UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	INF3470/4470 — Digital signal processing
Day of examination:	December 10th, 2018
Examination hours:	14:30 – 18.30
This problem set consists of 8 pages.	
Appendices:	None
Permitted aids:	None

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Note 1: All numbers and figure axes should have units.
Note 2: Read through the whole exercise set before you start!

Problem 1 Blood pressure measurements (13p)

Marit is a Master graduate student and has received a dataset from her supervisor. She is to analyze several blood pressure measurements on patients. Figure 1 shows the signal from one of the patients. The sampling

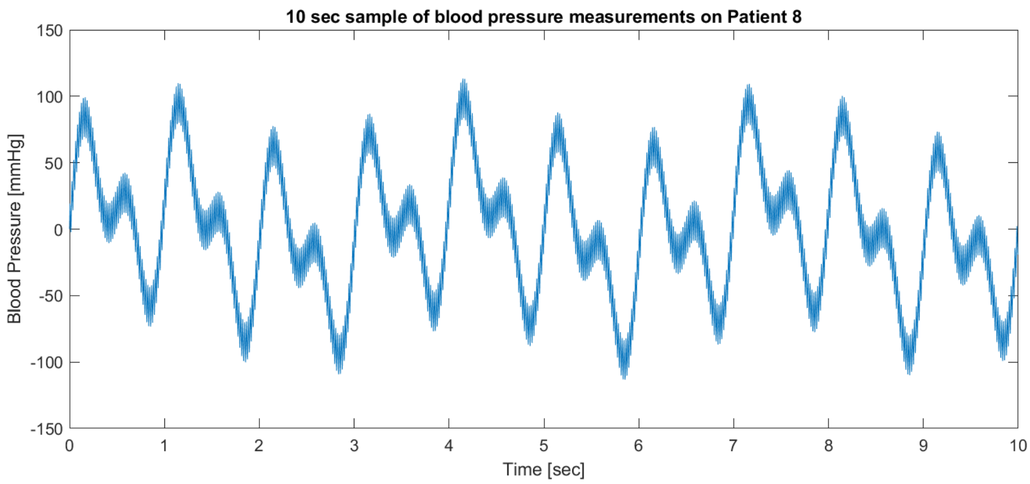

Figure 1: Excerpt from blood pressure measurements on one of the patients

frequency is 200 Hz. The measurements are noisy, and the supervisor explains that the signal probably has contributions from both the patient's respiration and noise from the measurements equipment itself. Respiratory rate or breathing rate means number of inhalations per minute. Marit takes a Fourier analysis of the signal. Figure 2 shows the result, where we can clearly see four distinct main frequencies.

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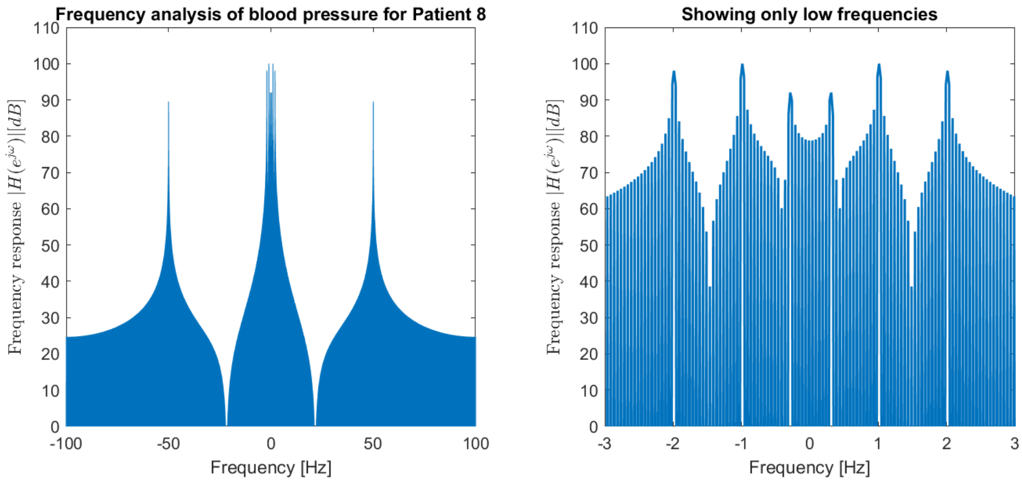

Figure 2: Frequency analysis of the blood pressure measurements in Figure 1. The four main frequencies are 0.3, 1, 2 and 50 Hz.

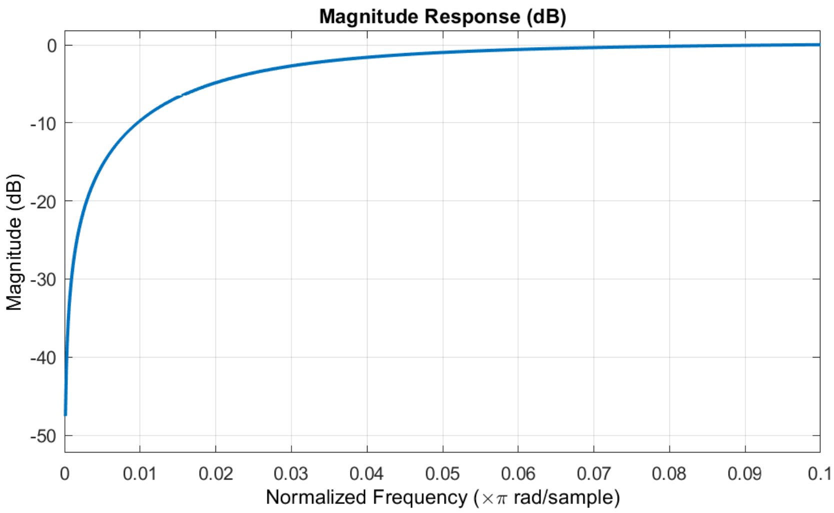

Figure 3: Magnitude response of Marit's filter in b). The figure is plotted in MATLAB. Note that MATLAB normalizes frequency with respect to the Nyquist frequency instead of the sampling frequency.

a) For an adult person the respiratory rate is normally around 12-16 inhalations per minute. The breathing rate is 0.3 Hz for this patient. How many inhalations per minute does this rate correspond to? Show calculation. 1 p.

b) To remove the contribution from the patient's respiration, Marit first looks at a simple, linear filter with difference equation $y[n]+ay[n-1] = x[n] + bx[n-1]$.

- Find the system's transfer function H(z). 1 p.
- For which values of a and b is the filter stable? When is the filter causal? When does the filter have minimum phase? Justify your

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answers. 3 p.

- Marit chooses values for a og b for her filter. She plots the magnitude response of the filter in MATLAB for positive normalized frequencies from 0 to 0.1 and gets Figure 3. Note that MATLAB normalizes using the Nyquist frequency, $F_s/2$, instead of the sampling frequency. In other words, normalized Nyquist frequency corresponds to 1 in MATLAB. How will this filter affect the dominant frequencies we see in Figure 2 and the resulting filtered signal? Justify your answer. 2 p.

c) In order for the signal to give meaning to doctors to analyze, the contribution from the unphysiological noise at 50 Hz should be removed. You are tasked with helping Marit design a notch filter that removes this 50 Hz noise. The filter is to have two poles and two zeros, and it shall have real coefficients.

- Determine the location of the poles and zeros. Include a pole-zero plot. Justify your choice of angles and radii for the poles and zeros. Have distinct axes values. 2 p.
- Determine the system transfer function H(z). 1 p.
- Sketch the magnitude response of your filter. Have distinct axes values. 1 p.
- Instead of making this notch filter, what is the simplest FIR filter you could have made to remove the 50 Hz noise? The filter shall have real coefficients. Find H(z) and draw a pole-zero plot. Justify your answer. 2 p.

Problem 2 Z-transform (6 p.)

a) What is the z-transform of the following sequence? What is the region of convergence?

$$x[n] = \left(\frac{4}{3}\right)^n u[1-n]$$

2 p.

b) A stable system has the following zeros and poles:

$$z_1 = j, \quad z_2 = -j, \quad p_1 = -\frac{1}{2} + j\frac{1}{2}, \quad p_2 = -\frac{1}{2} - j\frac{1}{2}$$

It is known that the frequency response function $H(e^{j\omega})$ evaluated at $\omega = 0$ is equal to 0.6, i.e. $H(e^{j0}) = 0.6$.

- What is the region of convergence? Justify your answer. 2 p.
- Determine the system function $H(z)$. Show full calculation. 2 p.

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Problem 3 Sampling (9 p.)

a) You have a measurement instrument with sampling rate of 2000 Hz.

- What is the folding frequency and what is its significance? 1 p.
- You wish to analyze a continuous 500 Hz cosine signal, $x_c(t)$. What is the period T of this signal? According to the Shannon criteria, what should the sampling frequency at least be for such a signal? 1 p.
- You sample this signal with your measurement instrument. Determine an expression for your sampled signal $x[n]$. You can assume no quantification errors and that the phase $\phi = 0$ at time $t = 0$. Is $x[n]$ periodic? Justify your answer. 2 p.

b)
- Draw the frequency response $X_c(e^{j2\pi F})$ of the continuous signal $x_c(t)$ given in a). Have distinct axes values for both the x- and y-axes. Let the x-axis be the frequency range $[-6000, 6000]$ Hz. 1 p.
- In a new figure, draw the frequency response of your sampled signal $x[n]$. Have distinct axes values for both the x- and y-axes. Let the x-axis be the frequency range $[-6000, 6000]$ Hz. 1 p.
- Comment on differences in the plots and explain why they arise. 1 p.

c) Which frequency would you register on your measurement instrument if you analyze a signal that has a frequency of 1500 Hz? What if the signal has a frequency of 4300 Hz? Justify your answers. 2 p.

Problem 4 Sample rate conversion (13 p.)

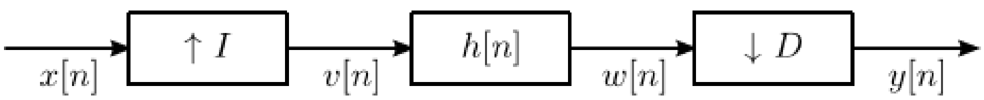

Figure 4: System for rate conversion.

Figure 4 illustrates a standard system for sample rate conversion (up- and downsampling) of discrete time signals. Assumes that the signal $x[n]$ is sampled from a continuous time signal $x_c(t)$ using a sampling frequence of $F_s = 10$ kHz. In the figure, I means the interpolation factor and D means the decimation factor. The filter $h[n]$ is an ideal lowpass filter with cutoff frequency at ω_c.

a) What is the purpose of this lowpass filter? Tip: There are *two* main reasons to have it. 2 p.

b) Express the cutoff frequency, ω_c, of the lowpass filter using I and D. Here, ω_c is digital angular frequency. 1 p.

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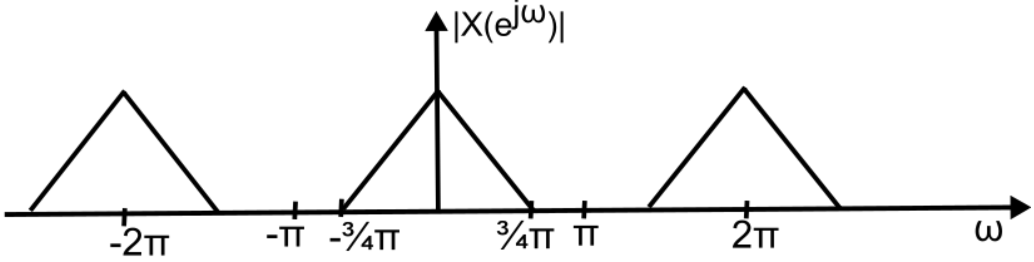

Figure 5: Magnitude response of the input signal.

c) Assume the cutoff frequency to be $\omega_c = \pi/3$ and that the magnitude response of the input signal $|X(e^{j\omega})|$ is as shown in figure 5. Sketch the magnitude response $|V(e^{j\omega})|$, $|W(e^{j\omega})|$ and $|Y(e^{j\omega})|$ for

- $I = 3$, $D = 2$ 4 p.
- $I = 2$, $D = 3$ 4 p.

Use the same frequency axis in all plots and indicate center frequencies and spectrum widths, as in figure 5. Tip: You may assume any changes to maximum magnitude in the process to be compensated by the gain in the lowpass filter, so just sketch the responses to have the same peak magnitude.

d) Assume that the lowpass filter is correctly configured, according to b). Find the largest ratio D/I that still enables perfect reconstruction of $x[n]$ from $y[n]$. Explain your answer. 2 p.

Problem 5 FIR filter design (10 p.)

a) Provided that the frequency response of an ideal lowpass filter, $H_{LP}(e^{j\omega})$, with cutoff frequency ω_c, show that the impulse response is:

$$h_{LP}[n] = \frac{sin(\omega_c n)}{\pi n}, -\infty < n < \infty \qquad (1)$$

1 p.

b) Explain why this filter is not practically realizable. 1 p.

c) One way to approximate the ideal filter, using a FIR filter, is to use a real window function to select only a section of the ideal impulse response. Denote the window function as $w[n]$ and assume it has length $2M + 1$ and is centered about $n = 0$. Explain qualitatively (by words) how the DTFT of the window function affects the properties of the lowpass filter. 2 p.

d) You have figured out that for your application, the transition band has to be less than 1 kHz. The sampling frequency is 10 kHz. You have

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looked up in a table and found that the rectangular window function has $\Delta\omega = 4\pi/L$ where $\Delta\omega$ is the transition band and L is the window length.

- Which filter order should you use? 1 p.
- Will your filter have linear phase? Explain your answer. 1 p.
- From the information provided so far, are you able to deduce whether your filter will be FIR type I, II, III or IV? Explain your answer. 1 p.

e) It turns out that the filter you design exhibits too much ripple in the pass band. You attempt to remedy this by increasing the filter order drastically. But it does not help. Explain why it does not help to increase the filter order and suggest a different solution to the problem. 1 p.

f) You have to abandon the idea of using a fixed window FIR filter design, as the filters meeting both the ripple and passband requirements turns out too long. What alternative design strategies would you suggest? Explain shortly (max 100 words) advantages/disadvantages of the different approaches and what considerations you have to do. 2 p.

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Formula sheet

Basic relations:

$$
\begin{aligned}
\sin(\alpha \pm \beta) &= \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\
\cos(\alpha \pm \beta) &= \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \\
\sin 2\alpha &= 2\sin\alpha\cos\alpha \\
\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\
\sin\alpha + \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\sin\alpha - \sin\beta &= 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos\alpha + \cos\beta &= 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \\
\cos\alpha - \cos\beta &= -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \\
\cos^2\alpha + \sin^2\alpha &= 1 \\
\cos\alpha &= \frac{1}{2}(e^{j\alpha} + e^{-j\alpha}) \\
\sin\alpha &= \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha}) \\
\sum_{n=0}^{N-1} a^n &= \begin{cases} N & \text{for } a = 1 \\ \frac{1-a^N}{1-a} & \text{otherwise} \end{cases} \\
ax^2 + bx + c = 0 &\Leftrightarrow x_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}
$$

Linear convolution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = h[n] * x[n]$$

Circular convolution:

$$y[n] = x[n] \text{Ⓝ} h[n] = \sum_{k=0}^{N-1} x[k]h[< n-k >_N] = \sum_{k=0}^{N-1} x[< n-k >_N]h[k] = h[n] \text{Ⓝ} x[n]$$

Continuous Time Fourier Transform (CTFT):

$$
\begin{aligned}
\text{Analysis:} \quad X(j\Omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt \\
\text{Synthesis:} \quad x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{j\Omega t}d\Omega
\end{aligned}
$$

Continuous Time Fourier Series (CTFS):

$$
\begin{aligned}
\text{Analysis:} \quad c_k &= \frac{1}{T_o}\int_{T_0} x(t)e^{-jk\Omega_0 t}dt \\
\text{Synthesis:} \quad x(t) &= \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t}
\end{aligned}
$$

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Discrete Time Fourier Series (DTFS):

$$\text{Analysis:} \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$\text{Synthesis:} \qquad x[n] = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}$$

Discrete Time Fourier Transform (DTFT):

$$\text{Analysis:} \qquad X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$\text{Synthesis:} \qquad x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

Discrete Fourier Transform (DFT):

$$\text{Analysis:} \qquad X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad 0 \leq k \leq N-1$$

$$\text{Synthesis:} \qquad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}, \quad 0 \leq n \leq N-1$$

z-transform:

$$\text{Analysis:} \qquad X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$