5 $$x(s) = \int_0^\infty e^{-st}\, x(t)\, dt \qquad (1)$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \int_0^\infty e^{-st}\, \frac{dx}{dt}\, dt$$

$$\int u'v = uv - \int uv'$$

$$u' = \frac{dx}{dt} \qquad u = x$$

$$v = e^{-st} \qquad v' = -s\, e^{-st}$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = e^{-st}\, x(t)\Big|_0^\infty - \int_0^\infty -s\, e^{-st}\, x(t)\, dt$$

$$= \underbrace{e^{-\infty t}\, x(\infty)}_{0} - \underbrace{e^{-0t}}_{1}\, x(0) + s\underbrace{\int_0^\infty e^{-st}\, x(t)\, dt}_{\text{se } (1) = x(s)}$$

$$= s\, x(s) - x(0)$$

6-7 $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$

$$\Downarrow \mathcal{L}$$

$$m\mathcal{L}\{\ddot{x}(t)\} + b\mathcal{L}\{\dot{x}(t)\} + k\mathcal{L}\{x(t)\} = \mathcal{L}\{f(t)\}$$

$$\Downarrow$$

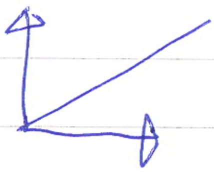
$$m\left(s^2X(s) - sx(0) - \dot{x}(0)\right) + b\left(sX(s) - x(0)\right) + kX(s) = F(s)$$

$$\Downarrow$$

$$(ms^2 + bs + k)X(s) = F(s) + \underbrace{m\dot{x}(0) + (ms+c)x(0)}_{\text{transient}}$$

$$\text{lim} \quad \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

9

		time	Laplace
step		1 if $t > 0$	$\frac{1}{s}$
rampe		ct	$\frac{c}{s^2}$
dirac-delta		$\delta(t)$	1

12

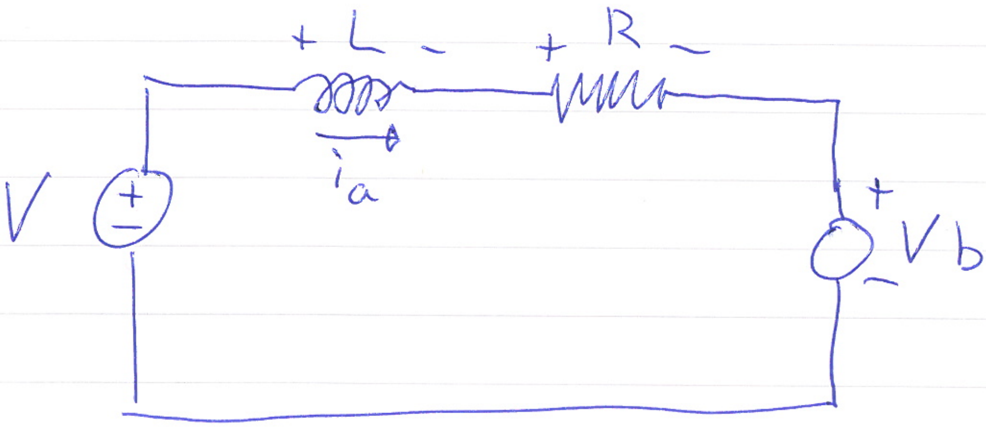
$$\tau = J\ddot{\theta} + B\dot{\theta}$$

$$\Downarrow \mathcal{L}$$

$$\tau = s^2 J \theta + s B \theta$$
$$= s(sJ + B)\theta$$

$$\frac{\theta}{\tau} = \frac{1}{s(sJ+B)}$$

16



$$V = L\frac{di_a}{dt} + Ri_a + V_b$$

$$L\frac{di_a}{dt} + Ri_a = V - V_b$$

$$V_b = K_2 \phi \omega_m$$

17 konstant flux: $K_m = K_1 \phi$

$K_b = K_2 \phi$

$\tau_m = K_m i_a$

18 $V_b = 0$ $\frac{di_a}{dt} = 0 \Rightarrow V = R i_a = \frac{R\tau_0}{K_m}$

19 $$\sum \tau = J\ddot{\theta}$$

$$J\ddot{\theta}_m = \tau_m - \tau_{damp} - \frac{\tau_L}{r} \qquad \tau_{damp} = B_m\dot{\theta}$$

$$J\ddot{\theta}_m + B\dot{\theta}_m = \tau_m - \frac{\tau_L}{r} \qquad (2)$$

$$= K_m i_a - \frac{\tau_L}{r}$$

20 Fra 16-17

$$L\frac{di_a}{dt} + Ri_a = V - k_b\dot{\theta}_m \qquad (1)$$

$$\Downarrow \mathcal{L}$$

$$LsI_a + RI_a = V - k_b s\Theta_m \qquad (1)$$

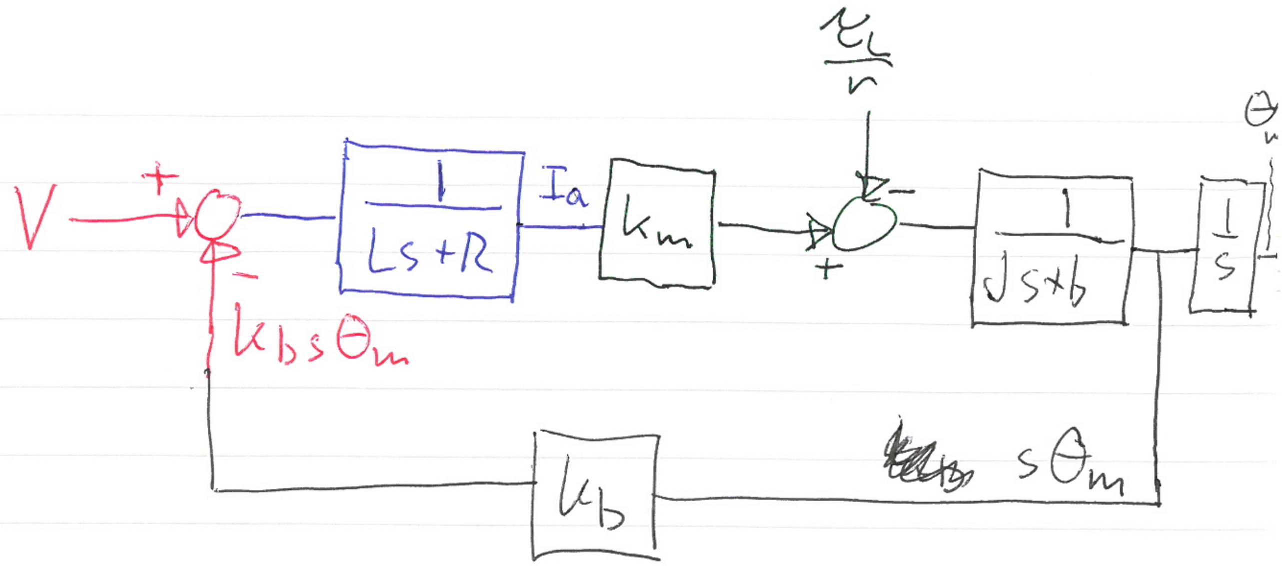
$$Js^2\Theta + Bs\Theta = k_m I_a - \frac{\tau}{r} \qquad (2)$$

$$\Downarrow$$

$$\underline{(Ls+R)I_a} = \underline{V - k_b s\Theta_m}$$

$$s(Js^{\cancel{2}} + B\cancel{s})\Theta_m = \underline{k_m I_a - \frac{\tau}{r}}$$

21



22

$$s(Js+B)\Theta_m = k_m I_a - \frac{\tau_L}{r}$$

$$\Downarrow \; \tau_L = 0$$

$$s(Js+B)\Theta_m = k_m I_a$$

$\Downarrow$ setter inn for I_a

$$s(Js+B)\Theta_m = k_m \frac{V - k_b s\Theta_m}{Ls+R}$$

$$\Downarrow$$

$$\frac{s(Js+B)(Ls+R)}{k_m}\Theta_m = V - k_b s\Theta_m$$

$$\frac{s(Js+B)(Ls+R) + k_m k_b s}{k_m}\Theta_m = V$$

$$\frac{\Theta_m}{V} = \frac{k_m}{s\left[(Js+B)(Ls+R) + k_m k_b\right]}$$

23

$$\cancel{s(J_m s + B)\theta_m = -\frac{\tau_L}{r}}$$

$$(Ls+R)I_a = \underset{0}{\underset{\shortparallel}{V}} - k_b s \Theta_m$$

$$\Downarrow$$

$$I_a = \frac{-k_b s \Theta_m}{(Ls+R)}$$

$\Downarrow$ setter inn for ~~Θ_m~~ I_a

$$\frac{s(sJ_m + B)\Theta_m + \frac{\tau_L}{r}}{K_m} = \frac{-k_b s \Theta_m}{(Ls+R)}$$

$$\Downarrow$$

$$s(sJ+B)\Theta_m = -\frac{K_m k_b s \Theta_m}{Ls+R} - \frac{\tau_L}{r}$$

$$\cancel{s(sJ+B)\Theta_m + K_m k_b s \Theta_m}$$

$$\Downarrow$$

$$\frac{s(sJ+B)(Ls+R)\cancel{\Theta_m} + K_m k_b s\cancel{\Theta_m}}{Ls+R}\Theta_m = -\frac{\tau_L}{r}$$

$$\Downarrow$$

$$\frac{\Theta_m}{\tau_L} = \frac{-(Ls+R)/r}{s\left[(sJ+B)(Ls+R) + K_m k_b\right]}$$

$$\frac{L}{R} = 0$$

24 $$\frac{\Theta_m}{V} = \frac{K_m}{s[(Ls+R)(Js+B)+k_b K_m]} \cdot \frac{\frac{1}{R}}{\frac{1}{R}}$$

$$= \frac{\frac{K_m}{R}}{s\left[(\underset{\overset{\shortparallel}{0}}{\frac{L}{R}}s+1)(Js+B)+\frac{k_b K_m}{R}\right]}$$

$$= \frac{\frac{K_m}{R}}{s\left[Js+B+\frac{k_b K_m}{R}\right]}$$

$$\frac{\Theta}{\tau_L} = \frac{-(Ls+R)/r}{s[(sJ+B)(sL+R)+K_m k_b]} \cdot \frac{\frac{1}{R}}{\frac{1}{R}}$$

$$= \frac{-(\frac{L}{R}s+1)/r}{s\left[Js+B+\frac{K_m k_b}{R}\right]}$$

$$= \frac{-\frac{1}{r}}{s\left[Js+B+\frac{K_m k_b}{R}\right]}$$

25 $$J\ddot{\theta} + (B_m + k_b k_m/R)\dot{\theta} = \frac{k_m}{R} V$$

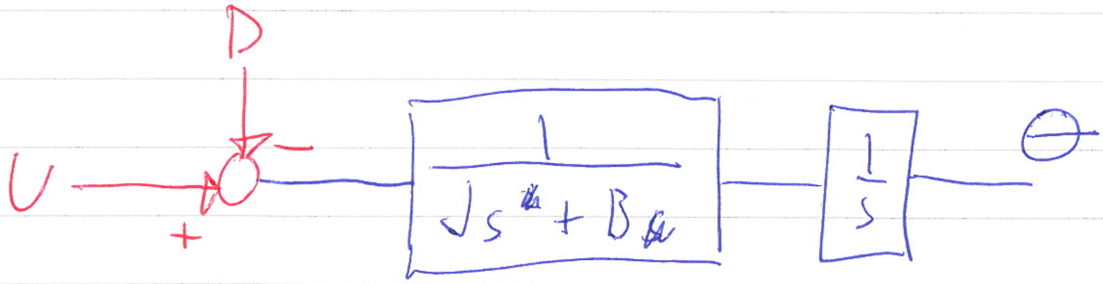
$$J\ddot{\theta} + (B_m + k_b k_m/R)\dot{\theta} = -\frac{1}{r} \tau_l$$

$$J\ddot{\theta} + B\dot{\theta} = \underbrace{\frac{k_m}{R} V}_{u(t)} - \underbrace{\frac{\tau_l}{r}}_{d(t)}$$

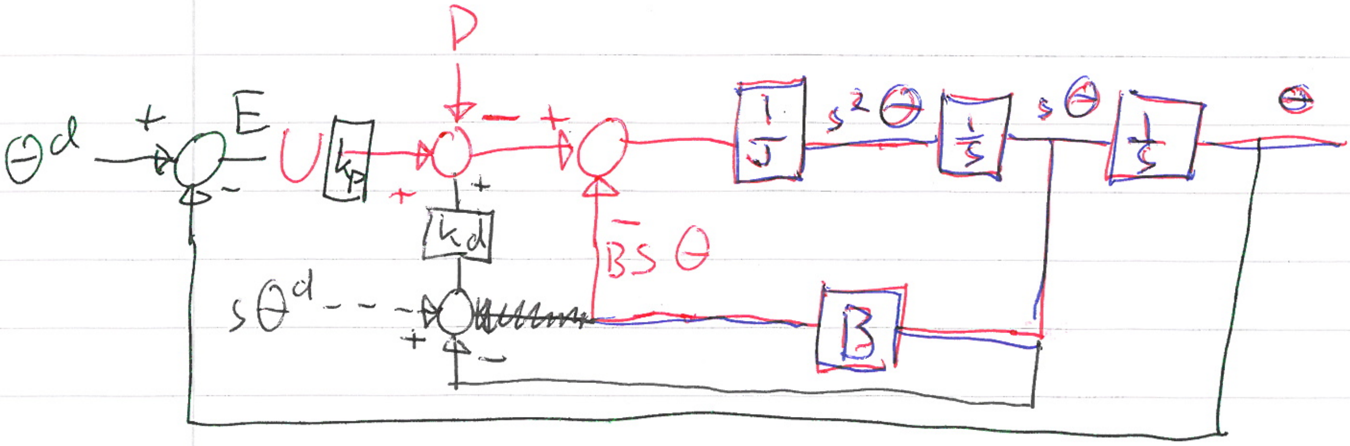
$$J\ddot{\theta} + B\dot{\theta} = u \mp d$$

$$\Downarrow \mathcal{L}$$

26 $$\underline{Js^2\Theta + Bs\Theta} = \underline{U \mp D}$$



$$Js^2\Theta = \underline{U - D - Bs\Theta}$$



grønn = P-kontroller

28 $u = K_p e \qquad e = \Theta^d - \Theta$

$$\Downarrow$$

$$\underline{U = k_p E}$$

$\boxed{\text{Simulering}}$

29 $u = K_p e + k_d e$

$$\Downarrow$$

$$U = K_p E + k_d s E$$

$$\underline{\underline{= (k_p + k_d s) E}}$$

30 $s(Js + B)\Theta = U - D$

$$U = (k_p + k_d s)(\Theta^d - \Theta)$$

$$\Downarrow$$

$$s(Js + B)\Theta = (k_p + k_d s)(\Theta^d - \Theta) - D$$

$$[s(Js + B) + (k_p + k_d s)]\Theta = (k_p + k_d s)\Theta^d - D$$

$$\Theta = \frac{(k_p + k_d s)\Theta^d - D}{\underline{s^2 J + (B + k_d)s + k_p}}$$

30 characteristish polynomial

$$s^2 + \frac{B+k_d}{J}s + \frac{k_p}{J} = 0$$

$$s^2 + 2\zeta\omega s + \omega^2 = 0$$

$$\Downarrow$$

$$2\zeta\omega \cancel{s} = \frac{B+k_d}{J} \Rightarrow k_d = 2\zeta\omega J - B$$

$$\omega^2 = \frac{k_p}{J} \Rightarrow k_p = \omega^2 J$$

$\zeta = 1$ $J = 1$ $B = 0,7$ $D = 0,5$

$\cancel{k_d}$ $k_p = 2\omega - 0,7$

$k_d = \omega^2$

$\omega =$	1	2
$k_p =$	1,3	3,3
$k_d =$	1	4

32 $\theta^d = \frac{C}{s}$ $D = \frac{D}{s}$ steg respons

sluttverditeoremet

$$\theta_{ss} = \lim_{t \to \infty} \theta(t) = \lim_{s \to 0} s\,\theta(s)$$

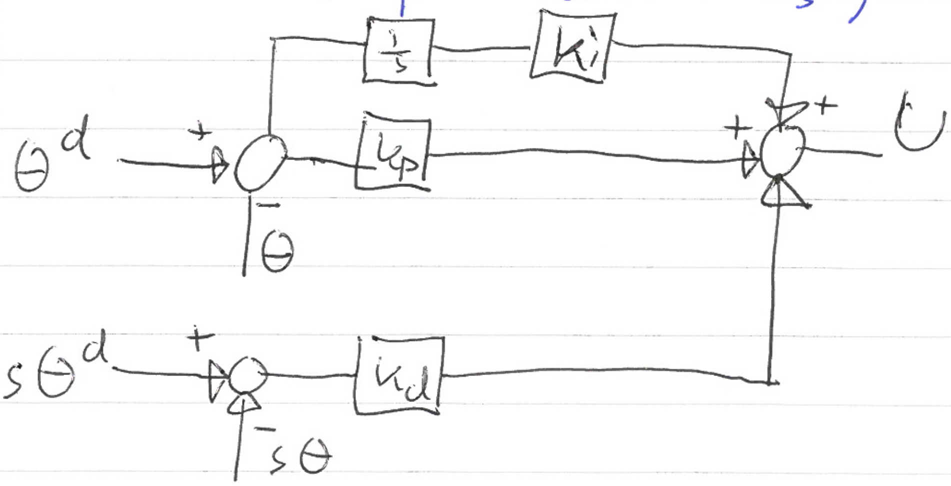
$$\lim_{s \to 0} s \frac{(k_p + k_d s)\frac{1}{s} C - \frac{1}{s} D}{s^2 J + (B + k_d)s + k_p} = \frac{k_p C - D}{k_p}$$

$$= C - \frac{D}{k_p}$$

33 $u = k_p e + k_d \dot{e} + k_i \int e\,dt$

$$\Downarrow$$

$$U = \left(k_p + k_d s + \frac{k_i}{s}\right) E$$



33 $$s(Js+B)\Theta = U - D$$

$$s(Js+B)\Theta = \left(k_p + k_d s + \frac{k_i}{s}\right)(\Theta^d - \Theta) - D \Big| \cdot s$$

$$s^2(Js+B)\cancel{\Theta} + \cancel{s}\left(sk_p + k_d s^2 + \frac{k_i}{\cancel{s}}\right)\Theta = (k_p s + k_d s^2 + k_i)\Theta^d - sD$$

$$\Theta = \frac{(k_d s^2 + k_p s + k_i)\Theta^d - sD}{Js^3 + (B+k_d)s^2 + k_p s + k_i}$$

34

$$\lim_{s \to 0} \cancel{s} \frac{(k_d s^2 + k_p s + k_i)\cancel{\frac{1}{s}} C - s\cancel{\frac{1}{s}} D}{Js^3 + (B+k_d)s^2 + k_p s + k_i}$$

$$= \frac{k_i C}{k_i} = C$$

35 $$Js^3 + (B+k_d)s^2 + k_p s + k_i = 0$$

$$\Downarrow$$

$$(s-\lambda_1)(s-\lambda_2)(s-\lambda_3) = 0$$