UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: Introduction to robotics (INF 3480)
Day of exam: 28th of may at 09:00
Exam hours: 09:00 – 12:00 (3 hours)
This examination paper consists of 4 page(s).
Appendices: None
Permitted materials:

- **a.** Mark W. Spong, Seth Hutchinson, M. Vidyasagar: *Robot Modeling and Control*, 2005. Wiley. ISBN: 978-0-471-64990-8.
- **b.** Karl Rottmann, Matematisk Formelsamling

Make sure that your copy of this examination paper is complete before answering.

Quest. 1	10%
Quest. 2	20%
Quest. 3	20%
Quest. 4	20%
Quest. 5	25%
Quest. 6	5%
totalt:	100%

1) (10%)
   a. Make a figure that shows the main elements that a robotic system typically can consist of. Explain with few sentences each element of such a system.
   b. Explain what a robots degrees of freedom tell us. How many degrees of freedom are necessary to reach any given position with an arbitrary orientation within the workspace of the robot?
   c. Prismatic joints are denoted by P and rotational joints by R. Which five different combinations of the first three joints for tool positioning are most common within robotic systems? What name do these combinations have and how are they characterized?
   d. How would you derive the orientation of the tool coordinate frame of a robot with reference to the robot base coordinate system in terms of a rotation matrix based on ZYZ-Euler representation?
   e. How is a total rotation matrix calculated out of a set of sucessive rotations about axes in the current coordinate system and fixed axes in the base coordinate system respectively?

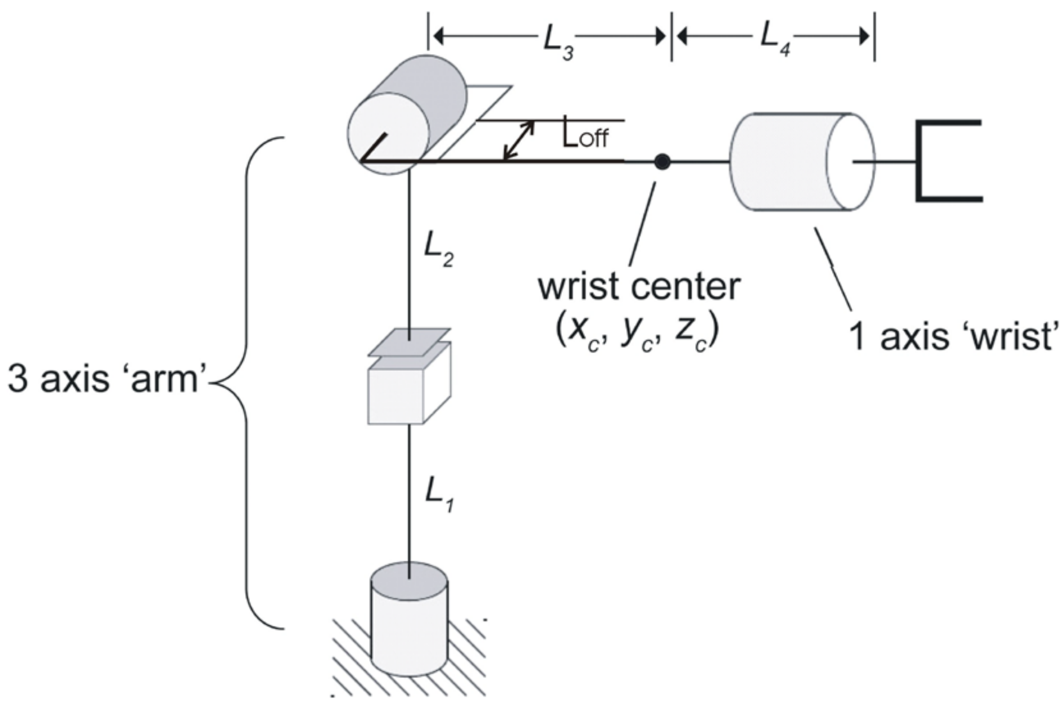

Figure 1: 4DOF robot

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2) (20%) Write the homogeneous transformation that represents the forward kinematics of the system shown in figure 1 above using the Denavit-Hartenberg convention. Label all origins, links, and joints as appropriate (label the joint variables q_1 through q_4 starting at the base). Make sure that you include your DH table.
3) (20%) Assume that you can kinematically decouple the arm in figure 1 in a position kinematics and an orientation kinematics.
   a. Write an expression for the inverse *position* kinematics. Be sure to use

the wrist center as defined in figure 1.

b. Without concern for joint limitations, how many solutions are there to the inverse position kinematics? Explain each case including considerations of the workspace boundary.

c. Find restrictions on q_2 and q_3 that ensure that there is only one solution to the inverse position kinematics (assuming that the desired wrist center position is always strictly inside the workspace).

4) (20%) Write an expression for the Jacobian of the system in figure 1 and determine the singular configurations and prove that they are singular based upon your Jacobian.

5) (25%) What is understood by a systems dynamics? Explain why it is important to know the dynamics of a robotic system in order to control it.

a. The Euler-Lagrange equation of motion is:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \tau_j$$

Explain the different elements of this equation.

b. Show/explain how the Euler-Lagrange equation, shown in question 5a, for a robotic system can be written on the form:

$$\frac{d}{dt}\frac{\partial k}{\partial \dot{q}_j} - \frac{\partial k}{\partial q_j} + \frac{\partial p}{\partial q_j} = \tau_j$$

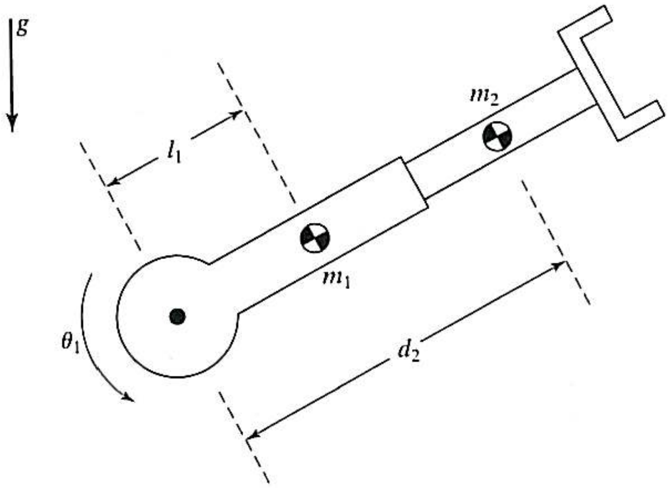
c. Derive the equations of motion for the robot in figure 2 by using the equation from question 5b. (Assume frictionless joints, and that the joint variables are θ_1 and d_2, centre of mass (C_1 og C_2- marked with circular black and white spot) to each arm link with a total mass of m_1 and m_2 respectively, the inertia tensor of link 1 and 2 is diagonal and expressed relative to the respective centre of masses as shown in figure 2)

d. Derive the equations of motion from question 5c in a compact matrix form as shown below. Show the details of the matrix D, V and G separately. What do the D, V and G matrix in physical terms express in the dynamic equation of motion?

$$D(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau$$

$V(q,\dot{q})$ equals $C(q,\dot{q})\dot{q}$

Explain how you would derive the C matrix isolating the first derivative of the joint angle as in $C(q,\dot{q})\dot{q}$?



$$^{C_1}I_1 = \begin{bmatrix} I_{xx1} & 0 & 0 \\ 0 & I_{yy1} & 0 \\ 0 & 0 & I_{zz1} \end{bmatrix},$$

$$^{C_2}I_2 = \begin{bmatrix} I_{xx2} & 0 & 0 \\ 0 & I_{yy2} & 0 \\ 0 & 0 & I_{zz2} \end{bmatrix},$$

Figure 2: 2DOF robot

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6) (5%) Derive the Laplacetransform of the equation of motion for the system in question 5d given on the form: $D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$

   Show schematically the closed loop control system using the Laplace transformed equations of motion and PDI feedback control. What is meant by a Single-Input Single-Output system (SISO-system). What makes the system at hand generally unsuited to be controlled as a Single-Input Single-Output system (SISO-system)?