UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in INF3480 – Introduction to Robotics
Day of exam: 8th June, 2015
Exam hours: 14:30, 4 hours
This examination paper consists of 3 page(s).
Appendices: None
Permitted materials:
Spong, Hutchinson and Vidyasagar, ***Robot Modeling and Control***, **2005**
Karl Rottman, Matematisk formelsamling (all editions)
Approved calculator

Make sure that your copy of this examination paper is complete before answering.

Exercise 1 (20 %)

a) (5 %) What are the benefits of using ROS and what does it provide?

b) (5 %) Describe pluses and minuses of the following locomotion techniques: wheels, legs, flying, swimming. What wheeled robot configurations do you know (you describe in writing and/or draw them)?

c) (5 %) What are the main differences between programmed robot using conventional methods and evolutionary robotics approach?

d) (5 %) Explain the basics of a closed loop system with PID control. Briefly explain the meaning of P, I and D (explain using words).

Exercise 2 (50 %)

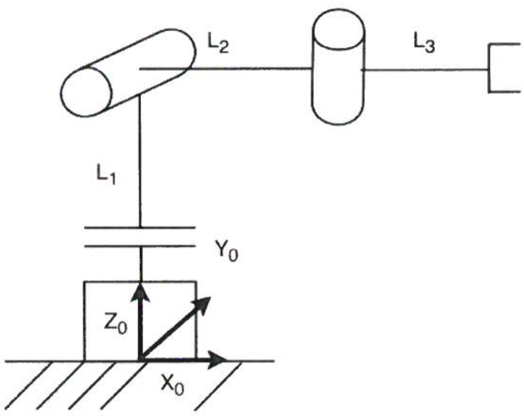


Figure 1:

Figure 1 shows the robot configuration that is being used. Joint constraints are the following: D1 = 0 to 30, theta2 = -135 deg to +135 deg, theta3 = -180 deg to +180 deg.

a) (10 %) Assign coordinate frames on the robot in Figure 1 using Denavit-Hartenberg convention. Write the Denavit-Hartenberg parameters in a table.

b) (5 %) Derive the forward kinematics for the robot from the base coordinate system to the tool coordinate system at the tip of the robot.

c) (10 %) Derive the Jacobian for the robot.

d) (10 %) Derive the inverse kinematics for the robot.

e) (5 %) Given L1 = 30, L2 = 20, L3 = 10, calculate joint variables needed to reach the following position P(25;20;8). Discuss the result.

f) (5 %) Describe and draw workspace of the robot shown in Figure 1. What are possible issues regarding reachability?

g) (5 %) Redesign by replacing one component, and draw the new robot and it's workspace to improve the reachability.

Exercise 3 (30 %)

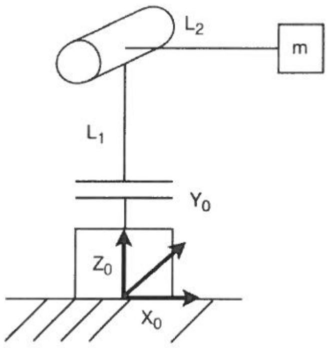


Figure 2:

Figure 2 shows a robot with two degrees of freedom. This a simplification of the robot in exercise 2. Assume that the only mass is a point mass of m at the tool.

a) (7.5 %) Find the Lagrangian $\mathcal{L}$ of the robotic system in Figure 2.

b) (7.5 %) Derive the dynamic equations for the robot using the Euler-Lagrange formulation.

For the rest of the exercise we assume that L_2 is fixed and approximate $\sin\theta_1$ to θ_1 to get the dynamic equation on the following form

$$J\ddot{\theta} + b\dot{\theta} + k\theta = \tau$$

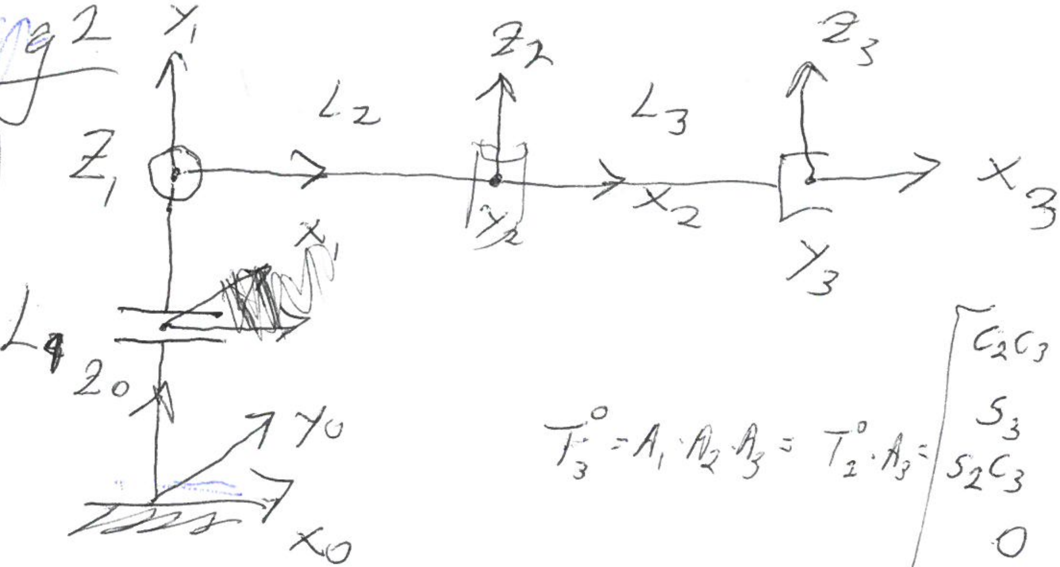
c) (7.5 %) Find the J, b and k and then transform the dynamic equation into the Laplace domain.

d) (7.5 %) Draw a closed-loop block diagram of the system using a PI-controller that has the desired angle θ_d as a setpoint.

~~Ödev~~ III

Ödg 2

a)



$$T_3^0 = A_1 \cdot A_2 \cdot A_3 = T_2^0 \cdot A_3 = \begin{bmatrix} c_2c_3 & -c_2s_3 & -s_2 & L_3c_2c_3 + L_2c_2 \\ s_3 & c_3 & 0 & L_3s_3 \\ s_2c_3 & -s_2s_3 & c_2 & L_3s_2c_3 + L_2s_2 + L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	a_i	α_i	d_i	θ_i
1	0	90	L_1^*	0
2	L_2	-90	0	θ_2^*
3	L_3	0	0	θ_3^*

b)

$$T_1^0 = A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & L_2c_2 \\ s_2 & 0 & c_2 & L_2s_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3c_3 \\ s_3 & c_3 & 0 & L_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad T_2^0 = A_1 \cdot A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & L_2c_2 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & L_2s_2 + L_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2)

c)

$$\begin{array}{cc} & \begin{array}{ccc} i: 1 & 2 & 3 \end{array} \\ \begin{array}{c} J_v \\ \\ J_w \end{array} & \begin{bmatrix} z_0 & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ \\ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} & z_1 & z_2 \end{bmatrix} \end{array}$$

$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \; z_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$z_2 = \begin{bmatrix} -S_2 \\ 0 \\ C_2 \end{bmatrix}; \; O_1 = \begin{bmatrix} 0 \\ 0 \\ L_1^* \end{bmatrix}$

$O_2 = \begin{bmatrix} L_2 C_2 \\ 0 \\ L_2 S_2 + L_1^* \end{bmatrix}$

$O_3 = \begin{bmatrix} L_3 C_2 C_3 + L_2 C_2 \\ L_3 S_3 \\ L_3 S_2 C_3 + L_2 S_2 + L_1^* \end{bmatrix} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$

$$\begin{matrix} i & j & k & i & j \\ 0 & -1 & 0 & 0 & -1 \\ A_x & A_y & A_z - L_1^* & A_x & A_y \end{matrix}$$

$\Rightarrow \begin{bmatrix} L_1^* - A_z \\ 0 \\ A_x \end{bmatrix} = z_1 \times (O_3 - O_1)$

$$\begin{matrix} i & j & k & i & j \\ -S_2 & 0 & C_2 & -S_2 & 0 \\ L_3 C_2 C_3 & L_3 S_3 & L_3 S_2 C_3 + L_1^* & L_3 C_2 C_3 & 0 \end{matrix}$$

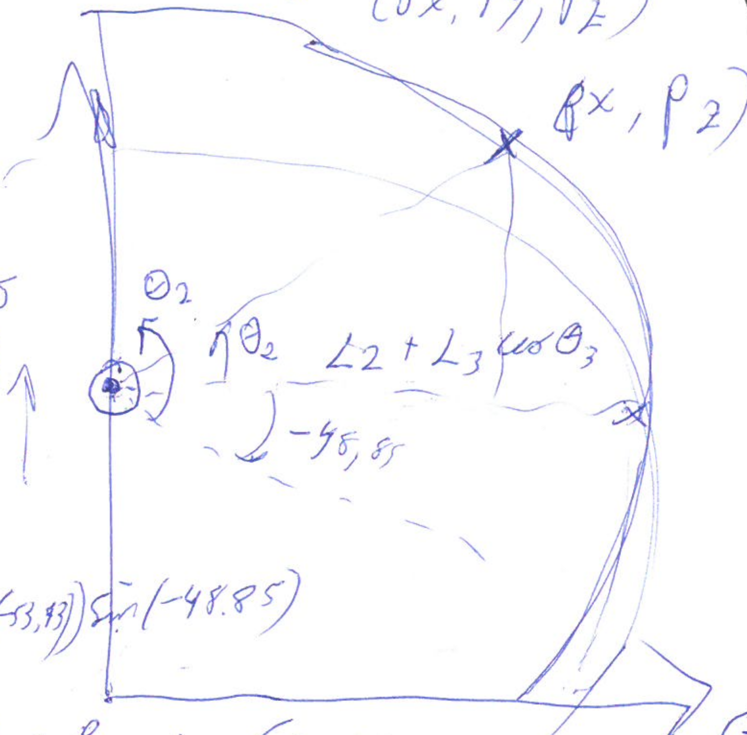
$\begin{bmatrix} -L_3 C_2 S_3 \\ L_3 C_2^2 S_3 + S_2 (L_3 S_2 C_3 + L_1^*) \\ 0 \end{bmatrix}$

$$J = \begin{bmatrix} J_v \\ \\ J_w \end{bmatrix} = \left[\begin{array}{c|c|c} 0 & \cancel{L_1^*} - L_3 S_2 C_3 - L_2 S_2 - \cancel{L_1^*} & -L_3 C_2 S_3 \\ 0 & 0 & L_3 C_2^2 S_3 + S_2 (L_3 S_2 C_3 + L_1^*) \\ 1 & L_3 C_2 C_3 + L_2 C_2 & 0 \\ \hline 0 & 0 & -S_2 \\ 0 & -1 & 0 \\ 0 & 0 & C_2 \end{array} \right]$$

d) Z

$P = (P_x, P_y, P_z)$

P_x, P_z



$\theta_1 = 19,58$
$\theta_2 = -\cancel{48},85$
$\theta_3 = -53,13$

$P_z = 30 + 19,58 + (20 + 10 \cdot \cos(-53,13)) \sin(-48.85)$

$P_z = \underline{30}$

$$\boxed{P_x = L_3 C_2 C_3 + L_2 C_2} \quad \text{OK}$$
$$\Rightarrow C_2 (L_3 C_3 + L_2) = P_x$$
$$\Rightarrow \cos\theta_2 = \frac{P_x}{L_3 C_3 + L_2} \quad \text{II}$$

L_3 θ_1

$L_1 = 30$
$L_2 = 20$; $L_3 = 10$

$P = (25, 8, 30)$

$\theta_3 = \underline{53,13°}$ ou $\underline{-53,13°}$

$\theta_2 = \pm \underline{48,85°}$

$\theta_1 = P_z - L_1 - (L_2 + L_3 \cos\theta_3) \cdot \sin\theta_2$: $-19,58$; $19,58$; $-19,58$; $19,58$

$P_z = L_1^{*} (+\theta_1) + (L_2 + L_3 \cos\theta_3) \cdot \sin\theta_2$

$$\boxed{\Rightarrow (L_2 + L_3 \cos\theta_3) = \frac{P_z - L_1}{\sin\theta_2}} \quad \text{I}$$

OK III

$\sin\theta_3 = \frac{P_y}{L_3}$

$\cos\theta_3 = \pm \sqrt{1 - \left(\frac{P_y}{L_3}\right)^2}$

$\Rightarrow \theta_3 = \pm \operatorname{atan}\left(\frac{P_y / L_3}{\sqrt{1 - \left(\frac{P_y}{L_3}\right)^2}}\right)$

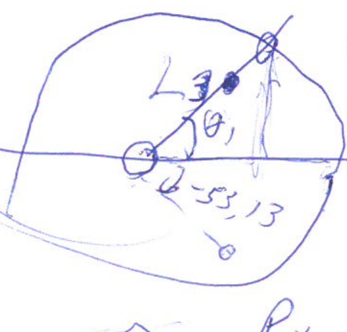
I

$P_z = L_3 S_2 C_3 + L_2 S_2 + L_1^{*}$

$\Rightarrow L_1^{*} = P_z - L_3 S_2 C_3 - L_2 S_2$

I $\boxed{P_y = L_3 \sin\theta_3} \Rightarrow \theta_3 = \operatorname{asin}\left(\frac{P_y}{L_3}\right)$

$P_x = \cancel{L_2 \cos\theta_2 + L_3}$
$(L_2 + L_3 \cos\theta_3) \cdot \cos\theta_2$



$$\boxed{\frac{P_x}{\cos\theta_2} = (L_2 + L_3 \cos\theta_3)} \quad \text{II}$$

$\frac{P_z}{P_x} =$

$$\Rightarrow \frac{P_x}{\cos\theta_2} = \frac{P_z - L_1}{\sin\theta_2} \Rightarrow \frac{\sin\theta_2}{\cos\theta_2} = \frac{P_z - L_1}{P_x}$$

$$\Rightarrow \theta_2 = \operatorname{atan}\left(\frac{P_z - L_1}{P_x}\right)$$

$\cos\theta_2 = \frac{P_x}{L_3 C_3 + L_2} \Rightarrow \theta_2 = \pm \text{atan2}\left(\frac{\frac{P_x}{L_3C_3+L_2}}{\sqrt{1-\left(\frac{P_x}{L_3C_3+L_2}\right)^2}}\right)$ (4)

e) $L_1^* = P_z - L_3 \sin\theta_2 \cos\theta_3 - L_2 \sin\theta_2$

$\sin\theta_3 = \frac{P_x}{L_3} \Rightarrow \theta_3 = \pm \text{atan2}\left(\frac{\frac{P_x}{L_3}}{\sqrt{1-\left(\frac{P_x}{L_3}\right)^2}}\right)$

$\left.\begin{array}{l} L_1 = 30 \\ L_2 = 20 \\ L_3 = 10 \end{array}\right\}$ L_1^*, θ_2 og θ_3 i punktet $P(25,20,8)$?

θ_3 : Kan ikke, komplekst tall ... neg. under kv. rot.

$\left(\frac{P_x}{L_3}\right)^2 < 1$

$\frac{P_x}{L_3} < 1 \Rightarrow L_3 > P_x$

$\Rightarrow$ Nytt punkt $P(25, 5, 8)$

$\theta_3 = \cancel{0,866} \pm \arctan(0,57735) \Rightarrow \theta_3 = \underline{\pm 30^\circ}$

$\theta_2 = \pm \text{atan2}\left(\frac{0,87228815}{0,35736794}\right) \Rightarrow \underline{\pm 67,72^\circ}$

$\theta_2 =$

θ_2 θ_3		
	$L_1^* = 8 - 10 \cdot \sin 67,72 \cdot \cos 30 - 20 \cdot \sin 67,72$	
+ +	$= 8 - 8,0137$	$-18,5070 = -18,52$
− +	$= 8 + 8,0137$	$+18,5070 = 34,52$
+ −		$= -18,52$
− −	$=$	$= \underline{34,52}$

$\Rightarrow \Delta L_1^* = 34,52 - L_1 = \underline{4,52}$

~~Oppg 3~~

Stryker elementer hvor θ_3 og L_3 er med

Ledd 3 tatt ut

a) $\boxed{\mathcal{L} = K - P}$

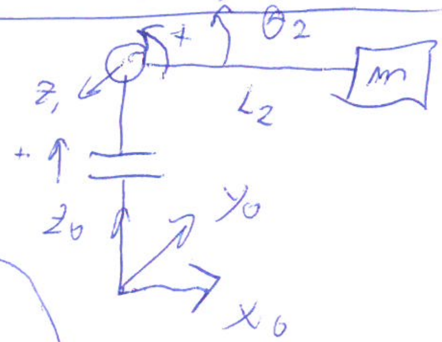
$$J_v = \begin{bmatrix} 0 & -\cancel{S_2} - L_2 S_2 & -C_2 \\ 0 & 0 & \\ 1 & \cancel{C_2} + L_2 C_2 & C_2^2 + S_2(S_2 + L_1^*) \end{bmatrix} \begin{bmatrix} \dot{L}_1^* \\ \dot{\theta}_2 \end{bmatrix}$$

$$v_x = -S_2(L_2 + \cancel{1})\dot{\theta}_2$$

$$v_y = 0$$

$$v_z = \dot{L}_1^* + C_2(L_2 + \cancel{1})\dot{\theta}_2$$

Se på robot:



$$\boxed{\begin{aligned} v_x &= -L_2 \sin\theta_2 \cdot \dot{\theta}_2 \\ v_y &= 0 \\ v_z &= \dot{L}_1^* + L_2 \cos\theta_2 \cdot \dot{\theta}_2 \end{aligned}}$$

$$h = L_1^* + L_2 \sin\theta_2$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = (L_2 \sin\theta_2)^2 \dot{\theta}_2^2 + \dot{L}_1^{*2} + (L_2 \cos\theta_2)^2 \cdot \dot{\theta}_2^2 + 2\dot{L}_1^* L_2 \cos\theta_2 \cdot \dot{\theta}_2$$

$$L_2^2 \dot{\theta}_2^2 \underbrace{(\sin^2\theta_2 + \cos^2\theta_2)}_{=1} = L_2^2 \cdot \dot{\theta}_2^2$$

$$\mathcal{L} = \frac{1}{2} m \left[L_2^2 \sin^2\theta_2 \cdot \dot{\theta}_2^2 + \dot{L}_1^{*2} + L_2^2 \cos^2\theta_2 \cdot \dot{\theta}_2^2 + 2\dot{L}_1^* L_2 \cos\theta_2 \cdot \dot{\theta}_2 \right]$$

$$- mg(L_1^* + L_2 \sin\theta_2)$$

$$= \frac{1}{2} m \left(\dot{L}_1^{*2} + L_2^2 \dot{\theta}_2^2 + 2\dot{L}_1^* L_2 \cos\theta_2 \cdot \dot{\theta}_2 \right) - mg(L_1^* + L_2 \sin\theta_2)$$

b)

$$\boxed{\tau_j = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j}}$$

$$L = \frac{1}{2}m\left(\dot{L}_1^{*2} + L_2^2\dot{\theta}_2^2 + 2\dot{L}_1^* \cdot L_2\cos\theta_2 \cdot \dot{\theta}_2\right) - mg\left(L_1^* + L_1\sin\theta_2\right)$$

$$\frac{\partial L}{\partial \dot{L}_1^*} = m\dot{L}_1^* + mL_2\cos\theta_2 \cdot \dot{\theta}_2 \qquad \frac{\partial L}{\partial L_1^*} = -mg$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = mL_2^2\dot{\theta}_2 + mL_2\dot{L}_1^*\cos\theta_2 \qquad \frac{\partial L}{\partial \theta_2} = -m\dot{L}_1^*\dot{\theta}_2\sin\theta_2 - mgL_2\cos\theta_2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{L}_1^*} = m\ddot{L}_1^* + mL_2\cos\theta_2 \cdot \ddot{\theta}_2 - mL_2\sin\theta_2 \cdot \dot{\theta}_2^2$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = mL_2^2\ddot{\theta}_2 - mL_2\dot{L}_1^* \cdot \sin\theta_2 \cdot \dot{\theta}_2$$

$$\tau_1 = m\ddot{L}_1^* + mL_2\cos\theta_2 \cdot \ddot{\theta}_2 - mL_2\sin\theta_2 \cdot \dot{\theta}_2^2 + mg$$

$$\tau_2 = mL_2^2\ddot{\theta}_2 - mL_2\dot{L}_1^*\sin\theta_2 \cdot \dot{\theta}_2 + m\dot{L}_1^*\dot{\theta}_2\sin\theta_2 + mgL_2\cos\theta_2$$

$$\begin{bmatrix}\tau_1 \\ \tau_2\end{bmatrix} = \begin{bmatrix} m & mL_2\cos\theta_2 \\ 0 & mL_2^2 \end{bmatrix}\begin{bmatrix}\ddot{L}_1^* \\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix}\dot{L}_1^* \\ \dot{\theta}_2\end{bmatrix}$$