UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	INF3470/4470 — Digital signal processing
Day of examination:	December 9th, 2011
Examination hours:	14.30 – 18.30
This problem set consists of 4 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 z-transformation

a) Find the z-transform $X(z)$ of the expression $x[n] = \alpha^n\, u[n] - \alpha^n\, u[n-5]$. 1 p.

b) Find the causal sequence $h[n]$ with z-transform $H(z) = \frac{z+3}{z-2}$. 1 p.

c) A signal $x[n]$ has the z-transform $X(z)$: Show that multiplying $x[n]$ with n corresponds to multiplying $X(z)$ with $-z\frac{d}{dz}$: 2 p.

$$n\, x[n] \overset{Z}{\longleftrightarrow} -z\frac{d}{dz}X(z)$$

How does this affect the ROC?

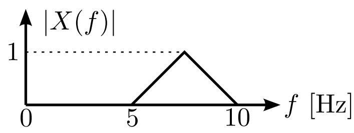
Problem 2 Sampling

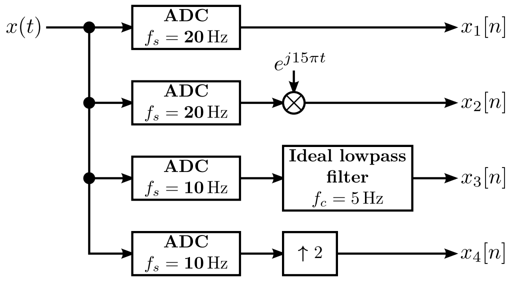
a) Explain briefly the meaning of the following terms: 2 p.

- Sampling frequency and Nyquist frequency
- Quantization and quantization error
- Uniform and non-uniform sampling
- Frequency aliasing

(Continued on page 2.)

b) A real signal $x(t)$ with the magnitude response $|X(f)|$ (as shown to the right) is sampled and filtered in four different ways:



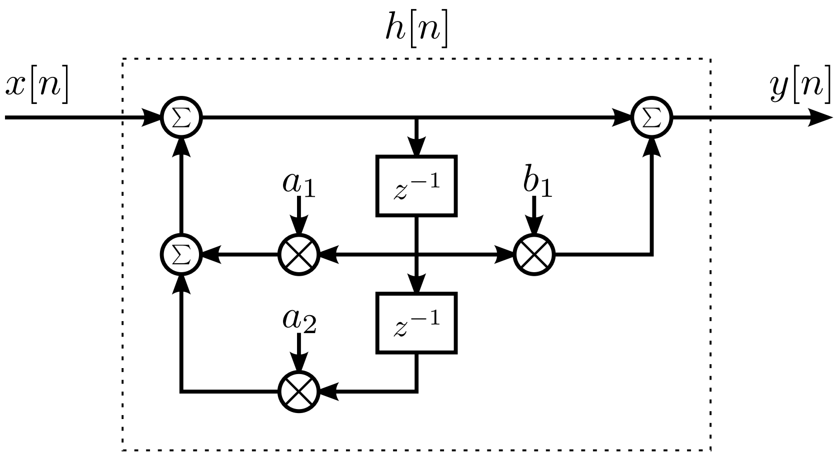


ADC	Analog-to-Digital Converter
f_s	Sample frequency
f_c	Filter cut-frequency
$\boxed{\uparrow}$	Upsampling by zero-expansion between samples

Sketch the magnitude responses $|X_1(f)|$, $|X_2(f)|$, $|X_3(f)|$ and $|X_4(f)|$ in the frequency range -10 Hz to 10 Hz. Assume that the amplitude of the signal is unaffected by the upsampling process. 2 p.

Problem 3 System analysis

A filter is described as:



where Σ indicates sum and $\otimes$ indicates multiplication.

a) Find the system function $H(z)$. 1 p.

In the following two exercises, assume that $H(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$:

b) Plot poles and zeros, and determine whether this filter is stable and/or causal. 2 p.

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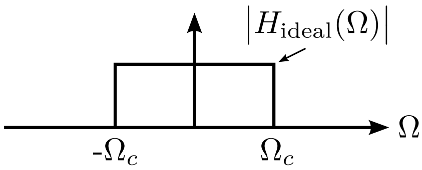
c) Find the expressions for the filter magnitude response $\left|H(\Omega)\right|$ and phase response $\Theta_H(\Omega)$. The final expressions should not contain complex numbers. Avoid spending time trying to simplify the terms. 2 p.

Problem 4 FIR filter design

The magnitude response of an ideal lowpass filter is given as:



a) Find the impulse response $h_{\text{ideal}}[n]$ when the phase is 0, and sketch it. 1 p.

b) This filter can not be realised. Why? What must be done with the impulse response to make the filter realisable, and how does this affect the frequency response? 1 p.

c) Two common ways to design FIR filters is by means of the "frequency sampling method" and the "window method". Explain the concept of these two methods, and list pros and cons. 2 p.

Problem 5 Filters

Given the system function:

$$H(z) = (1 - 1.25e^{j0.8\pi}z^{-1})(1 - 1.25e^{-j0.8\pi}z^{-1})$$

a) Find poles and zeros and plot them. 1 p.

b) Find the minimum-phase filter $H_{\text{MF}}(z)$ that has the same frequency response:

$$H_{\text{MF}}(\Omega) = H(\Omega)$$

1 p.

c) Find the allpass filter $H_{\text{AP}}(z)$ that transforms between them, i.e.

$$H_{\text{MF}}(\Omega) = H_{\text{AP}}(z)\,H(\Omega),$$

and determine whether this filter is stable. 1 p.

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Formulas

Basic relations:

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{\jmath\alpha} + e^{-\jmath\alpha}) \\
\sin\alpha &= \frac{1}{2\jmath}(e^{\jmath\alpha} - e^{-\jmath\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 &\Leftrightarrow x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Discrete convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = h[n] * x[n]$$

Discrete-time Fourier transformation (DTFT):

$$\begin{aligned}
\text{Analysis:} \qquad X(\Omega) &= = \sum_{n=-\infty}^{\infty} x(n)e^{-\jmath\Omega n} \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{\jmath\Omega n}d\Omega
\end{aligned}$$

Discrete Fourier transformation (DFT):

$$\begin{aligned}
\text{Analysis:} \qquad X[k] &= \sum_{n=0}^{N-1} x[n]e^{-\jmath 2\pi kn/N}, \quad 0 \le k \le N-1 \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{\jmath 2\pi kn/N}, \quad 0 \le k \le N-1
\end{aligned}$$

z-transformation:

$$\text{Analysis:} \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$