Differensialligninger

$$\frac{dx}{dt} = kx \qquad x(0) = x_0$$

$$\frac{dx}{x} = k\,dt$$

$$\int \frac{1}{x}\,dx = \int k\,dt$$

$$\ln x = kt + C$$

$$x = e^{kt+C}$$

$$x = e^{C} e^{kt}$$

$$x = Ae^{kt}$$

Setter inn for startbetingelsen

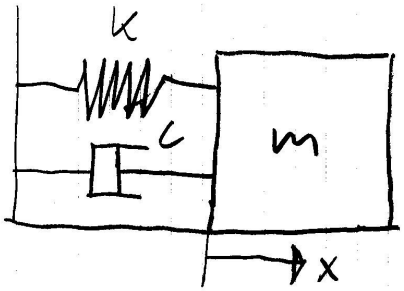
$$x_0 = Ae^{k\cdot 0}$$

$$x_0 = A$$

Svar

$$x(t) = x_0 e^{kt}$$

Masse - fjær - demper system



Newtons 2. lov

$\Sigma F_i = ma$

$F_s = -kx$ Fjærkraft

$F_D = -c\dot{x}$ Dempingskraft

Antar friksjonsfritt

$$m\ddot{x} = -kx - c\dot{x}$$

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

Skriver om til matriseform

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$\dot{x}_1 = \dot{x} = x_2$$

$$\dot{x}_2 = -\frac{c}{m}\dot{x} - \frac{k}{m}x$$

$$= -\frac{c}{m}x_2 - \frac{k}{m}x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = Ax$$

Må finne egenverdiene

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -\frac{k}{m} & -\frac{c}{m} - \lambda \end{vmatrix} = \lambda\left(\frac{c}{m} + \lambda\right) + \frac{k}{m}$$

$$= \lambda^2 + \frac{c}{m}\lambda + \frac{k}{m} = 0$$

Samme form som

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m} = 0$$

$$x(t) = e^{At} = Q e^{\Lambda t} Q^{-1}$$

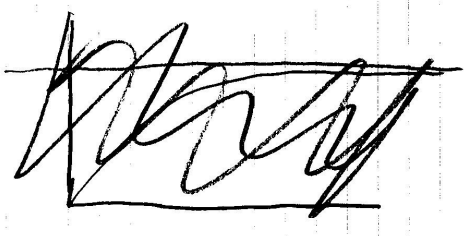
Tver til tellar

$$\lambda_{1,2} = -\frac{c}{2m} \pm \sqrt{\frac{c^2 - 4k^2}{4m^2}}$$

Tre tilfeller

λ_1 og λ_2 har to reelle røtter

$$x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$



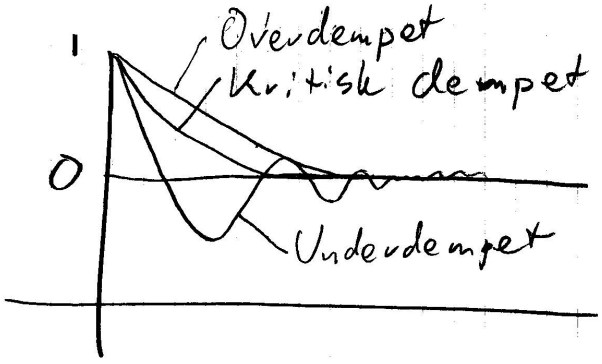
Over dempet

$\lambda_1 = \lambda_2$ Kritisk dempet

$$x(t) = (A + Bt)e^{\lambda t}$$

λ_1 og λ_2 er kompleks konjugerte

$$x(t) = e^{Re(\lambda)t}\left(A\cos(Im(\lambda)t) + B\sin(Im(\lambda)t)\right)$$



Setter $m = \frac{1}{2}$

Overdempet

$$\lambda = -c \pm \sqrt{c^2 - 4k^2}$$

$$c^2 - 4k^2 > 0$$

$\boxed{c = 4 \quad k = 1}$

$$16 - 4 = 12$$

Kritish dempet

$$c^2 = 4k^2$$

$$c = 2k$$

$\boxed{c = 2 \quad k = 1}$

Underdempet

$$c^2 - 4k^2 < 0$$

$\boxed{c = 1 \quad k = 1}$

$$1 - 4 = -3$$