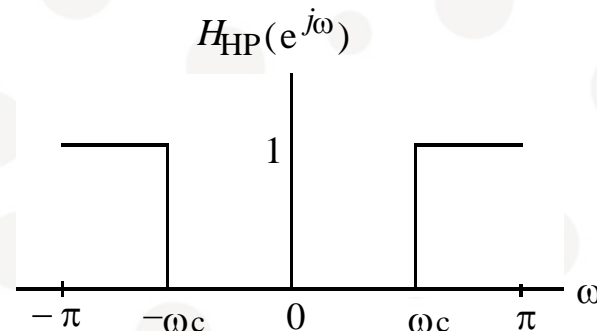
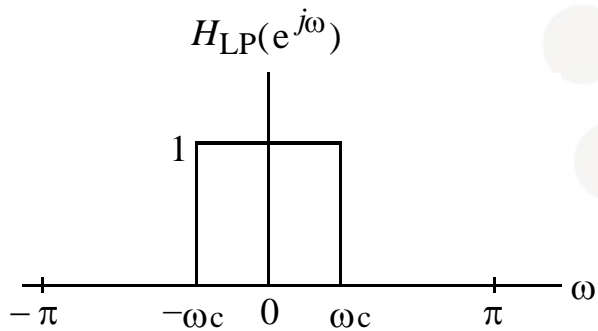


**FIGURE 5.3** The equivalence between the time domain and frequency domain. Note that convolution in the time domain transforms to multiplication in the frequency domain

- Konvolusjon  $\Leftrightarrow$  produkt  $\Leftrightarrow$  sum av dB-verdier:
  - $20\log[Y(F)] = 20 \log[X(F)] + 20 \log[H(F)]$

# Ideelle filtre

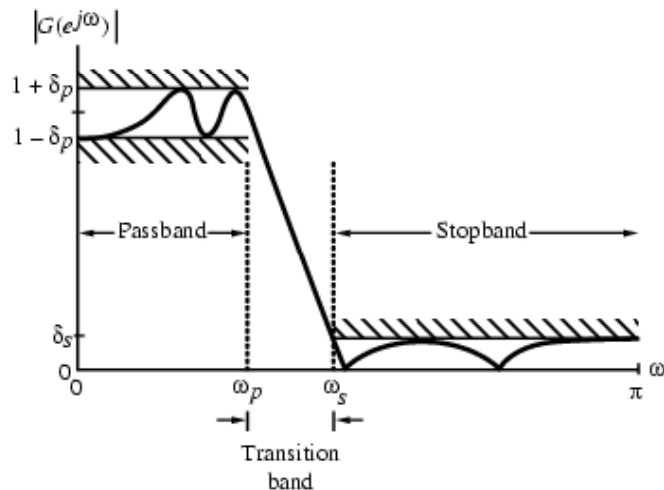
- Lavpass, høypass, båndpass, båndstopp





# Filterspesifikasjon

- Magnituderrespons:
  - Pass-, stopp-, transisjonsbånd



- Faserespons:
  - Lineær fase
  - Ikke-lineær fase

- **Lineær forvrengning:**

- Amplitude distorsjon: Ikke-flat respons i passbånd
- Fasedistorsjon: Avvik fra lineær fase
- Veier eller forsinker forskjellige frekvenser ulikt

- **Ulineær forvrengning:**

- Genererer nye frekvenskomponenter
- Ikke-lineært system
- Ingen LTI-filtre kan gi ulineær forvrengning
- Høres lett!

# Forsterkning

$$H(F) = H(z) \Big|_{z=e^{j2\pi F}} = \sum_{m=-\infty}^{\infty} h[m] e^{-j2\pi m F}$$

- DC (direct current = likestrøm) gain  $\Leftrightarrow F=0$ :
  - $H(0) = \sum h[m]$
  - $\approx 0$  for høypass
- Halve samplingsfrekvens,  $F=0.5$ :
  - $H(0.5) = \sum (-1)^m h[m]$
  - $\approx 0$  for lavpass

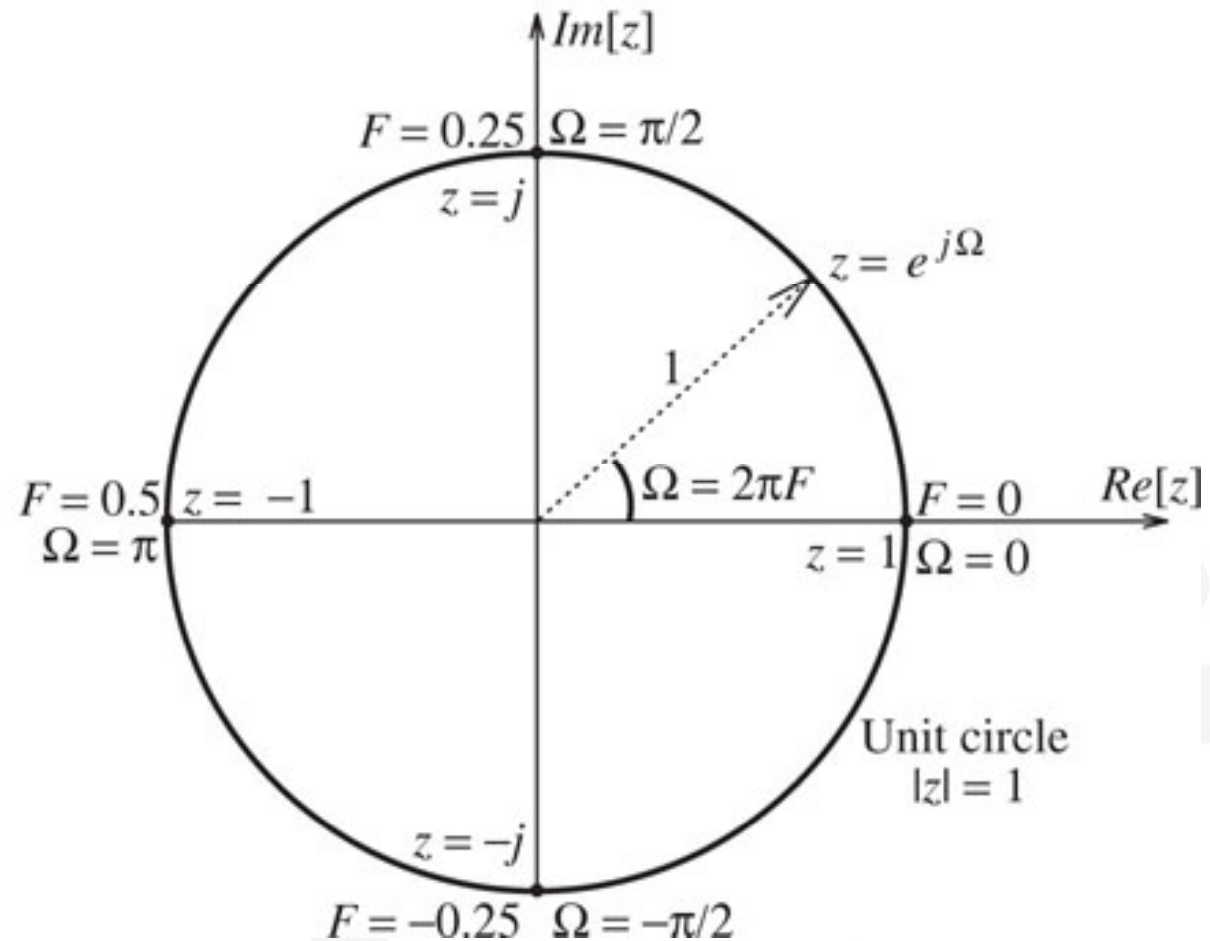
# Lineær fase $\Leftrightarrow$ Ren tidsforsinkelse

- Cosinus som faseforsinkes med  $\phi$ :
  - $\cos(\omega t + \phi) = \cos(\omega(t + \phi/\omega)) = \cos(\omega(t - t_p))$
  - Hvis  $\phi = -\omega t_p$  – lineær fase – så er det det samme som konstant tidsforsinkelse
- Viktig sammenheng: fase og tidsforsinkelse!
- Faseforsinkelse:  $t_p(F_0) = -\phi/\omega = -\angle H(F_0)/(2\pi F)$
- Gruppeforsinkelse:  $t_g(F_0) = -d\angle H(F_0)/(2\pi dF)$ 
  - Forsinkelse av en informasjon som er modulert inn på signalet
  - I det enkle cos-eksemplet er  $t_p = t_g$

## 6.1.5. Grafisk metode for frekvensrespons

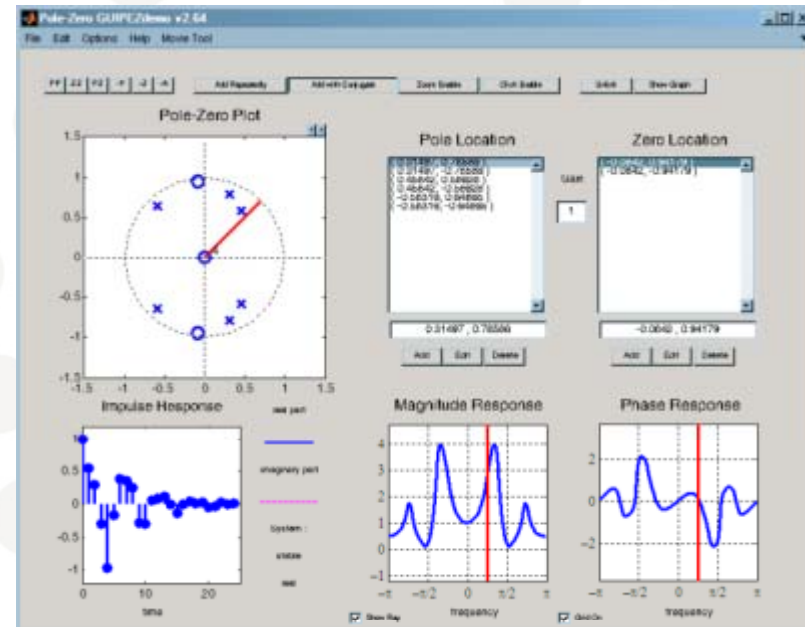
**FIGURE 6.1**

Relating the variables  $z$ ,  $F$ , and  $\Omega$  through the unit circle in the  $z$ -plane. The angle in radians equals the digital frequency  $\Omega$ . On the unit circle,  $F = 0$  corresponds to  $z = 1$  while  $F = 0.5$  corresponds to  $z = -1$



# Pol-nullpunktseeditor for Matlab

- PEZdemo allows the user to select the poles and zeros of a system and then change those poles and zeroes dynamically.
- The z-plane, Fourier domain and time-domain are all updated simultaneously.
- <http://users.ece.gatech.edu/mcclella/matlabGUIs/>
- Georgia Tech



Legges i `..\toolbox\pezdemo`

Legg til i `..\toolbox\local\startup.m`:

`- path(path,'C:\Program Files\MATLAB\R2008a\toolbox\pezdemo');` (eksempel)



# PEZDemo - tricks

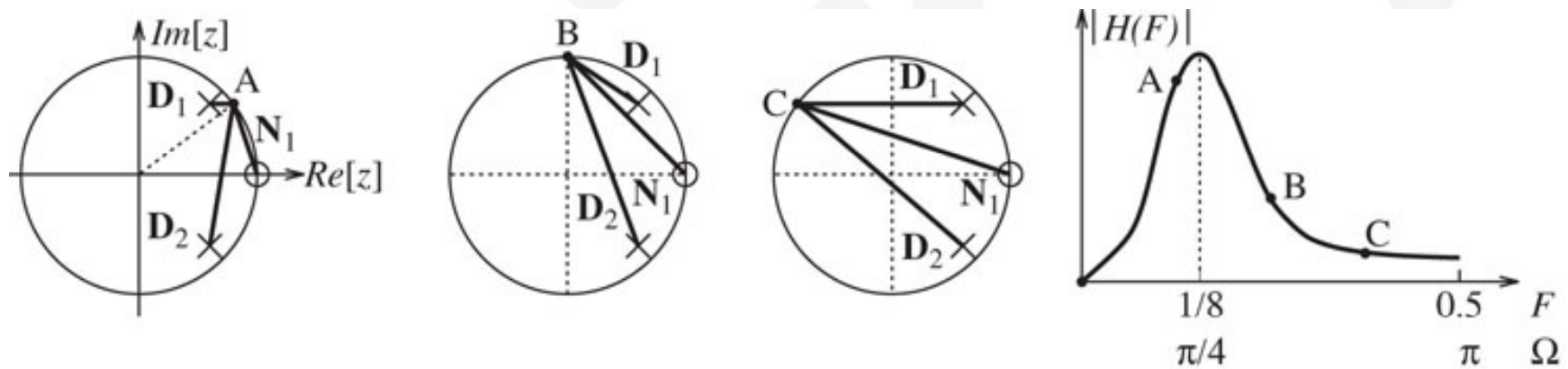
- 'Move on real axis' i Options – i komplekse plan eller på reell akse
- Skru av og på 'Conjugate' etter behov - Conjugate pole/zero må til for å få reell respons
- Får ikke plassert pol/null hvis det klikkes nær den røde vinkelmarkøren

# Eks, grafisk analyse

- $H(z) = 8(z-1) / (z-0.6-j0.6)(z-0.6+j0.6)$ 
  - Null:  $z=1$ , poler:  $0.6 \pm j0.6$
- $H(\Omega_0) =$   
 $8(\exp(j\Omega_0)-1) / (\exp(j\Omega_0)-0.6-j0.6)(\exp(j\Omega_0)-0.6+j0.6)$
- $H(\Omega_0) = 8 N_1 / (D_1 D_2)$
- $|H(\Omega_0)| = 8 |N_1| / (|D_1| |D_2|)$ :
  - Gain ·  $II$  avstander fra nuller /  $II$  avst. fra poler
- $\angle H = \angle N - \angle D_1 - \angle D_2$

# Grafisk analyse: Kvalitativ metode

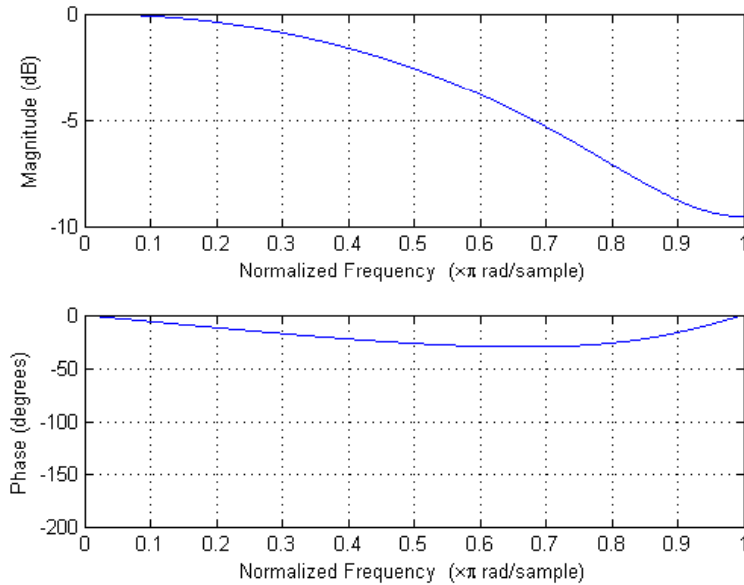
- Evaluerer for  $z = \exp(j\Omega_0)$  i  $z=1$ , A, B og C:  $8 |N_1| / (|D_1| |D_2|)$



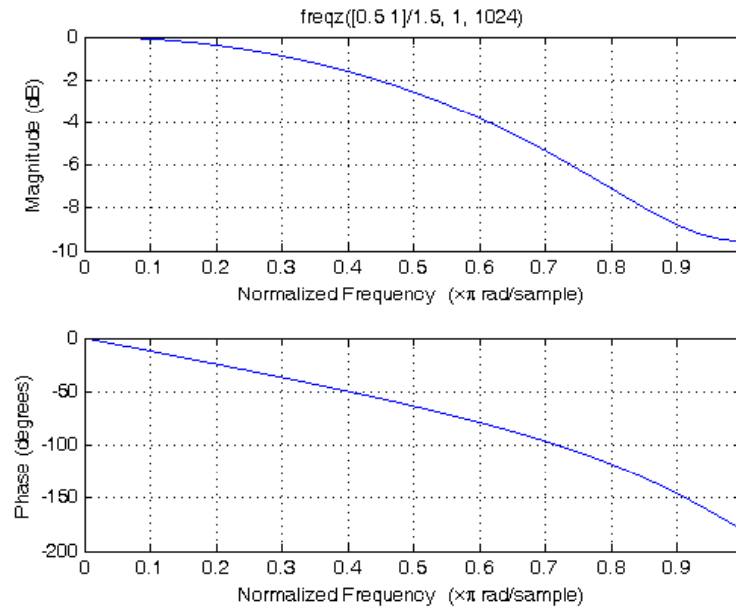
**FIGURE 6.2** Graphical interpretation of the frequency response. At the three frequencies indicated by the points A, B and C on the unit circle (in the first three panels), the gain is estimated by computing the ratio of the products of the distances to the zeros and products of distances to the poles. The corresponding values of the gain are labeled on the gain plot

- Fint for å finne verdier der filteret er 0 eller har topper

# 6.1.3 Minimum fase, FIR filter, 2 koeff



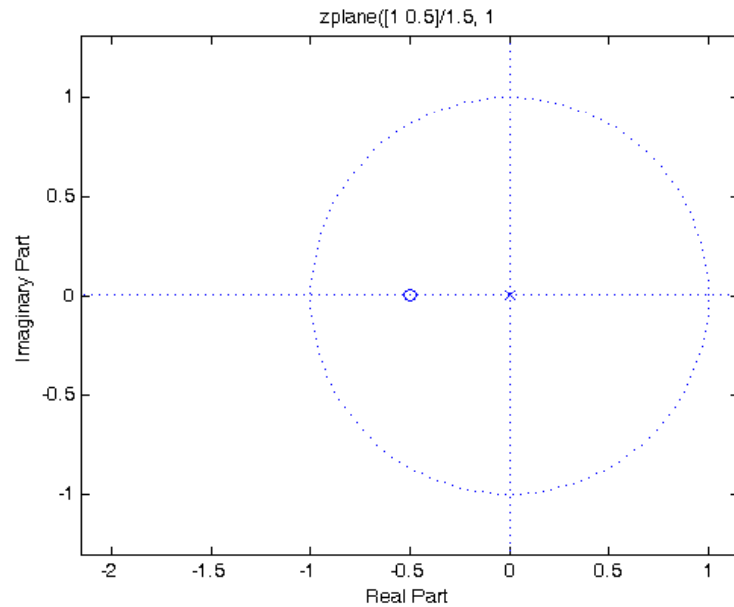
$$h_1 = [1 \ 0.5]/1.5$$



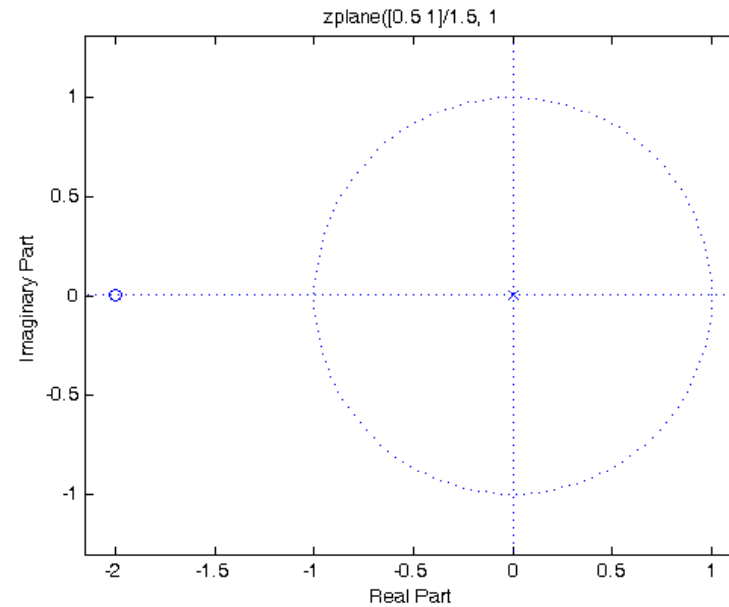
$$h_2 = [0.5 \ 1]/1.5$$

- Normalisert med 1.5 for å få DC-gain på 1 (=0 dB)
- Reversert filter. Samme magnitude. Minimum og maksimum fase

# Minimum fase, FIR filter, 2 koeffisienter



$$h_1 = [1 \ 0.5]/1.5$$
$$\text{Nullpunkt } 1+0.5z^{-1}=0$$



$$h_2 = [0.5 \ 1]/1.5$$
$$0.5+z^{-1}=0$$

Minimum fase systemer har alle nullpunkter på eller innenfor enhetssirkelen



# Minimum fase filtre fra magnitude

- Ofte er spesifikasjonen bare gitt for magnitude (lav-, høy-pass etc):  $H_p(F)$
- Ofte ønskelig med minst mulig forsinkelse
  - $\Rightarrow$  minimum fase
- Anvendelse:
  - Filtre med minst mulig forsinkelse er ofte en fordel: påvirker signalet minst mulig (hørbarhet)
  - Bare minimum fase systemet blir stabilt og kausalt
    - Invers  $\Rightarrow$  poler  $\rightarrow$  nullpunkter og omvendt
    - Særlig viktig innen seismikk

## Eks 6.2: Finne minimum-fase filtret

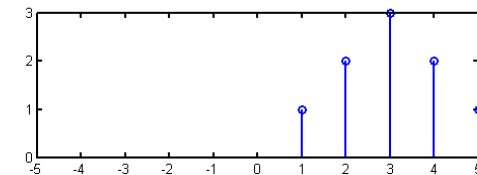
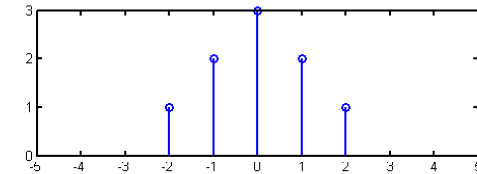
- Finn minimum fase  $H(z)$  når  $H_p(\Omega)$  er gitt:
  - $|H_p(\Omega)|^2 = (5+4\cos\Omega)/(17+8\cos\Omega)$
- Uttrykt med exp-funksjoner:
  - $|H_p(\Omega)|^2 = (5+2\exp(j\Omega)+2\exp(-j\Omega)) / (17+4\exp(j\Omega)+4\exp(-j\Omega))$
- Uttrykt med  $z=\exp(j\Omega)$ :
  - $H(z)H(1/z) = (5+2z+2/z) / (17+4z+4/z)$   
 $= (2z^2+5z+2) / (4z^2+17z+4) = ((2z+1)(z+2)) / ((4z+1)(z+4))$
- Nullpunkt for  $|z|<1 \Rightarrow (2z+1)$ : min fase
- Pol for  $|z|<1 \Rightarrow (4z+1)$ : stabil & kausal
- $H(z) = (2z+1) / (4z+1) = \underline{0.5(z+0.5)/(z+0.25)}$
- Max fase  $H(z) = (z+2)/(4z+1) = 0.25(z+2)/(z+0.25)$

# Hvordan å finne minimum fase $H(z)$ ?

- Hvordan finne transferfunksjonen?
  - $|H_p(F)|^2 = H_p(F) H_p(-F) = H_p(z) H_p(1/z)|_{z=\exp(j2\pi F)}$
- Konjugert symmetri:  
for hvert nullpunkt  $r_k$  er det en  $1/r_k^*$ .
- Nullpunkter: Plukk ut alle de som er innenfor enhetssirkelen for å finne minimum fase  $H(z)$
- Poler: Pga stabilitet og kausalitet må en ta de som er innenfor enhetssirkelen

## 6.2 FIR filtre og lineær fase

- Anta  $a[n]=a[-n]$ , symmetrisk (like) om 0:
  - $\Rightarrow$  reell Fourier transform  $A(j\Omega)=A^*(j\Omega)$
  - Ingen faseforvrengning (fase=0 for alle f.)
  - Ikke kausalt
- Så gjør vi det til et kausalt filter
  - Forsinker så det starter i  $n=0$
  - Ren forsinkelse er en liten form for distorsjon
  - Forsinkelse:  $h[n]=a[n-M/2] \Leftrightarrow$ 
$$H(j\Omega) = e^{-j\Omega M/2} A(j\Omega)$$
  - Et ledd med lineær fase i Fouriertransformen
- Fordel nr 2 med slike filtre
  - Bare ca 50% av multiplikasjonene



# FIR lineær fase – nullpunkter

- Ren forsinkelse endrer ikke nullpunkter
  - $h[n]$  har samme egenskaper som den ikke-kausale, symmetriske  $a[n]$

$$H(z) = \sum_{m=-\infty}^{\infty} h[m]z^{-m}$$

## 1. Reell $h[n]$

- $\Leftrightarrow H(z) = H^*(z)$ : nullpunkt i  $r \Rightarrow$  nullpunkt i  $r^*$

## 2. Symmetri $a[n] = a[-n]$ , $h[n+M/2] = h[n-M/2]$

- $\Leftrightarrow H(z) = H(1/z)$ : nullpunkt i  $r \Rightarrow$  nullpunkt i  $1/r$

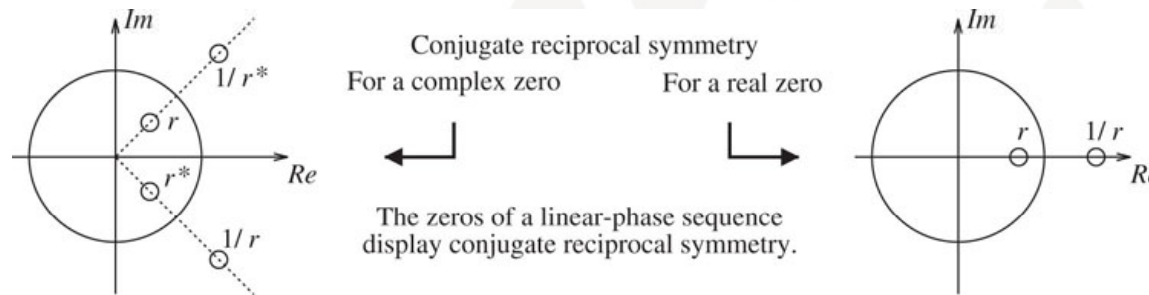
## 3. Altså $H(z) = H^*(z) = H^*(1/z) = H(1/z)$ : reell



# Lineær fase FIR filtre

Nullpunkt i  $r \Rightarrow$  nullpunkt i  $r^*$ ,  $1/r$ , og  $1/r^*$ :

- Komplekse nullpunkt kommer 4 i slengen
- Hvis de havner oppå hverandre:
  - Reelle nullpunkt i par:  $r$  og  $1/r$  (for  $r \neq 1$ )
  - Nullpunkter på enhetssirkelen  $|z|=1$  i par
  - Nullpunkt i  $z=\pm 1$  ( $r = \pm 1$ ) kan være alene



**FIGURE 6.4** Illustrating conjugate reciprocal symmetry. Each zero not on the real axis and not on the unit circle forms part of a quadruple, being paired with its reciprocal and its conjugate. A zero on the real axis but not on the unit circle need not be conjugated and is paired with just its reciprocal. A zero on the unit circle (but not at  $z = 1$  or  $z = -1$ ) forms its own reciprocal and is paired with just its conjugate. A zero at  $z = 1$  or  $z = -1$  can occur singly, because it forms its own reciprocal and its own conjugate

## Eks 6.4a

- Er  $H(z)=1+2z^{-1}+2z^{-2}+z^{-3}$  et lineær fase filter?
- Ser at  $h[n] = \{1,2,2,1\}$
- Like symmetri, like lengde, symmetrisk om  $n=1.5$

# Lineær fase – minimum fase

- Kan et lineær fase filter være minimum fase?

# Symmetri & like/odde antall koeffisienter

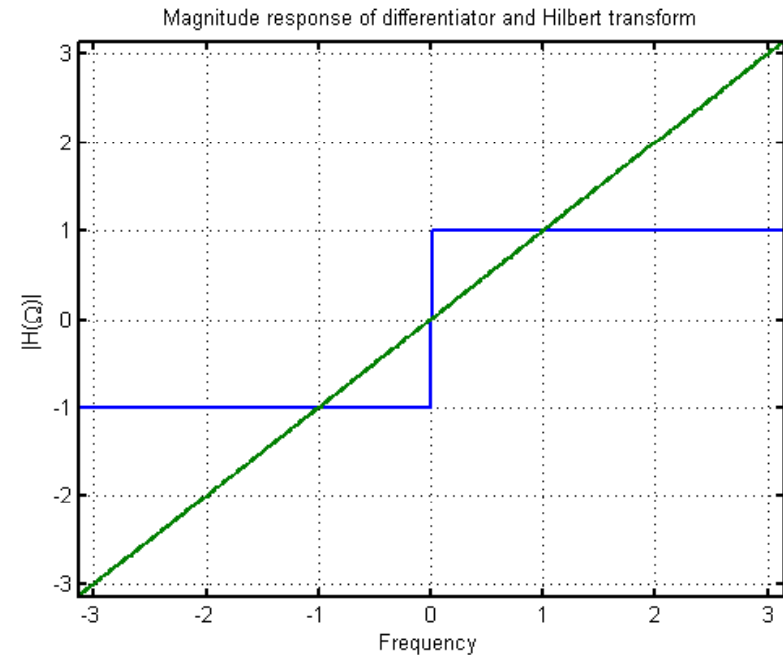
- Type 1: Filter med odde antall koeffisienter:
  - Eks: {0.25 0.5 0.25}
  - Like antall nullpunkt bestående av 4-er grupper + 2-er grupper, ingen enslige nullpunkt i  $z = \pm 1$ 
    - Analogi: 2. grads ligning har 3 koeffisienter (odde antall) og 2 løsninger (like antall)
- Type 2: Filter med like antall koeffisienter:
  - Eks: {0.5 0.5}
  - Odde antall nullpunkt bestående av 4-er grupper, 2-er grupper + *et obligatorisk enslig nullpunkt.*
  - *Det enslige nullpunktet må være i  $z = -1$  da nullpunkt i  $z = 1$  krever asymmetri (se neste slide) - (ikke HP-filter)*

# Odde symmetri i lineær fase filter

- Antisymmetrisk filter:  $h[n] = -h[-n] \Rightarrow$  imaginær  $H(F)$ 
  - $H(F) = jA(F)e^{-j2\pi\alpha F} \Leftrightarrow A(F)e^{-j(2\pi\alpha F - \pi/2)}$  der  $A(F)$  er reell
- Fasen består av konstant  $e^{j\pi/2}$  som får et lineær fase tillegg pga forsinkelse med halve lengden
- Vil alltid ha nullpunkt i  $z=1$  da  $H(0)=0$  pga  $\sum h[n]=0$
- Mye mindre viktig enn de to like-symmetriske filtrene

# Anti-symmetriske filtre

- Hilbert transform:
  - $H(\Omega) = -j \operatorname{sgn}(\Omega)$
  - Faseskift med 90 grader for negative frekvenser og -90 grader for positive
- Differentiator:
  - $H(\Omega) = j \Omega$
  - Fra differensierings-egenskapen til Fouriertransform



# 4 typer lineær fasefiltre

Type 1: Like symmetri, odde lengde:

$$\{\underline{0.25} \ 0.5 \ 0.25\}$$

Type 2: Like symmetri, like lengde:

$$\{\underline{0.5} \ 0.5\}$$

Type 3: Ulike symmetri, odde lengde:

$$\{\underline{-0.5} \ 0 \ 0.5\}$$

Type 4: Ulike symmetri, like lengde:

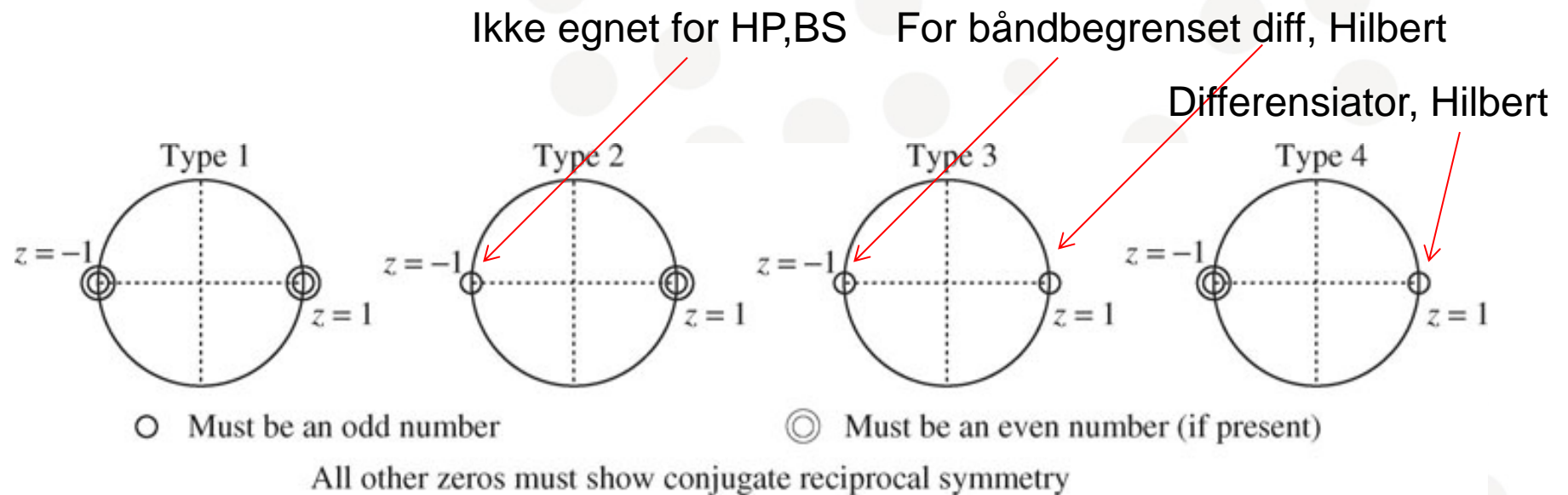
$$\{\underline{-0.5} \ 0.5\}$$



## Eks 6.5b

- Finn alle nullpunkter til en type 2 sekvens (av korteste mulige lengde) når det er kjent at det er et nullpunkt ved  $z=0.5e^{j\pi/3}$  og et nullpunkt i  $z=1$ 
  - Type 2 nullpunkt: 4-er grupper, 2-er grupper + et obligatorisk enslig nullpunkt i  $z=-1$ .
- Komplekst nullpunkt speiles i 4:  
 $z = 0.5e^{+/-j\pi/3}$  og  $z=2e^{+/-j\pi/3}$
- Nullpunktet i  $z=1$  må være dobbelt
- Må ha nullpunkt i  $z=-1$
- Altså 7 nullpunkter:  $z = 0.5e^{+/-j\pi/3}$  og  $z=2e^{+/-j\pi/3}$ ,  
dobbelt i  $z=1$ , enkelt i  $z=-1$

# Lineær fase filtre: Oppsummering

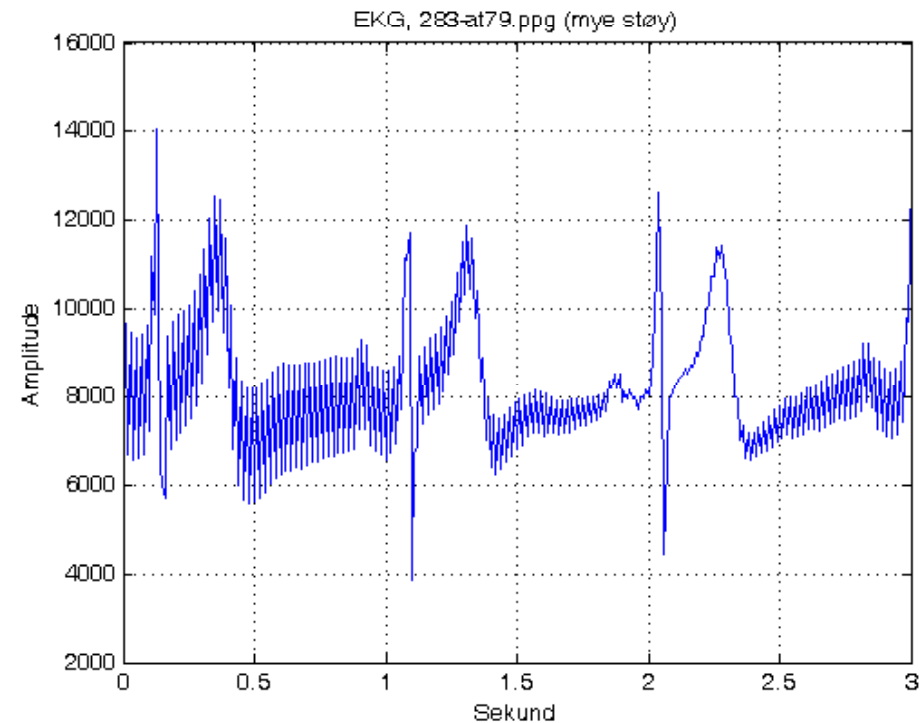


**FIGURE 6.5** Identifying the sequence type from its zeros at  $z = \pm 1$ . Type 2 sequences *require* an odd number of zeros at  $z = -1$ , type 4 sequences *require* an odd number of zeros at  $z = 1$ , and type 3 sequences *require* an odd number of zeros at both  $z = 1$  and  $z = -1$ . The number of zeros (if present) at the locations  $z = \pm 1$  (other than the ones required) must be even. Naturally, any zeros elsewhere must satisfy conjugate reciprocal symmetry

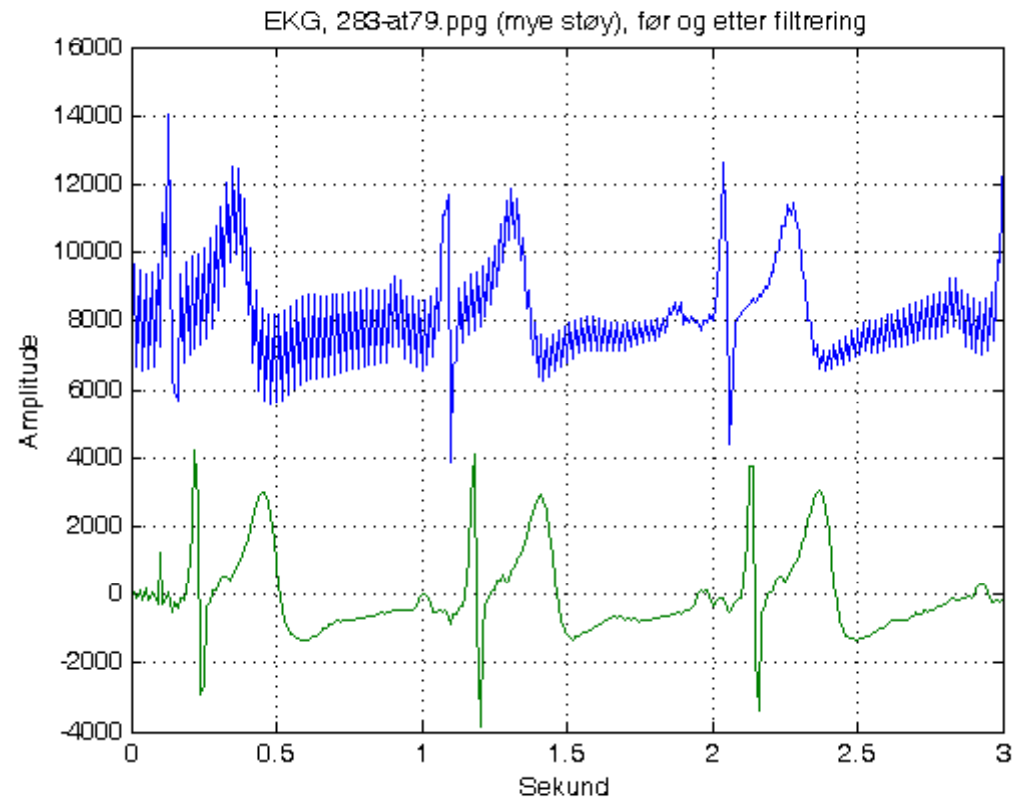
# Støyfiltrering på EKG-signal

- Matlab: [Aase.m](#) - Filtrere bort 50 Hz fra EKG-signal

- Data fra Ottar Aase, Kardiologi, Ullevål Universitetssykehus



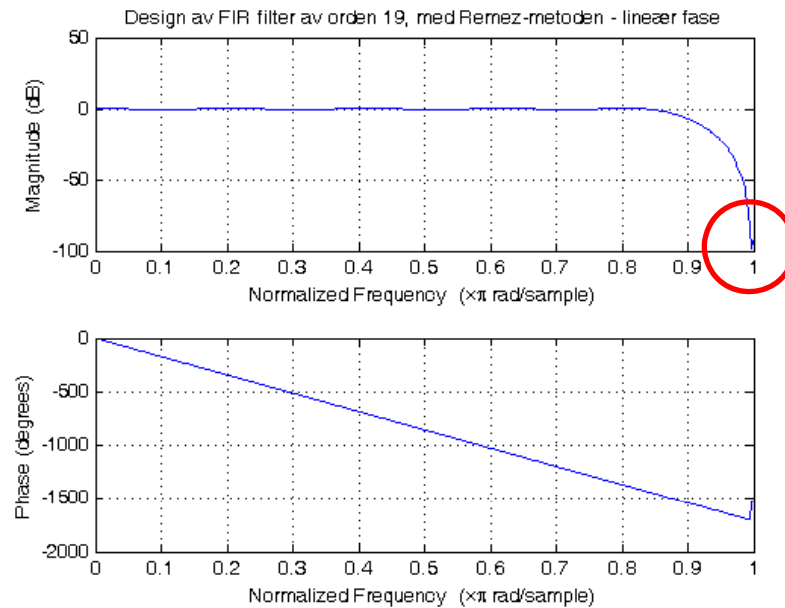
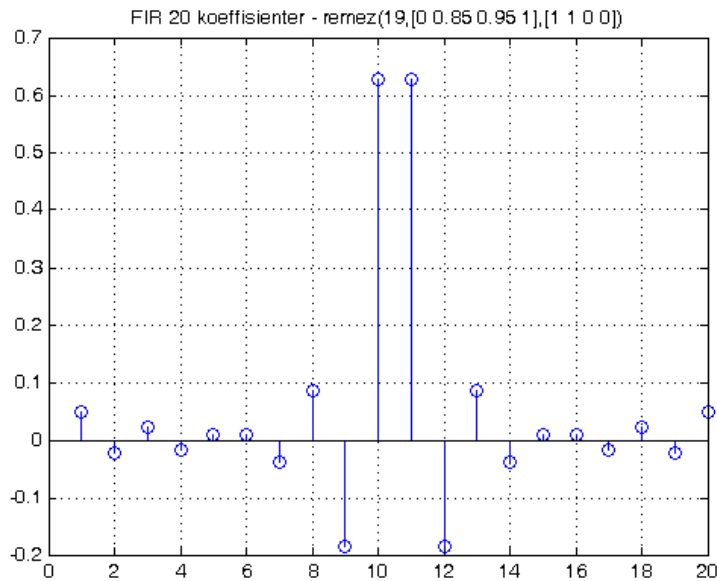
# EKG: Før og etter filtrering



9. oktober 2013

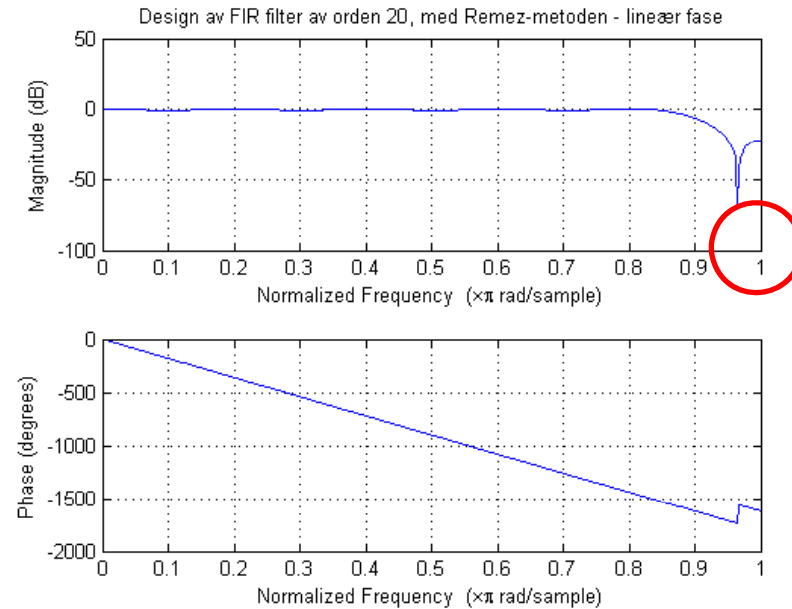
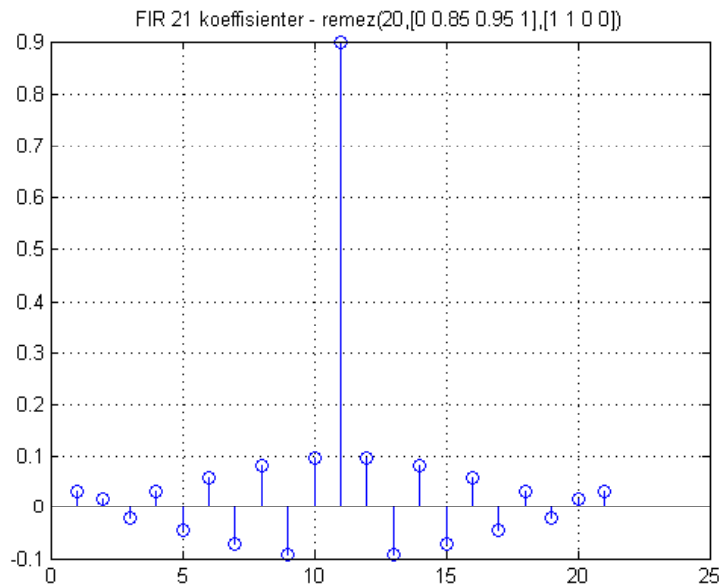
34

# EKG: Filter for støyfiltrering, N=20



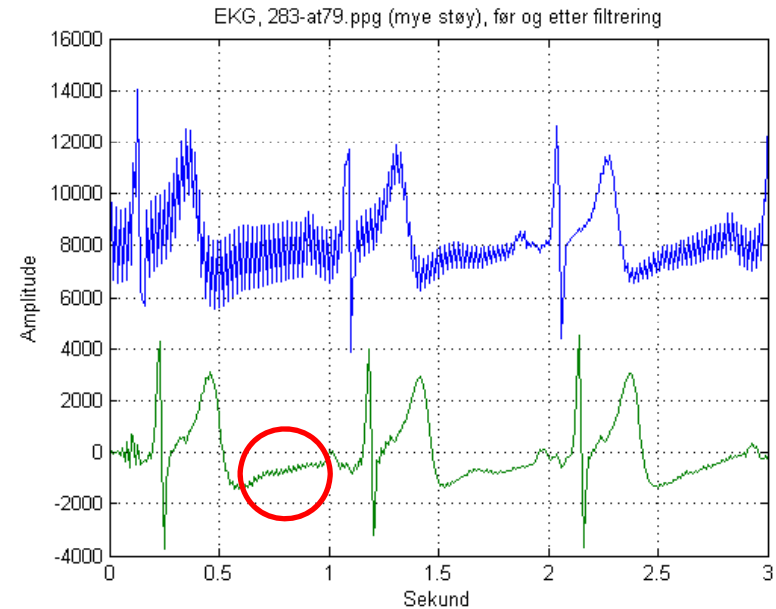
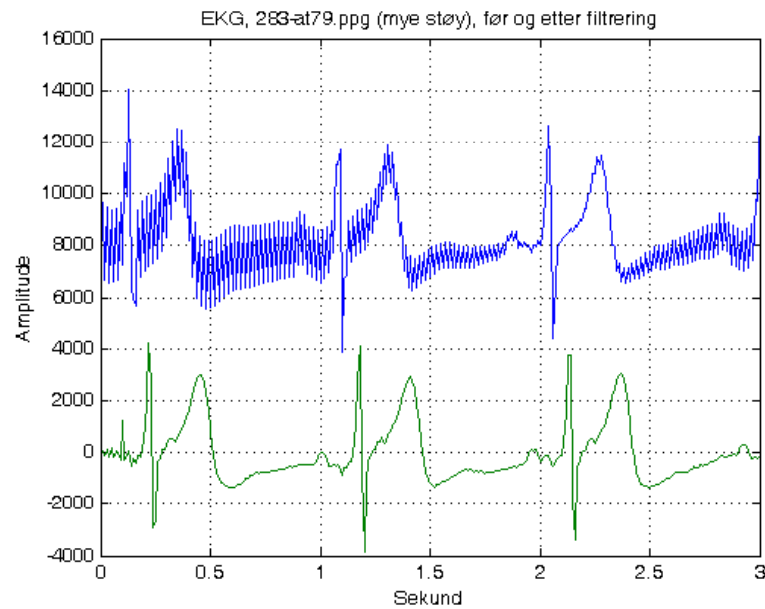
Symmetrisk, like antall koeffisienter (N=20): type 2.  
Alltid nullpunkt i  $z=-1$ , dvs  $F=0.5$ . Velegnet som lavpassfilter

# EKG: Filter for støyfiltrering, N=21



Symmetrisk, ulike antall koeffisienter (N=21): type 1.  
Ikke nullpunkt i  $z=-1$ , dvs  $F=0.5$ . Ulempe i denne anvendelsen

# EKG-filtrering med type 2 vs type 1 FIR



Mer støy med et filter med høyere orden, fordi man ikke får fordel av nullpunktet i  $z=-1$ .

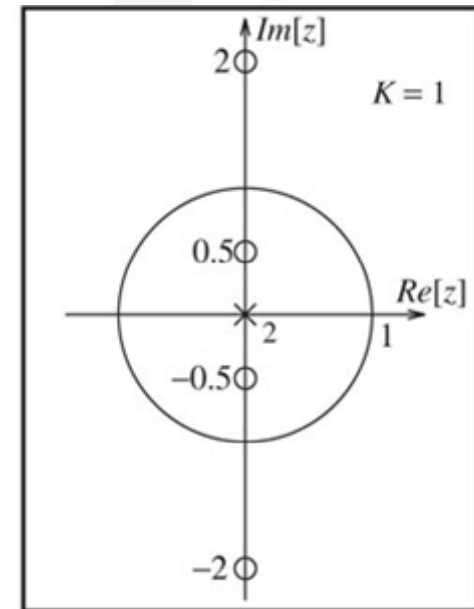


## Eks 6.4b

- $h[n]=\{1, 0, \underline{4.25}, 0, 1\}$ . Poler og nullpunkter?
- Like symmetrisk om  $n=0$
- $H(z) = z^2+4.25+z^{-2} = (z^4+4.25z^2+1)/z^2$
- Nullpunkter, løs 2. gradsligning:  $z^2 = (-4.25 \pm (4.25^2-4)^{0.5})/2 = (-4.25 \pm 3.75)/2 = -0.25, -4$ 
  - Altså nullpunkter for  $z = \pm j0.5$  og  $\pm j2$
- Dobbel pol for  $z=0$
- $H(z)=(z+j0.5)(z-j0.5)(z+j2)(z-j2)/z^2$

# Pol-nullpunktsploott for eks 6.4b

- $H(z) = (z+j0.5)(z-j0.5) (z+j2)(z-j2)/z^2$
- $h[n]=\{1, 0, \underline{4.25}, 0, 1\}$ .



(a)

- Eks på type 1 lin. fase filter: Like symmetri, odde lengde (=5), like antall nullpunkter (4)

## 6.2.3 Midlingsfiltre

- Kausalt N-punkts midlingsfilter (moving average)
  - $y[n] = (x[n] + x[n-1] + \dots + x[n-(N-1)]) / N$
  - Like symmetri: Type 1 eller 2

- Transferfunksjon:

$$H(F) = \sum_{k=0}^{N-1} h[k] e^{-j2\pi kF} = \frac{1}{N} \sum_{k=0}^{N-1} (e^{-j2\pi F})^k = \frac{1}{N} \frac{1 - e^{-j2\pi NF}}{1 - e^{-j2\pi F}}$$

- Ikke sentrer sum nr 2 og legg sammen til cosinuser!
- Trick, ta ut halve eksp.faktoren i teller/nevner:

$$H(F) = \frac{1}{N} \left( \frac{e^{-j\pi NF}}{e^{-j\pi F}} \right) \cdot \left( \frac{e^{j\pi NF} - e^{-j\pi NF}}{e^{j\pi F} - e^{-j\pi F}} \right) = e^{-j\pi(N-1)F} \cdot \frac{\sin(\pi NF)}{N \sin(\pi F)}$$

# Midlingsfiltre

$$H(F) = e^{-j\pi(N-1)F} \cdot \frac{\sin(\pi NF)}{N \sin(\pi F)}$$

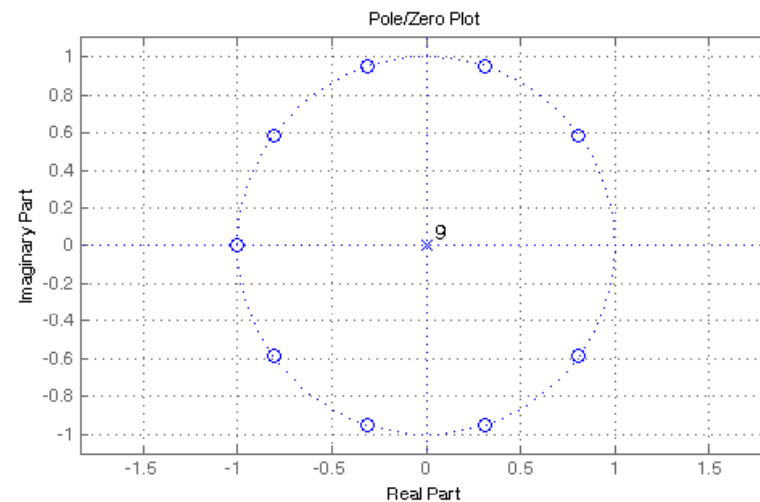
Lineær fase + et ledd som er reelt

- Siste ledd: Dirichlet kjerne, aliased sinc-funksjon
  - $\text{sinc}(x) = \sin(\pi x)/(\pi x)$
- Midlingsfilter, egenskaper:
  - $H(0) = 1$  (l'Hôpitals regel eller Taylor-rekke)
  - Nullpunkter:  $\sin(\pi NF)=0 \Leftrightarrow \pi NF=k\pi \Rightarrow F=k/N$
  - $N-1$  nullpunkter innenfor  $0.5 < F \leq 0.5$ 
    - Egentlig  $N$  stk, men nullpunktet i  $F=0$  kanselleres av en pol
  - $|H(0.5)|=0$  for  $N$  like – type 2 filter
    - $k=N/2 \Rightarrow F=(N/2 / N)= 0.5$

# Nullpunkter og poler

$$H(F) = \frac{1}{N} \frac{1 - e^{-j2\pi NF}}{1 - e^{-j2\pi F}} \Rightarrow H(z) = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}} = \frac{1}{N} \frac{z^N - 1}{z^{N-1}(z - 1)}$$

- $z^N=1 \Rightarrow N$  løsninger spredt rundt enhets-sirkelen, avstand  $2\pi/N$  radianer
- $N-1$  nullpkt da pol i  $z=1$  kansellerer null i  $z=1$
- $N-1$  poler i  $z=0$

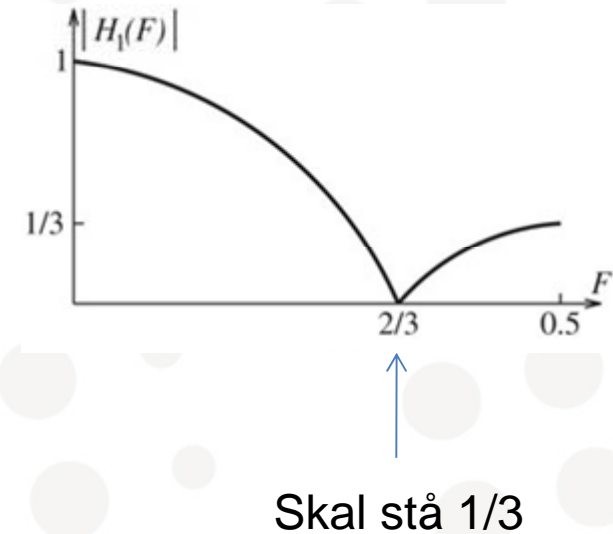


N=10

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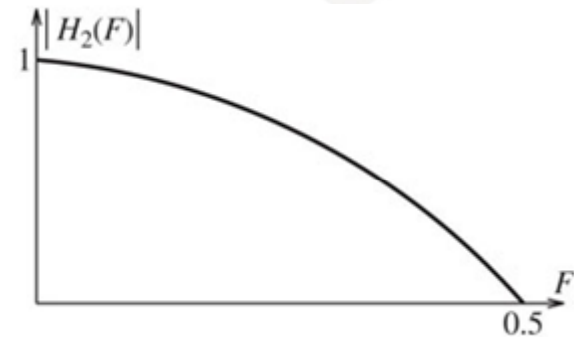
# Eks 6.6b 3-punkts midling

- $h[n]=\{1/3, \underline{1/3}, 1/3\}$ ,
  - $\Sigma=1 \Rightarrow$  DC-gain = 1
- $H(F) =$   
 $[\exp(j2\pi F)+1+\exp(-j2\pi F)]/3 =$   
 $[1+2\cos(2\pi F)]/3$
- Null:  $\cos(2\pi F)=-0.5 \Leftrightarrow$   
 $2\pi F=\pm 120^\circ = \pm 2\pi/3 \Leftrightarrow F=\pm 1/3$
- Midler alle frekvenser opp til og med  $F \approx 1/3$



# En bedre 3-punkts midler

- $h[n]=\{1/4, \underline{1/2}, 1/4\}$
- $H(F) =$   
 $[\exp(j2\pi F)+2+\exp(-j2\pi F)]/4$   
 $= [1+\cos(2\pi F)]/2$
- Null:  $\cos(2\pi F)=-1$   
 $\Leftrightarrow 2\pi F=\pm\pi \Leftrightarrow F=\pm 0.5$
- Midler alle frekvenser opp til og med  $F \approx 0.5$

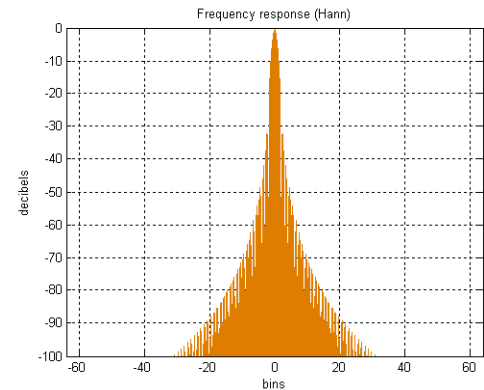
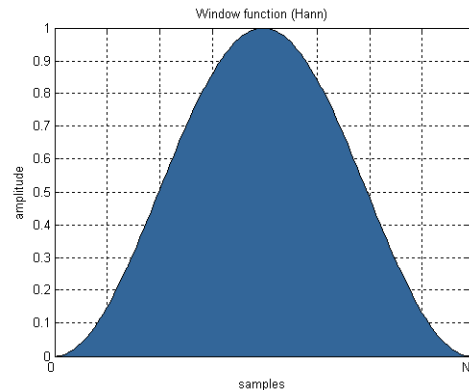




# Hann eller Hanning vindu

- $h[n]=\{1/4, \underline{1/2}, 1/4\}$
- Generell formel:  $w[n] = 0.5\left(1 + \cos\left(\frac{2\pi n}{N-1}\right)\right)$
- Sett inn  $N=3$  og  $n=-1,0,1$   
 $\Rightarrow \{0.5, 1, 0.5\}$  som normalisert til  $\sum=1$  blir  $h[n]$  over

- Julius Ferdinand von Hann (1839 -1921) was an Austrian meteorologist. He is seen as the father of modern meteorology.



- Wikipedia
- Viktig i spektralanalyse og filterdesign

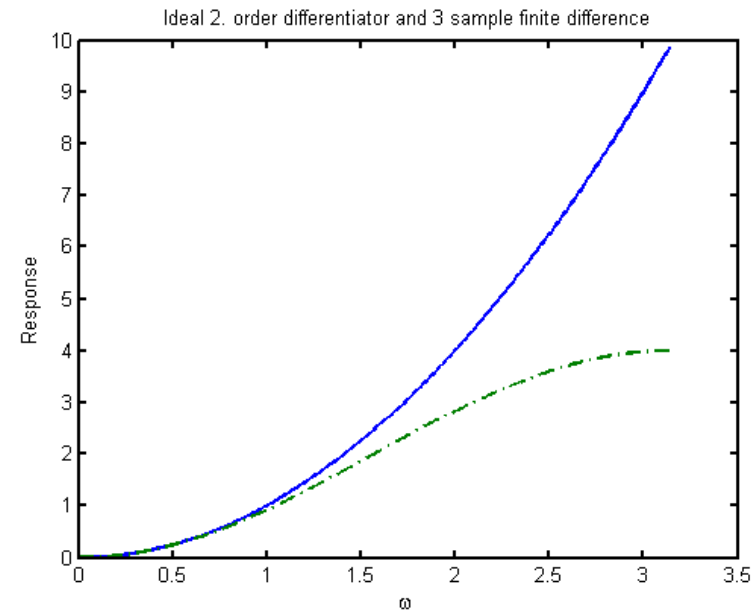
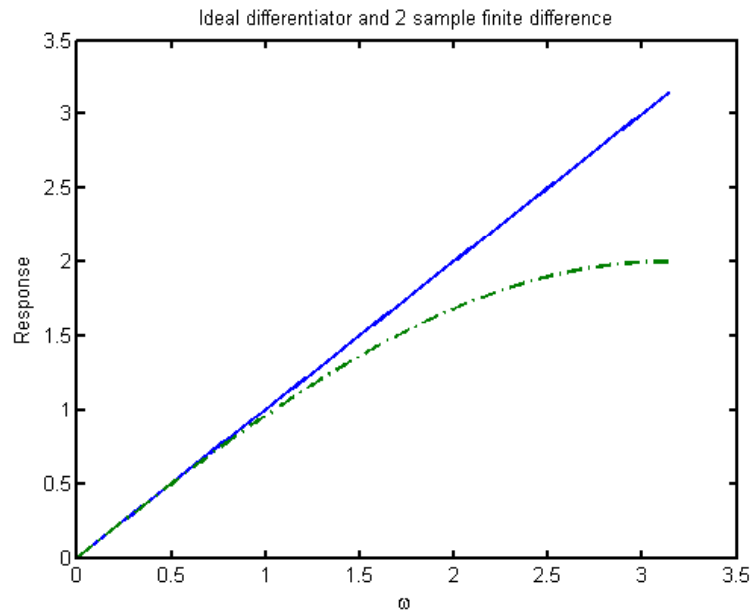
## Eks 6.6d Differansefilter

- 2 punkts midlingsfilter med snudd fortegn  
 $h[n]=\{1,-1\}$
- Type 4: asymmetrisk, like lengde: null for  $z=1$
- $H(\Omega)=1-e^{-j\Omega} = e^{-j\Omega/2}(e^{j\Omega/2}-e^{-j\Omega/2}) = 2je^{-j\Omega/2}\sin(0.5\Omega)$   
 $= 2 e^{-j(\pi/2 - \Omega/2)} \sin(0.5\Omega)$
- Lineær fase:  $\pi/2 - \Omega/2$
- Amplitude: 0 ved  $\Omega=0$ , øker til 2 ved  $\Omega=\pi$
- Differensiering = høypassfilter: støyfølsomt
- Matlab: `fvtool([1 -1],1)`

# 1. & 2. ordens derivasjon vs diskret approx

1. ordens differanse  $\{1,-1\}$

2. ordens diff.  $\{1,-2,1\}$



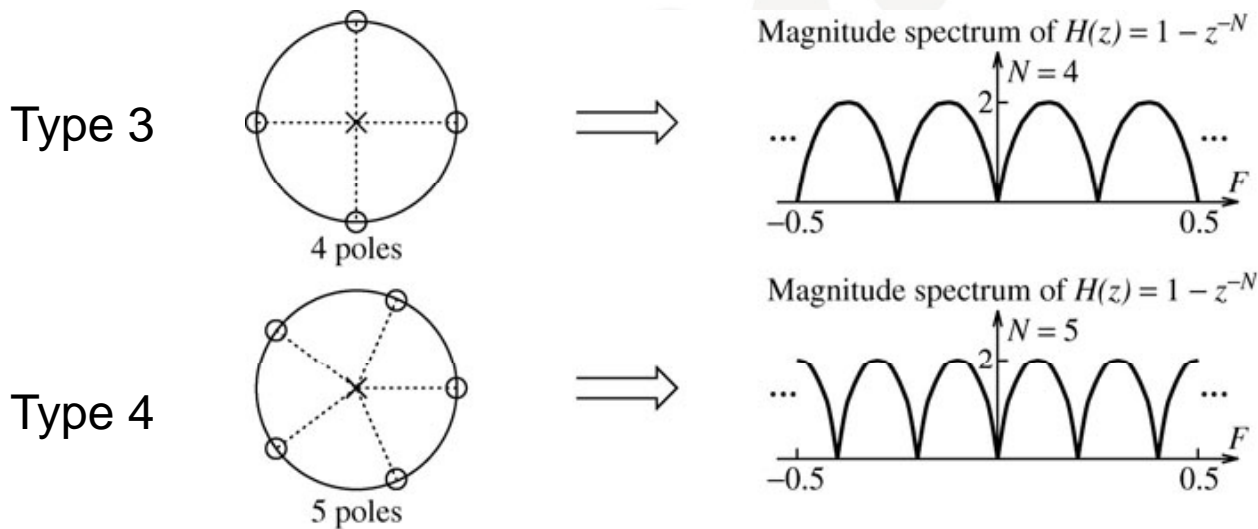
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# Lydeffekter

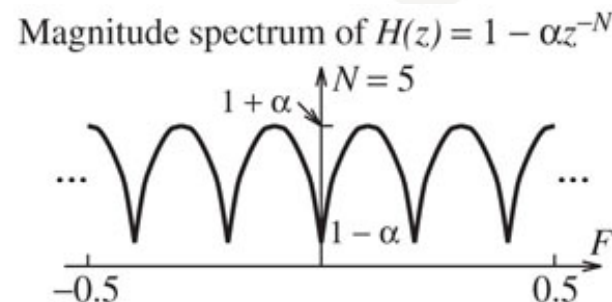
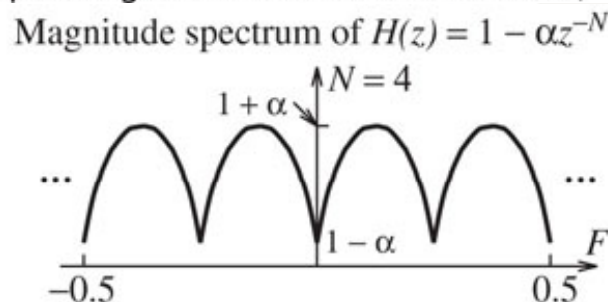
- [3470-kap7-9-Musikkeffekter.ppt](#)
- Demo at kam/allpassfiltre brukes i musikk

# FIR Kamfiltre: $1 - z^{-N}$ og $1 - \alpha z^{-N}$



**FIGURE 6.6** Pole-zero plot and frequency response of the comb filter  $H(z) = 1 - z^{-N}$ . The number of lobes in one period corresponds to the number of zeros. The dc gain is always zero, but the high-frequency gain (at  $F = 0.5$ ) is zero only if the filter length is odd (corresponding to an even number of zeros)

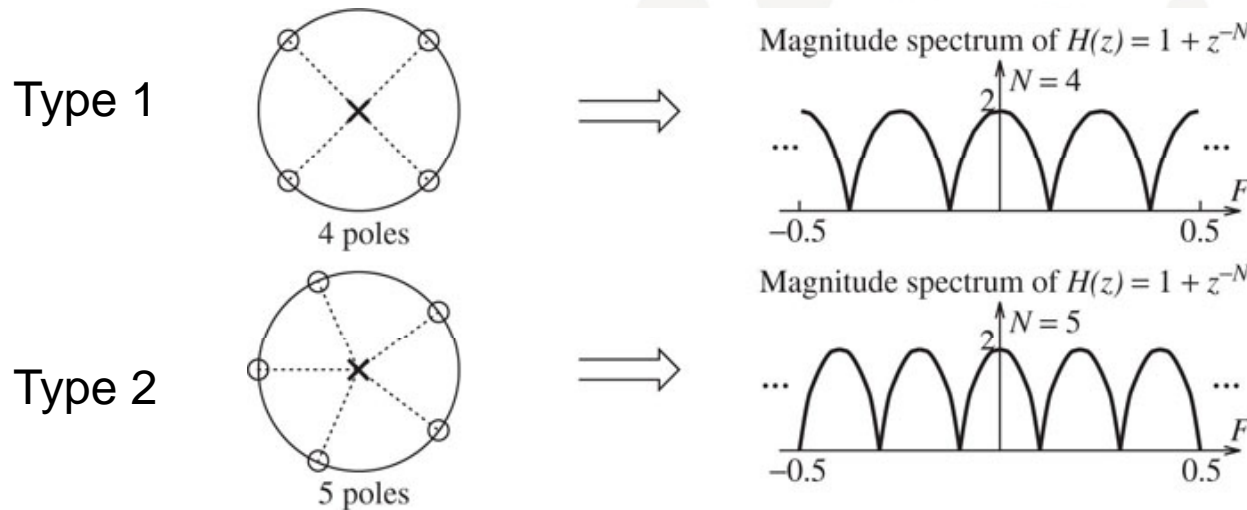
**FIGURE 6.7**  
Frequency response  
of the comb filter  
 $H(z) = 1 - \alpha z^{-N}$



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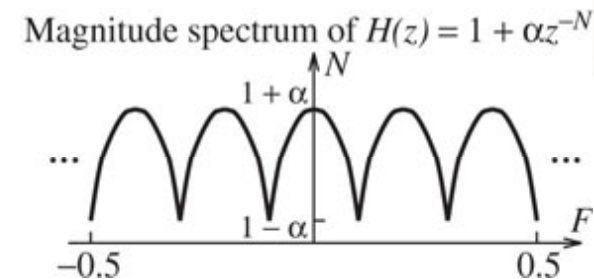
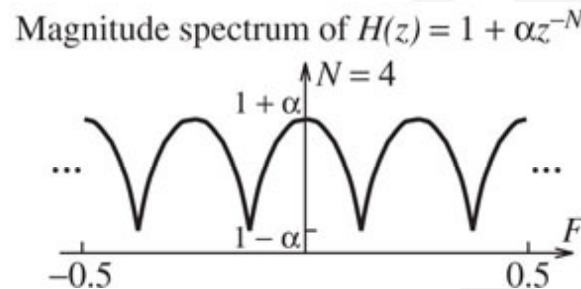
# FIR kamfiltre: $1+z^{-N}$ og $1+\alpha z^{-N}$



Forskjell fra forrige slide:  
DC respons ( $F=0$ )

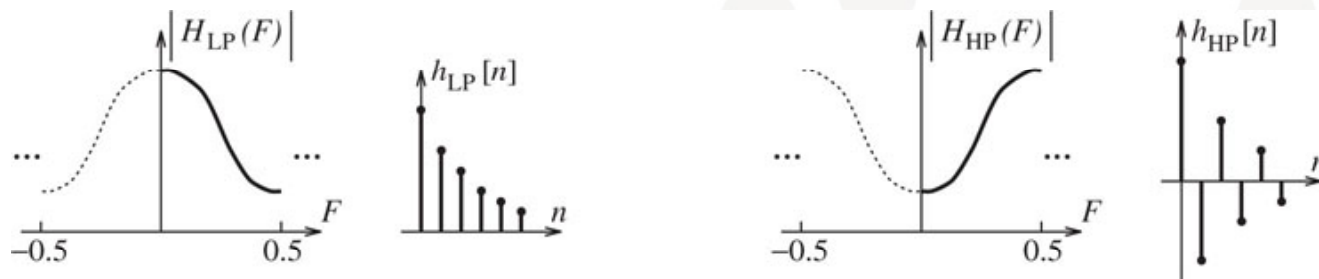
**FIGURE 6.8** Pole-zero plot and frequency response of the comb filter  $H(z) = 1 + z^{-N}$ . The number of lobes in one period corresponds to the number of zeros. The dc gain is always equal to the peak gain. The high-frequency gain (at  $F = 0.5$ ) is zero if the filter length is even (corresponding to an odd number of zeros)

**FIGURE 6.9**  
Frequency response  
of the comb filter  
 $H(z) = 1 + \alpha z^{-N}$



## 6.3 IIR filtre: 1. ordens lav/høy-pass

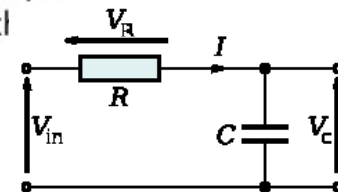
- $y[n] = x[n] + \alpha y[n-1] \Rightarrow h[n] = \alpha^n u[n], |\alpha| < 1$ 
  - $Y(z) = X(z) + \alpha z^{-1} Y(z), H(z) = Y(z)/X(z) = 1/(1 - \alpha z^{-1})$
  - $H(F) = 1/(1 - \alpha e^{-j2\pi F})$ , pol i  $z = \alpha$ 
    - $H(0) = (1 - \alpha)^{-1}, H(0.5) = (1 + \alpha)^{-1}$
  - Lavpass:  $0 < \alpha < 1$
  - Høypass:  $-1 < \alpha < 0$



**FIGURE 6.10** Spectrum and impulse response of first-order lowpass and highpass filters. The impulse response of both filters shows an exponential decay with sample values that keep getting smaller. The difference is that, for the highpass form, the samples alternate in sign

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Digital ekvivalent til analogt RC/CR filter



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# Filterbegreper

- Grensefrekvens, halv effekt, -3 dB

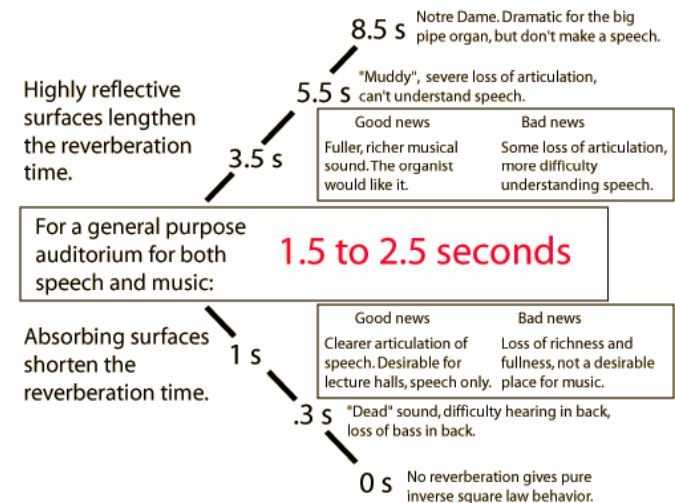
- Løs  $|H_{LP}(F)|^2 = 0.5 |H_{LP}(0)|^2$ :  $F_{-3dB} = \frac{1}{2\pi} \cos^{-1} \left( \frac{4\alpha - 1 - \alpha^2}{2\alpha} \right)$ 
    - Merk feil i boka!

- Impulsrespons, tidskonstant

- Når er  $h[n]$  falt til  $\epsilon\%$ ?  $\alpha^\eta = \epsilon \Rightarrow \eta = \ln\epsilon / \ln\alpha$
  - 1%  $\Leftrightarrow 20\log(0.01) = -40$  dB
  - 0.1%  $\Leftrightarrow 20\log(.001) = -60$  dB

- Etterklangstid:  $\tau_{-60\text{ dB}} = \eta_{-60\text{ dB}} / f_s$   
impulsresponsen, konserthall

- <http://hyperphysics.phy-astr.gsu.edu/hbase/acoustic/revtim.html>



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# Regler for pol-nullpunktsplassering

1. Kompleks konjugerte par  $\Leftrightarrow$  reell  $h[n]$
  2. Kausalitet: Antall nullpunkt  $\leq$  antall poler
  3. Poler/nullpunkter i origo påvirker ikke  $|H(F)|$
  4. Stabilitet: Alle poler for  $|z| < 1$ 
    - Jo nærmere  $|z|=1 \Rightarrow$  Jo større og smalere topp
    - **Passbånd:** poler nær  $|z|=1$
    - Tommelfingerregel, smalbåndsfiler: Båndbredde  $\Delta\Omega \leq 0.2 \Rightarrow$  poler ved  $z = R_p \cdot \exp(\pm j \Omega_0)$  der  $R_p \approx 1 - 0.5\Delta\Omega$
  5. Minimum fase: Nullpunkter kan være overalt, men bare  $|z| \leq 1$  gir minimum fase
    - Nullpunkter på  $|z|=1$  gir nullpunkter i  $|H(F)|$
    - **Stoppbånd:** nullpunkter nær  $|z|=1$
  6. **Transisjonsbånd:** pol og null nær hverandre
  7. Pol-null interaksjon: jo nærmere hverandre jo mer
- Begrensning: Kan bare designe enkle filtre med disse reglene

# 1/3-octave equalizer



- 31-band Behringer 1/3-octave graphic equalizer with LEDs that show signal presence per frequency band
- 1/3 oktav: 20,25,31.5,40,50,63,80,...,20k Hz

## 2. ordens IIR: Notch & resonator

$$H(z) = K \frac{(z - R_z e^{j\Omega_z})(z - R_z e^{-j\Omega_z})}{(z - R_p e^{j\Omega_p})(z - R_p e^{-j\Omega_p})}$$

- Samme frekvens for poler/nuller:  $\Omega_0 = \Omega_p = \Omega_z$
- Stabilitet og kausalitet:  $R_p < 1$

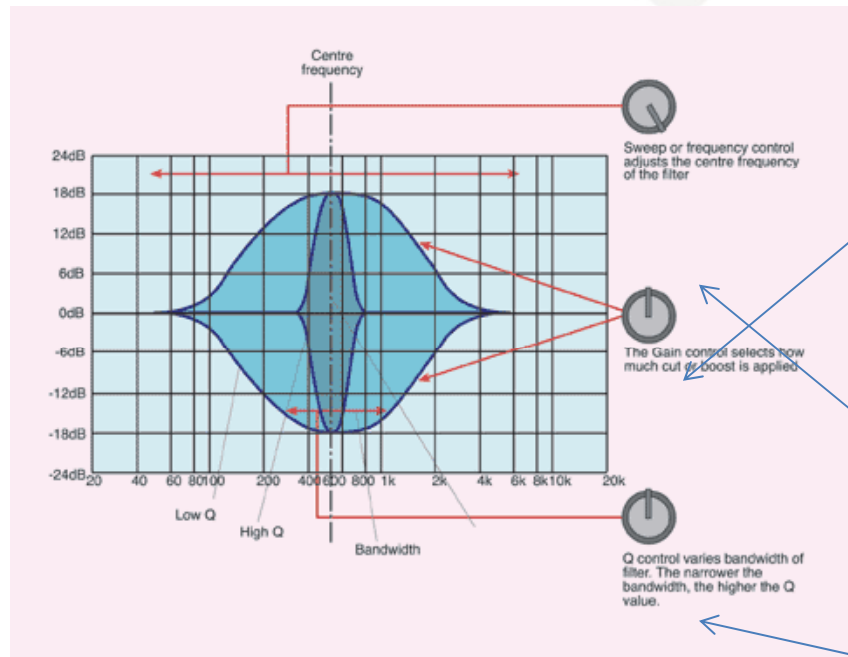
Minimum fase:  $R_z < 1$ :

- $R_z > R_p$ : Minimum ved  $\Omega_0$ :  
Notchfilter (60 Hz notch)
  - Også for  $R_p = 0 \Rightarrow$  FIR
- $R_z < R_p$ : Max i  $\Omega_0$ : Resonator
  - Også for  $R_z = 0$  (50 Hz res.)

Ikke minimum fase:

- $R_z = 1$ : Notch:  $H(F_0) = 0$
- $1 < R_z < 1/R_p$ : Svakere notch
- $R_z = 1/R_p$ : Allpass
- $R_z > 1/R_p$ : Resonator i  $\Omega_0$

# Equalizers



- Graphic equalizer:
  - one *gain control* per center frequency spaced by octaves, 1/3-octaves, etc
- Parametric:
  - may also vary *center frequency*
  - Option: may also vary *Q-factor (bandwidth)* (*eks på neste slide*)

# Shelving/parametric EQ

- The equaliser section from the Audient ASP8024 Mixing console.
- The upper section has high and low shelving EQ
- The lower section has fully parametric EQ.

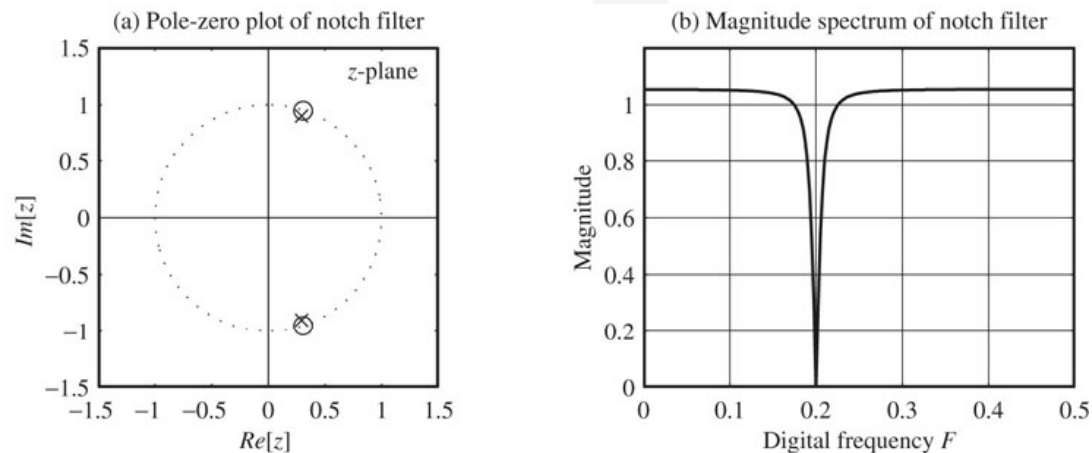




# Eks 6.7b: 2. ordens notch

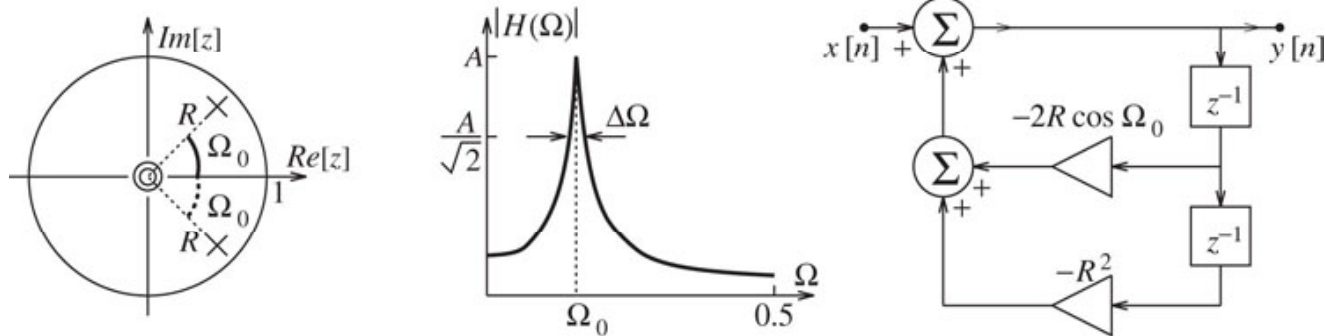
- Sentrert om 60 Hz, stoppbånd (båndbredde) 5 Hz,  $f_s=300$  Hz
  - Nettstøy (USA) – eller dempe bassresonans i rom
  - $\Omega_0=2\pi\cdot 60/300 = 2\pi/5$ ,  $\Delta\Omega=2\pi\cdot 5/300=\pi/30 (\leq 0.2)$ , bruker tommelfingerregel:  $R_p \approx 1-\Delta\Omega/2 = 0.9476$
  - Nullpunkter på samme frekvens, på  $|z|=1$

**FIGURE E.6.7B**  
Frequency response  
of the bandstop  
filter for Example  
6.7(b)



# Eks 6.8: Digital resonator

- Resonans: 50 Hz,  $f_s=300$  Hz, båndbredde 6 Hz
  - $\Omega_0=2\pi\cdot 50/300 = \pi/3$ ,  $\Delta\Omega=2\pi 6/300=0.04\pi$ ,  
 $R_p \approx 1-\Delta\Omega/2 = 0.9372$
- Nullpunktspaar i  $z=0$ , poler nær enhetssirkelen ved resonansen



**FIGURE 6.11** A second-order digital resonator. The conjugate poles closer to the unit circle ensure a sharper peak. The zeros at the origin are not a requirement and may be removed or even moved elsewhere in order to change the shape of the response. A realization requires two delay elements.



## 6.4 Allpassfiltre

$$H_A(F) = \frac{A + Be^{j2\pi F}}{B + Ae^{j2\pi F}}$$

- To unike koeffisienter, egentlig bare én: B/A
- $H_A(0) = (A+B)/(B+A) = 1$ ,  $H_A(0.5) = (A-B)/(B-A) = -1$ 
  - Altså  $|H_A(0)| = |H_A(0.5)| = 1$
  - Gjelder alle frekvenser: Allpass
- Brukes for å modifisere fasen
  - Kan gjøre et ikke-minimum fase system om til minimum fase
  - Kan brukes til å approksimere ikke-heltallige forsinkelser

# Analyse av enkelt allpassfilter

$$H(z) = \frac{A + Bz}{B + Az} = \frac{1 + \alpha z}{z + \alpha}, \alpha = B/A, |\alpha| < 1$$

$$H(F) = \frac{1 + \alpha e^{j2\pi F}}{\alpha + e^{j2\pi F}} = \frac{e^{-j\pi F} + \alpha e^{j\pi F}}{\alpha e^{-j\pi F} + e^{j\pi F}}$$

- Teller og nevner = kompleks konjugerte  $\Rightarrow$  samme tallverdi
- Fasen til  $H(F)$  er 2 x fasen til telleren
- Telleren:

$$\cos(\pi F) - j \sin(\pi F) + \alpha \cos(\pi F) + j\alpha \sin(\pi F) = (1 + \alpha) \cos(\pi F) - j(1 - \alpha) \sin(\pi F)$$

$$\angle H(F) = -2 \tan^{-1} [\text{Im}(\text{teller}) / \text{Re}(\text{teller})] = -2 \tan^{-1} \left[ \frac{1 - \alpha}{1 + \alpha} \tan(\pi F) \right]$$

$$t_p(F) = -\frac{\angle H(F)}{2\pi F} = \frac{1}{\pi F} \tan^{-1} \left[ \frac{1 - \alpha}{1 + \alpha} \tan(\pi F) \right] \approx \frac{1 - \alpha}{1 + \alpha}, F \ll 1$$

- For  $\alpha < 1$  blir forsinkelsen  $t_p < 1$ : ikke-heltallig forsinkelse (lav  $F$ )

# Allpassfiltre, generelt

$$H_{AP}(z) = \frac{C_N + C_{N-1}z^{-1} + \dots + C_1z^{N-1} + z^{-N}}{1 + C_1z^{-1} + \dots + C_{N-1}z^{N-1} + C_Nz^{-N}}$$

Hvordan kjenne igjen et allpass filter?

1. Koeffisientene i teller og nevner er like, men i omvendt rekkefølge
2. Hver pol har et konjugert resiprokt nullpunkt
3. Differanselikning for høyre og venstre side er speilvendte av hverandre

# Lydeffekter

- [3470-kap7-9-Musikkeffekter.ppt](#)
- Se også kapittel 7.9 Digital Audio Effects