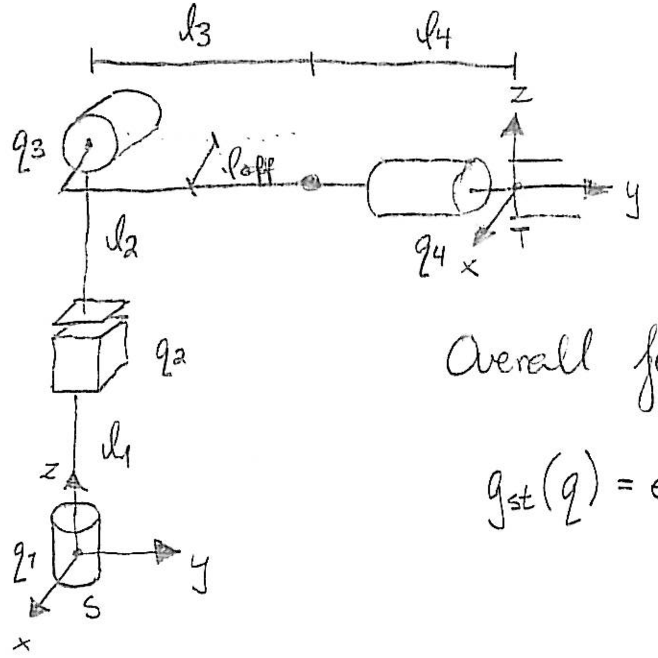
Eksamen i Introduksjon til robotikk (INF3480)

Oppgave 2



$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & l_{off} \\ 0 & 1 & 0 & l_3+l_4 \\ 0 & 0 & 1 & l_1+l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Twists:

Joint 1: $q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $\xi_1 = \begin{bmatrix} -\omega_1 \times q_1 \\ \omega_1 \end{bmatrix}$

Joint 2: $\xi_2 = \begin{bmatrix} v_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Joint 3: $q_3 = \begin{bmatrix} 0 \\ 0 \\ l_1+l_2 \end{bmatrix}$, $\omega_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\xi_3 = \begin{bmatrix} -\omega_3 \times q_3 \\ \omega_3 \end{bmatrix}$

Joint 4: $q_4 = \begin{bmatrix} l_{off} \\ l_3+l_4 \\ l_1+l_2 \end{bmatrix}$, $\omega_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\xi_4 = \begin{bmatrix} -\omega_4 \times q_4 \\ \omega_4 \end{bmatrix}$

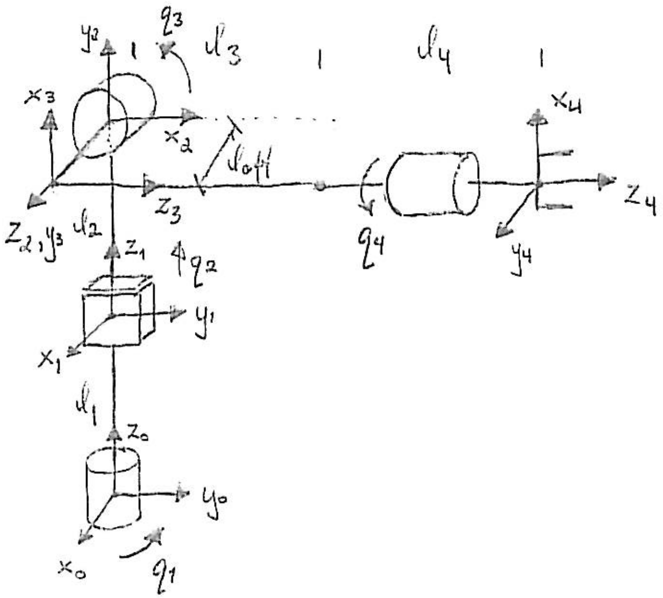
Overall forward kinematics:

$$g_{st}(q) = e^{\hat{\xi}_1 q_1} e^{\hat{\xi}_2 q_2} e^{\hat{\xi}_3 q_3} e^{\hat{\xi}_4 q_4} g_{st}(0)$$

With MATLAB this gives ('Oppgave2_Kinematics.m')

$$g_{st}(q) = \begin{bmatrix} \cos(q_1)\cos(q_4) - \sin(q_1)\sin(q_3)\sin(q_4) & -\sin(q_1)\cos(q_3) & \cos(q_1)\sin(q_4) + \sin(q_1)\sin(q_3)\cos(q_4) & -(l_3+l_4)\sin(q_1)\cos(q_3) + l_{off}\cos(q_1) \\ \sin(q_1)\cos(q_4) + \cos(q_1)\sin(q_3)\sin(q_4) & \cos(q_1)\cos(q_3) & \sin(q_1)\sin(q_4) - \cos(q_1)\sin(q_3)\cos(q_4) & (l_3+l_4)\cos(q_1)\cos(q_3) + l_{off}\sin(q_1) \\ -\cos(q_3)\sin(q_4) & \sin(q_3) & \cos(q_3)\cos(q_4) & (l_3+l_4)\sin(q_3) + l_1 + l_2 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DENAVIT - HARTENBERG :



i	a_i	d_i	α_i	θ_i
1	0	l_1	0	q_1
2	0	d_2^*	$+90°$	$+90°$
3	0	l_{off}	$+90°$	$+90°+q_3$
4	0	l_3+l_4	0	q_4

$$d_2^* = l_2 + q_2$$

$$A_1 = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2+q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -\sin(q_3) & 0 & \cos(q_3) & 0 \\ \cos(q_3) & 0 & \sin(q_3) & 0 \\ 0 & 1 & 0 & l_{off} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} \cos(q_4) & -\sin(q_4) & 0 & 0 \\ \sin(q_4) & \cos(q_4) & 0 & 0 \\ 0 & 0 & 1 & l_3+l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Total kinematics:

$$T = \begin{bmatrix} \cos(q_1)\sin(q_4)+\sin(q_1)\sin(q_3)\cos(q_4) & \cos(q_1)\cos(q_4)-\sin(q_1)\sin(q_3)\sin(q_4) & -\sin(q_1)\cos(q_3) & -(l_3+l_4)\sin(q_1)\cos(q_3)+l_{off}\cos(q_1) \\ \sin(q_1)\sin(q_4)-\cos(q_1)\sin(q_3)\cos(q_4) & \sin(q_1)\cos(q_4)+\cos(q_1)\sin(q_3)\sin(q_4) & \cos(q_1)\cos(q_3) & (l_3+l_4)\cos(q_1)\cos(q_3)+l_{off}\sin(q_1) \\ \cos(q_3)\cos(q_4) & -\cos(q_3)\sin(q_4) & \sin(q_3) & l_1+l_2+q_2+(l_3+l_4)\sin(q_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Oppgave 3

a) Assume we are given the position of the wrist center as (x_c, y_c, z_c). These coordinates are found from the forward kinematics as

$$\left.\begin{aligned} x_c &= -l_3 \sin(q_1)\cos(q_3) + l_{off}\cos(q_1) \\ y_c &= l_3\cos(q_1)\cos(q_3) + l_{off}\sin(q_1) \end{aligned}\right\} \Rightarrow \begin{aligned} x_c\cos(q_1) + y_c\sin(q_1) &= l_{off} \\ \cos(q_3) &= \frac{1}{l_3}\left(y_c\cos(q_1) - x_c\sin(q_1)\right) \end{aligned}$$

$$z_c = l_3\sin(q_3) + l_1 + l_2 + q_2$$

The first two equations can be solved separately for q_1 and q_3. Then the last equation is solved for q_2

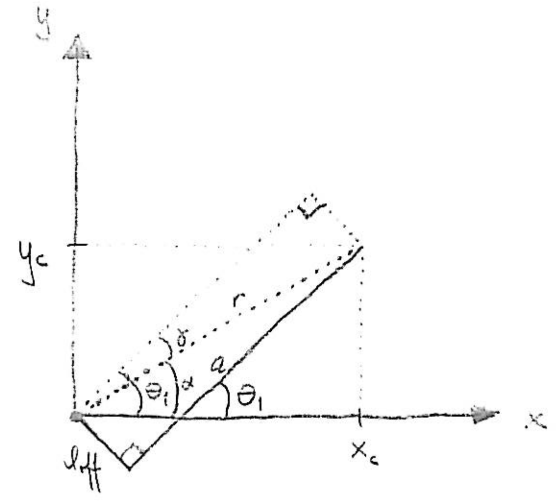
$$q_1 = \operatorname{atan2}\left(l_{off}\, y_c - x_c\sqrt{x_c^2 + y_c^2 - l_{off}^2},\; x_c l_{off} + y_c\sqrt{x_c^2 + y_c^2 - l_{off}^2}\right)$$

$$q_1 = \operatorname{atan2}\left(-\text{"}- \; + \; -\text{"}-,\; -\text{"}- \; - \; -\text{"}-\right)$$

$$q_3 = \operatorname{acos}\left(\sqrt{x_c^2 + y_c^2 - l_{off}^2}\,/\,l_3\right)$$

$$q_3 = \pi - \operatorname{acos}\left(\sqrt{x_c^2 + y_c^2 - l_{off}^2}\,/\,l_3\right)$$

$$q_2 = z_c - l_1 - l_2 - l_3\sin(q_3)$$



$$\alpha = \operatorname{atan2}(y_c, x_c)$$

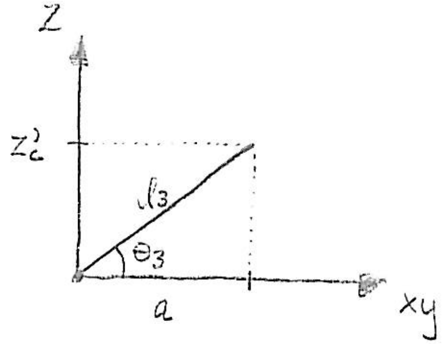
$$r = x_c^2 + y_c^2$$

$$a^2 = r^2 - d_{off}^2$$

$$\tan(\gamma) = \frac{d_{off}}{a}$$

$$\gamma = \operatorname{atan2}(d_{off}, a)$$

$$\theta_1 = \alpha + \gamma$$



$$\theta_3 = \operatorname{acos}(a, d_3)$$

$$\theta_2 = z_c - d_1 - d_2 - d_3 \sin(\theta_3)$$

INDIKASJON PÅ HVORDAN LØSE

Oppg. 4 i eksamen INF 3480 - 2009

Jacobian: $J = [J_1 \; J_2 \; J_3 \; J_4]$

- Se robotkonfigurasjon i fig 1.
- Bruk likningene på side 133 i læreboka. (4.56 - 59)
- For manipulatoren i fig 1 blir dette da:

$$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix}; \quad J_1 = \begin{bmatrix} z_0 \times (O_4 - O_0) \\ z_0 \end{bmatrix}; \; J_2 = \begin{bmatrix} z_1 \\ 0 \end{bmatrix}; \; J_3 = \begin{bmatrix} z_2 \times (O_4 - O_2) \\ z_2 \end{bmatrix}; \; J_4 = \begin{bmatrix} z_3 \times (O_4 - O_3) \\ z_3 \end{bmatrix}$$

- Fra oppg 2. har man mulighet til å finne:

$$T_1^0 = A_1; \; T_2^0 = A_1 \cdot A_2; \; T_3^0 = A_1 \cdot A_2 \cdot A_3; \; T_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4$$

For eksempel:

$$T_1^0 = A_1 = \begin{bmatrix} C_1 & -S_1 & \overbrace{0}^{z_1} & \overbrace{0}^{O_1} \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$T_2^0 = A_1 \cdot A_2 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

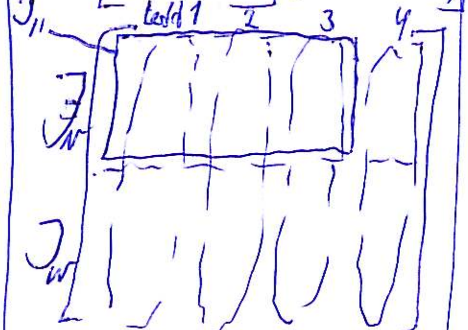
$$= \begin{bmatrix} -S_1 & 0 & \overbrace{C_1}^{z_2} & \overbrace{0}^{O_2} \\ C_1 & 0 & S_1 & 0 \\ 0 & 1 & 0 & l_1 + l_2 + q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_3^0 = A_1 \cdot A_2 \cdot A_3 \Rightarrow z_3$ og O_3

$T_4^0 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \Rightarrow z_4$ og O_4

Singularitetene:

$$J = \begin{bmatrix} J_{v_i} \\ J_{w_i} \end{bmatrix} = \begin{bmatrix} J_{v_1} J_{v_2} J_{v_3} J_{v_4} \\ J_{w_1} J_{w_2} J_{w_3} J_{w_4} \end{bmatrix}$$



Finn $Det(J_{11}) = 0$

Fordi: $J = [\underbrace{J_{Pos}}_{J_{v_1}, J_{v_2}, J_{v_3}} : J_{Orient}]$

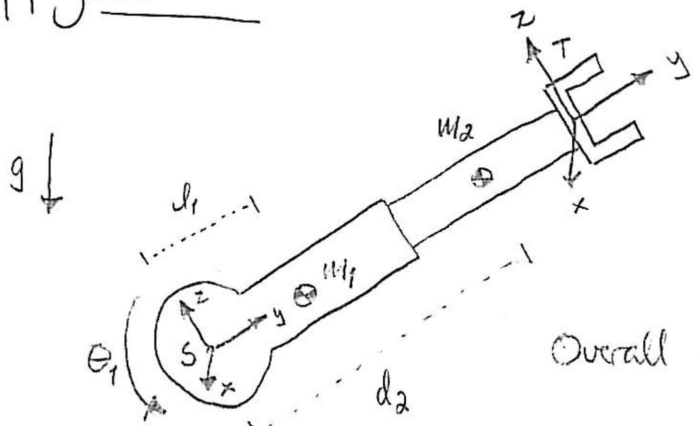
- Foreta så matrisesubtraksjonene $\Rightarrow O_4 - O_0, \; O_4 - O_2, \; O_4 - O_3$
- Til slutt kryssproduktene og sett inn i Jacobian.

Oppgave 4

With MATLAB the Jacobian (spatial) is given as ('Oppgave4_Jacobian.m')

$$J_{st}^{s}(q) = \begin{bmatrix} 0 & 0 & -(d_1+d_2+q_2)\sin(q_1) & -(d_1+d_2+q_2)\cos(q_1)\cos(q_3) + d_{off}\sin(q_1)\sin(q_3) \\ 0 & 0 & (d_1+d_2+q_2)\cos(q_1) & -(d_1+d_2+q_2)\sin(q_1)\cos(q_3) - d_{off}\cos(q_1)\sin(q_3) \\ 0 & 1 & 0 & d_{off}\cos(q_3) \\ 0 & 0 & \cos(q_1) & -\sin(q_1)\cos(q_3) \\ 0 & 0 & \sin(q_1) & \cos(q_1)\cos(q_3) \\ 1 & 0 & 0 & \sin(q_3) \end{bmatrix}$$

Oppgave 5



Forward kinematics:

$$g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Twists:

Joint 1: $\omega_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $q_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\xi_1 = \begin{bmatrix} -\omega_1 \times q \\ \omega_1 \end{bmatrix}$

Joint 2: $\xi_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Overall forward kinematics:

$$g_{st}(\theta_1, d_2) = e^{\hat{\xi}_1 \theta_1} e^{\hat{\xi}_2 d_2} g_{st}(0)$$

$$\Rightarrow g_{st}(\theta_1, d_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_1) & -\sin(\theta_1) & d_2\cos(\theta_1) \\ 0 & \sin(\theta_1) & \cos(\theta_1) & d_2\sin(\theta_1) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The dynamics of the robot are computed with the MATLAB script 'Oppgave5_Dynamics.m'.

Jacobian: $J = [J_1 \; J_2 \; J_3 \; J_4]$

$$J_1 = \begin{bmatrix} z_0 \times (o_4 - o_0) \\ z_0 \end{bmatrix}, \quad J_2 = \begin{bmatrix} z_1 \\ 0 \end{bmatrix}, \quad J_3 = \begin{bmatrix} z_2 \times (o_4 - o_2) \\ z_2 \end{bmatrix}, \quad J_4 = \begin{bmatrix} z_3 \times (o_4 - o_3) \\ z_3 \end{bmatrix}$$

J_{11}

$$J = \left[\begin{array}{cc:c|c} -(l_3+l_4)\cos(q_1)\cos(q_3) - d_{off}\sin(q_1) & 0 & (l_3+l_4)\sin(q_1)\sin(q_3) & 0 \\ -(l_3+l_4)\sin(q_1)\cos(q_3) + d_{off}\cos(q_1) & 0 & -(l_3+l_4)\cos(q_1)\sin(q_3) & 0 \\ 0 & 1 & (l_3+l_4)\cos(q_3) & 0 \\ \hdashline 0 & 0 & \cos(q_1) & -\sin(q_1)\cos(q_3) \\ 0 & 0 & \sin(q_1) & \cos(q_1)\cos(q_3) \\ 1 & 0 & 0 & \sin(q_3) \end{array}\right]$$

$$\det(J_{11}) = -(l_3+l_4)^2 \sin(q_3)\cos(q_3)$$

dvs. vi har singulære konfigurasjoner for $q_3 = 0$, $q_3 = \pi/2$, $q_3 = \pi$