UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	INF3470/4470 — Digital signal processing
Day of examination:	December 11th, 2012
Examination hours:	14.30 – 18.30

This problem set consists of 4 pages.

Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1

a) Find the z-transform (and ROC) to the data sequence

$$x[n] = \begin{cases} \frac{1}{n} & \text{for} \quad n \in [-2, 2], \qquad n \in \mathbb{Z}, \\ 0 & \text{for} \quad n = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $\mathbb{Z}$ represents integers. 1 p.

b) Find the z-transform (and ROC) to the function

$$x[n] = n\, 2^{n-1}\, u[n-1].$$

Hint: Some z-transform properties can be used to significantly simplify this calculation. 1 p.

c) Consider two finite data sequences $x[n]$ and $h[n]$. Show that:

$$x[n] * h[n] \stackrel{Z}{\longleftrightarrow} X(z)\, H(z),$$

where $*$ denotes the convolution operator. Briefly explain why this property is useful. 2 p.

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Problem 2

The two main categories of filters are: FIR and IIR.

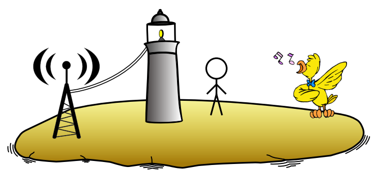
a) What do these acronyms mean, and how are these filter classes fundamentally different? Compare impulse responses, stability, causality, and filter performance (amplitude and phase responses). 2 p.

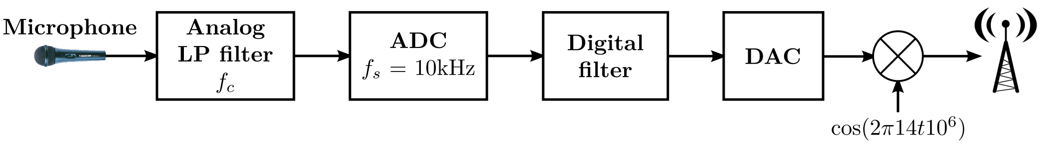
b) Mention the four types of exact linear phase FIR filters, and sketch each filter's magnitude response. What restrictions apply when we wish to implement lowpass, highpass, bandstop and bandpass filters? 1 p.

c) Four common IIR filters are Butterworth, Chebyshev type 1 and 2, and elliptic filters. What characterizes these filters? Compare magnitude responses in pass-band, stop-band and transition-band. Which filter has the narrowest transition from pass-band to stop-band? 1 p.

Problem 3

Your current residence is in a lighthouse on an island. To be able to communicate you have bought a digital amateur radio as a do-it-yourself kit. The assembly manual contains a sketch showing how the components are to be connected:





where LP is short for low-pass filter, ADC for analog-to-digital converter and DAC for digital-to-analog converter, f_c is the cut frequency, and f_s is the sampling frequency.

a) Explain what each component does and why they are essential in this system, except ignore the digital filter for now (it will be handled in exercise **b**). What do you think is most likely, that the ADC and DAC is 1bit, 10bit or 24bit? Why do we sample at a rate of 10kHz, and not on a frequency substantially higher (or lower) than this? What restrictions apply to the low-pass filter cut-frequency f_c, and what is a suitable value for it in this scenario? 2 p.

On the island there is a noisy bird that interferes with the radio transmission. Therefore a digital Notch-filter is implemented to remove the bird noise:

$$H(z) = \frac{(z - e^{j\pi/4})(z - e^{-j\pi/4})}{(z - 0.95e^{j\pi/4})(z - 0.95e^{-j\pi/4})}$$

The bandwidth Ω_Δ of the Notch-filter may in this case be found by utilizing the following rule-of-thumb: $R = 1 - 0.5\Omega_\Delta$, where R is the absolute value of the Notch-filter poles.

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b) Plot poles and zeros for this filter. 1 p.

c) What is the center frequency and bandwidth of the bird song? Will the filter affect the speech quality notably? 1 p.

Problem 4

A causal filter is described by the difference equation:

$$y[n] = c\,y[n-1] + (1-c)\,x[n] \qquad c \in \mathbb{R} \quad \text{and} \quad 0 < c < 1.$$

a) Make a block diagram for the filter directly from this equation. 1 p.

b) Show that the system function for this filter is given by:

$$H(z) = \frac{1-c}{1-c\,z^{-1}}.$$

1 p.

c) Assume that the input signal is a unit-step function, $x[n] = u[n]$. Start by finding the z-transform $Y(z)$ of the output signal $y[n]$, then find $y[n]$ using partial fractions and the inverse z-transform. 1 p.

d) Show that $\lim_{n\to\infty} y[n] = 1$. Can you find this answer from $H(z)$ as well? 1 p.

Problem 5

An all-pass filter has the form:

$$H(z) = \frac{z^{-1}-a}{1-a\,z^{-1}} \qquad a \in \mathbb{R}$$

a) Find poles and zeros for $H(z)$. 1 p.

b) Show that $|H(\Omega)| = 1$ for all Ω. 1 p.

c) Show that $\angle H(\Omega) = \pm\pi$ when $\Omega = \pi$. 1 p.

d) Show that when $0 < a < 1$ then $\angle H(\Omega)$ is negative for all Ω. 1 p.

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Formulas

Basic relations:

$$\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{\jmath\alpha} + e^{-\jmath\alpha}) \\
\sin\alpha &= \frac{1}{2\jmath}(e^{\jmath\alpha} - e^{-\jmath\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 \quad &\Leftrightarrow \quad x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Discrete convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = h[n] * x[n]$$

Discrete-time Fourier transformation (DTFT):

$$\begin{aligned}
\text{Analysis:} \qquad X(\Omega) &= = \sum_{n=-\infty}^{\infty} x(n)e^{-\jmath\Omega n} \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{\jmath\Omega n}d\Omega
\end{aligned}$$

Discrete Fourier transformation (DFT):

$$\begin{aligned}
\text{Analysis:} \qquad X[k] &= \sum_{n=0}^{N-1} x[n]e^{-\jmath 2\pi kn/N}, \quad 0 \le k \le N-1 \\
\text{Synthesis:} \qquad x[n] &= \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{\jmath 2\pi kn/N}, \quad 0 \le k \le N-1
\end{aligned}$$

z-transformation:

$$\text{Analysis:} \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$