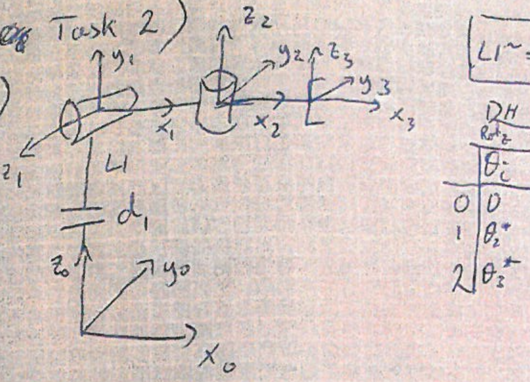
Task 2)

a)



$\boxed{L1^{\sim} = L1 + d_1^{+}}$

DH - table:

	Rot z	Trans z	Trans x	Rot x
	θ_i	d_i	a_i	α_i
0	0	$L1^{\sim}$	0	90
1	θ_2^{*}	0	L2	-90
2	θ_3^{*}	0	L3	0

b)

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L1^{\sim} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & L2c_2 \\ s_2 & 0 & c_2 & L2s_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L3c_3 \\ s_3 & c_1 & 0 & L3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1A_2 = \begin{bmatrix} c_2 & 0 & -s_2 & L2c_2 \\ 0 & 1 & 0 & 0 \\ s_2 & 0 & c_2 & (L2s_2 + L1^{\sim}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1A_2A_3 = \begin{bmatrix} c_2c_3 & -c_2s_3 & -s_2 & L3c_3c_2 + L2c_2 \\ s_3 & c_3 & 0 & L3s_3 \\ c_3s_2 & -s_2s_3 & c_2 & (L3c_3s_2 + L2s_2 + L1^{\sim}) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Jacobian. 2 solutions using the cross product or take the partial derivative of the position.

$$J_v = \frac{\partial O_n}{\partial q_i} \qquad \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} L3C_3C_2 + L2C_2 \\ L3S_3 \\ L3C_3S_2 + L2S_2 + L1 \end{bmatrix}$$

$$J_v = \begin{bmatrix} \frac{\partial p_x}{\partial L1} & \frac{\partial p_x}{\partial \theta_2} & \frac{\partial p_x}{\partial \theta_3} \\ \frac{\partial p_y}{\partial L1} & \frac{\partial p_y}{\partial \theta_2} & \frac{\partial p_z}{\partial \theta_3} \\ \frac{\partial p_z}{\partial L1} & \frac{\partial p_z}{\partial \theta_2} & \frac{\partial p_z}{\partial \theta_3} \end{bmatrix}$$

this gives:

$$J_v = \begin{bmatrix} 0 & -L3C_3S_2 - L2S_2 & -L3S_3C_2 \\ 0 & 0 & L3C_3 \\ 1 & L3C_3C_2 + L2C_2 & -L3S_3S_2 \end{bmatrix}$$

$$J_\omega = \begin{cases} Z_i & \text{if revolute} \\ 0 & \text{if prismatic} \end{cases}$$

$$J_\omega = \begin{bmatrix} 0 & 0 & -S_2 \\ 0 & -1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \qquad J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

d) We have the position of the end-effector described as

(1) $p_x = L3c_3c_2 + L2c_2$

(2) $p_y = L3\,s_3$

(3) $p_z = L3c_3s_2 + L2s_2 + L1 + d_1^*$

from (2) we can express θ_3 which is the only joint that has any impact of the position in y-direction.

$$s_3 = \frac{p_y}{L3}, \quad c_3 = \sqrt{1-\left(\frac{p_y}{L3}\right)^2}$$

$$\underline{\theta_3 = \operatorname{atan2}\left(\frac{p_y}{L3}, \sqrt{1-\left(\frac{p_y}{L3}\right)^2}\right)}$$

this makes (1) solvable:

$$c_2(L3c_3 + L2) = p_x$$

$$c_2 = \frac{p_x}{(L3c_3 + L2)}, \quad s_3 = \sqrt{1-\left(\frac{p_x}{(L3c_3+L2)}\right)^2}$$

$$\underline{\theta_2 = \operatorname{atan2}(s_3, c_2)}$$

which leads to (3)

$$\underline{d_1 = p_z - L3c_3s_2 - L2s_2 - L1}$$

e) $\theta_2 = \operatorname{atan2}\left(\frac{20}{10}, \sqrt{1-2^2}\right) = \operatorname{atan2}(2, \sqrt{-3})$

this equation is undefine since the point is out of the manipulators workspace

in y-direction the point must lies inside

$y \in [-L3, L3]$ | $\frac{P_y}{L3} \leq 1$

which implies that

$L3 \leq P_y$

f) $\theta_3 = \operatorname{atan2}\left(\frac{5}{10}, \sqrt{1-\left(\frac{5}{10}\right)^2}\right) = 30°$

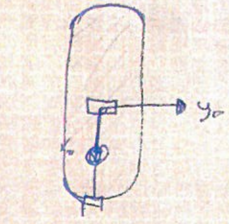
$\theta_2 = \operatorname{atan2}\left(\sqrt{1-\left(\frac{25}{10c30+20}\right)^2}, \frac{25}{10c30+20}\right) = 29.27°$

$d_1 = 8 - 10\cos(30)\sin(29.27) - 20 \cdot \sin(29.27) - 30$
$= \underline{-36}$

this means that the position is unreachable due to the limitations of d_1

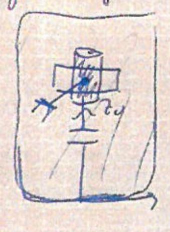
9) Area of reachability is marked

from top: x_0-y_0 plane



from side x_0-z_0 plane

from front : y_0-z_0 plane



The volume is still a bit small since θ_3 is the only factor in y_0. ~~By switching~~ By replacing θ_1 with a revolute joint we can achieve a bigger volume.

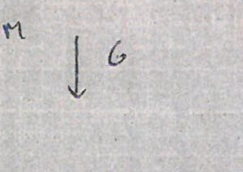
The configuration of the robot can result in 2 points thats aside eachother to have completely different joint properties. this makes trajectory/path planning hard for this robot.

Task 3)

a) can use the jacobian from (2c) with θ_3 joints set to 0 ~~noch~~ ~~...~~

~~...~~ $v = J_v \cdot \dot{q}$ $\Rightarrow v^2 = v^T \cdot v$

v_r:



Position:

$x = L2 \cdot \cos(\theta_2)$

$y = L1 + d_1 + L2 \sin(\theta_2)$

velocity is given by the time derivative of the position:

$v_x = -\dot{\theta}_2 L2 \sin(\theta_2)$

$v_y = \dot{d}_1 + \dot{\theta}_2 L2 \cos(\theta_2)$

$v^2 = v_x^2 + v_y^2 = \dot{d}_1^2 + 2\dot{d}_1 \dot{\theta}_1 L2 \cos(\theta_2) + \dot{\theta}_2^2 L2^2$

$K = \frac{1}{2} m (\dot{d}_1^2 + 2\dot{d}_1 \dot{\theta}_1 L2 \cos(\theta_2) + \dot{\theta}_2^2 L2^2)$

$P = m \cdot g (L1 + d_1 + L2 \sin(\theta_2))$

$\mathcal{L} = \frac{1}{2} m (\dot{d}_1^2 + 2\dot{d}_1 \dot{\theta}_2 L2 \cos(\theta_2) + \dot{\theta}_2^2 L2^2) - mg(L1 + d_1 + L2 \sin(\theta_2))$

b) $\frac{d}{dt}\frac{\partial L}{\partial \dot{d}_1} = \frac{d}{dt}\left(m\dot{d}_1 + m\dot{\theta}_2 L2\cos(\theta_2)\right) = m\ddot{d}_1 + \ddot{\theta}_2 L2 c_2 m - \dot{\theta}_2 L2 s_2 m$

$\frac{\partial L}{\partial d_1} = -mg$

$\underline{\tau_1 = m\ddot{d}_1 + \ddot{\theta}_2 L2 c_2 m - \dot{\theta}_2 L2 s_2 m - (-mg)}$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} = \frac{d}{dt}\left(m\dot{d}_1 L2\cos(\theta_2) + m\dot{\theta}_2 L2^2\right) = m\ddot{d}_1 L2 c_2 - m\dot{d}_1\dot{\theta}_2 L2 s_2 + m\ddot{\theta}_2 L2^2$

$\frac{\partial L}{\partial \theta} = -\dot{d}_1 \dot{\theta}_2 L2 s_2 m - mgL2c_2$

$\tau_2 = m\ddot{d}_1 L2 c_2 - \cancel{m\dot{d}_1\dot{\theta}_2 L2 s_2} + m\ddot{\theta}_2 L2^2 - (-\cancel{m\dot{d}_1\dot{\theta}_2 L2 s_2} - mgL2c_2)$

$\underline{= m\ddot{d}_1 L2 c_2 + m\ddot{\theta}_2 L2^2 + mgL2c_2}$

c) $c_2 \approx s_2$ Siden d_1 er konstant ser vi bort fra τ_1 $\Rightarrow$ $\dot{d}_1 = 0$, $\ddot{d}_1 = 0$

Vi får da:

$\tau_2 = 0 + m\ddot{\theta}_2' L2^2 + mgL2\theta_2'$

$\cancel{\ddot{\theta}_2' J + \dot{\theta}' b + k}$

$mL2^2 \cdot \ddot{\theta}_2' + mgL2 \cdot \theta_2' = \tilde{\tau}$

This result in

$J = mL2^2$ and $b = 0$ and $K = mgL2$

$J\ddot{\theta}' + k\dot{\theta}' = \tau$

$\mathcal{L}\{\tau\} = J\mathcal{L}\{\ddot{\theta}'\} + k \cdot \mathcal{L}\{\dot{\theta}'\}$

$\tau(s) = Js^2\theta(s) + k\theta(s)$

d) PI-Controller is described as:

~~[illegible]~~

Look at the highest nominator of $\tau(s)$

$Js^2\theta(s) = \tau(s) - k\theta(s)$

then draw the block diagram: