UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in **INF4380 – Introduction to Robotics**
Day of exam: **31th May, 2017**
Exam hours: **14:30, 4 hours**
This examination paper consists of 7 pages + 3 pages appendix.
Appendices: **Rules & Formulas INF3480/INF4380**
Permitted materials:

Spong, Hutchinson and Vidyasagar, *Robot Modeling and Control*, 2005
Karl Rottman, Matematisk formelsamling (all editions)
Approved calculator

Make sure that your copy of this examination paper is complete before answering.

Exercise 1 (20 %)

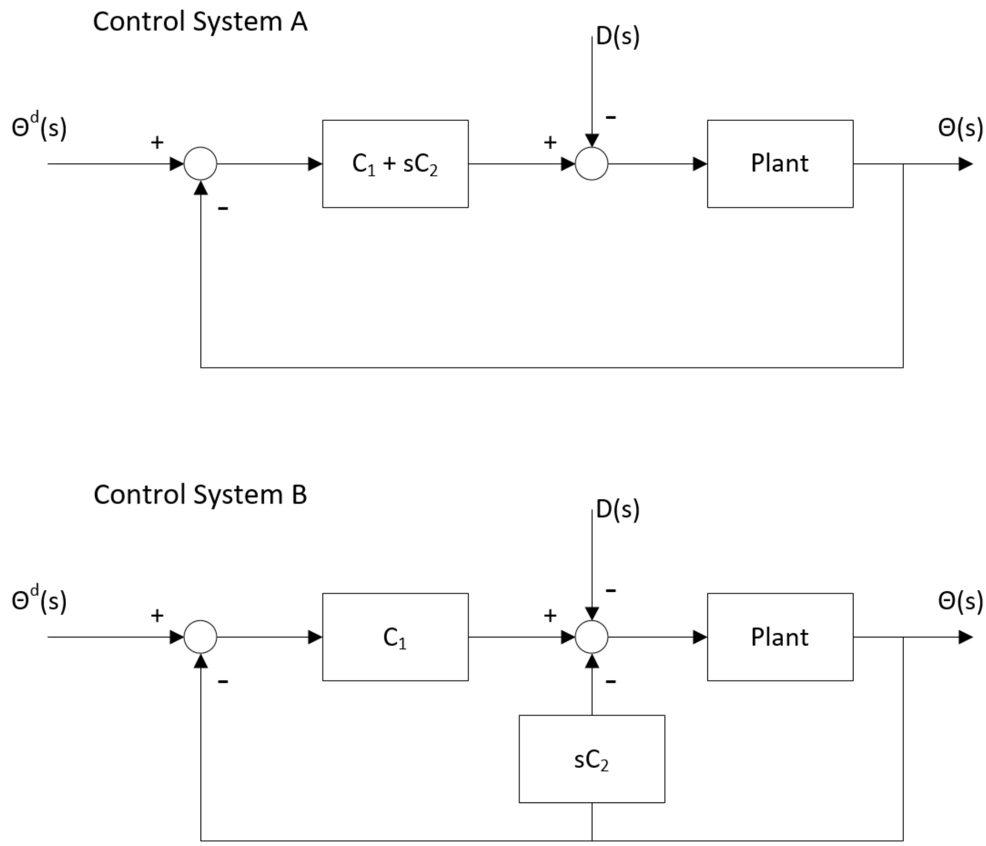

Figure 1: Control Systems

a) (5 %) Figure 1 shows two set-point tracking control systems, which in practise do the same. Find the transfer function for the controller $U_a(s)$ in control system A, and $U_b(s)$ in control system B, and use these to show that it is valid to draw the controllers in both ways. Use your knowledge of the different parts of a PID controller to draw a logical conclusion based on your algebraic result.

b) (5 %) For a camera with focal length λ=10, find the image plane coordinates for the following physical 3D points P(x,y,z) given with respect to the camera frame:

1. (25, 25, 50)
2. (-25, -25, 50)
3. (20, 5, -50)
4. (15, 10, 25)
5. (0,0,50)
6. (0,0,100)

Draw an illustration of the "Perspective projection", and indicate if any of the 3D points will not be visible to a physical camera. Show and explain why using the illustration.

c) (5 %) What is the "Extrinisc" and "Intrinsic" parameters of a camera? Explain the fundamental difference between them. What do you call the process of finding these parameters of a camera system?

d) (5 %) What is the first- and second order moments of an object? Why are these parameters useful in order to make a visual servo system where a robot manipulate objects?

Exercise 2 (45 %)

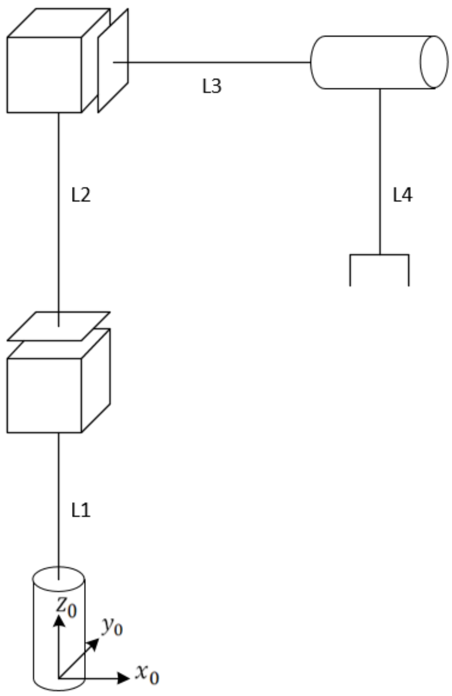

Figure 2: Robot

Figure 2 shows the robot configuration that is being used. In the initial position, shown in Figure 2, the rotational joint is rotating about the Z_0 axis, the first prismatic joint moves along the Z_0 axis and the second prismatic joint moves perpendicular to the Z_0 axis (along X_0), the second rotational joint (joint 4) rotates about an axis parallel to X_0 in this initial position. L_1, L_2, L_3 and L_4 are fixed lengths. The first rotational joint is considered to be in the base of the robot with zero position as shown in the figure.

a) (10 %) Assign coordinate frames on the robot in Figure 2 using Denavit-Hartenberg convention. Write the Denavit-Hartenberg parameters in a table.

b) (5 %) Derive the forward kinematics for the robot from the base coordinate system to the tool coordinate system at the tip of the robot.

c) (10 %) Derive the Jacobian for the robot.

d) (10 %) To simplify, assume that the angle of the first rotational joint is given. Derive the inverse kinematics for the robot, using the fact that you already know the first rotational joint.

e) (5 %) How would you proceed to find the singularities of the robot? What is the difference between workspace and joint space singularities? Mention different consequences with each of them.

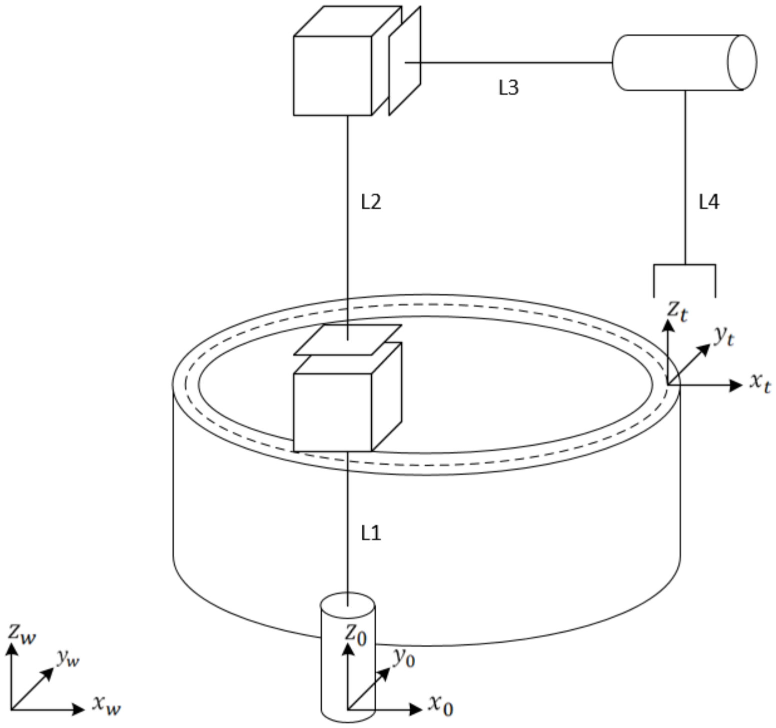

Figure 3: Robot in a welding station

f) (5 %) We will now use our robot in a real world application. The environment where the robot shall work is shown in figure 3. A welding tool is attached to the end effector, and the robot shall weld two pipes together starting in (0,0,0) in the target t coordinate frame. The target coordinate frame is located at $P_t^W = (x_t, y_t, z_t)$, where W is the world coordinate frame. The robots base coordinate frame is located at $P_b^W = (x_b, y_b, z_b)$. Write the formulas which will describe the joint configuration that puts the TCP (Tool Center Point, at the tip of the tool/end effector) at (0,0,0) in the target coordinate frame. Describe your approach thoroughly.

Exercise 3 (15 %)

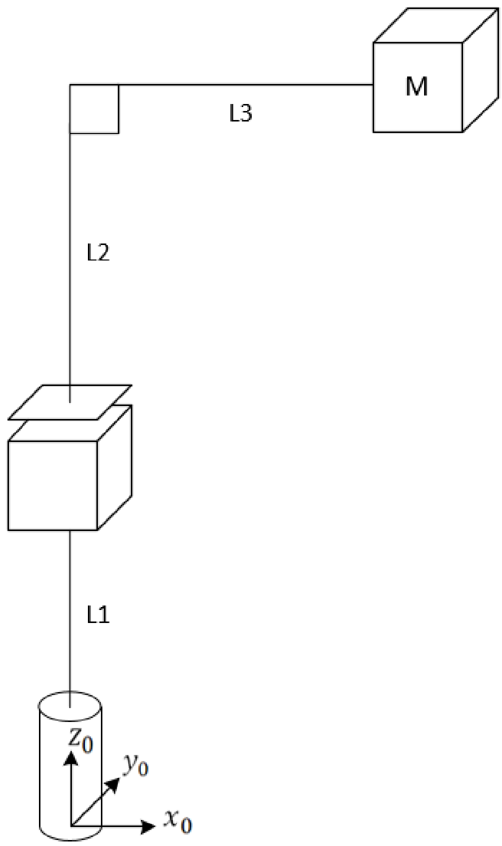

Figure 4: Simplified robot

Figure 4 shows a robot with two degrees of freedom. This is a simplification of the robot in exercise 2. Assume that the only mass is a point mass of M at the tool of the robot. We will not be considering the forces generated by the systems inertia.

a) (10 %) Find the Lagrangian $\mathcal{L}$ of the robotic system in Figure 4.

b) (5 %) Derive the dynamic equations for the robot using the Euler-Lagrange formulation. Formulate the Euler-Lagrange equations of the form $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau$

Exercise 4 (20 %)

We have the system $J\ddot{\theta}(t) + B\dot{\theta}(t) + K\theta(t) = \tau$. When we use the Laplace transform on this system we get $Js^2\theta(s) + Bs\theta(s) + K\theta(s) = \tau$.

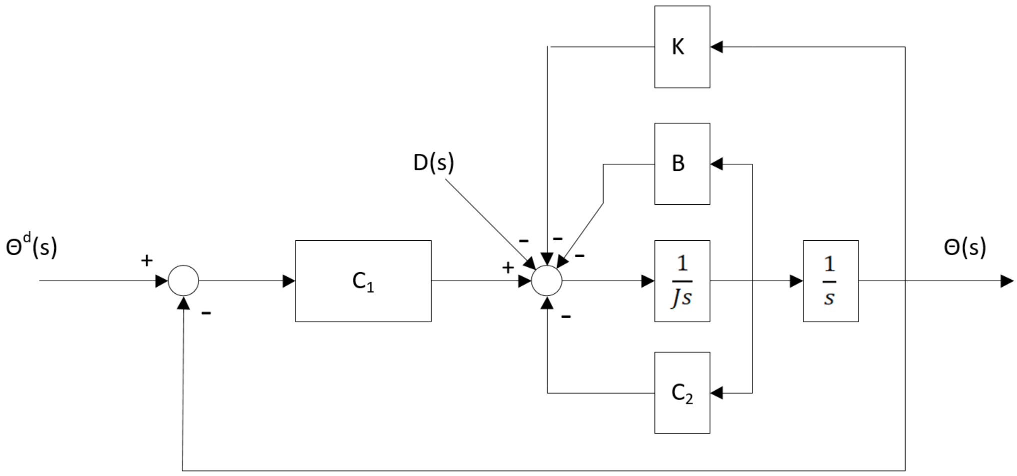

Figure 5: Control system

a) (2.5 %) Figure 5 shows the system with controller in Laplace domain. What is the name of the controller used here?

b) (10 %) Working further with the controller in figure 5, how can we remove the steady state error, and still have a fast responsive system that reacts to the rate of change of the process value? What is the name of this new controller? Find the closed loop transfer function between the input value ($\Theta^d(s)$ - desired angle) and output value ($\Theta(s)$ - actual/measured angle) for the system with this new improved controller. Use the final value theorem to calculate the steady state error for the closed loop control system with this new improved controller, when both the desired angle $\Theta^d(s)$ and the disturbance $D(s)$ are "step inputs". Comment on the result.

c) (2.5 %) In general, how would you examine the stability of a control system like the one in task a? What is required to get a stable system?

d) (5 %) We analyze the step response of a closed loop control system;

$$s^2 + 2\zeta\omega s + \omega^2 \tag{1}$$

It is an under damped second order system ($\zeta < 1$) that gives us fast response, but unfortunately oscillations, see figure 6

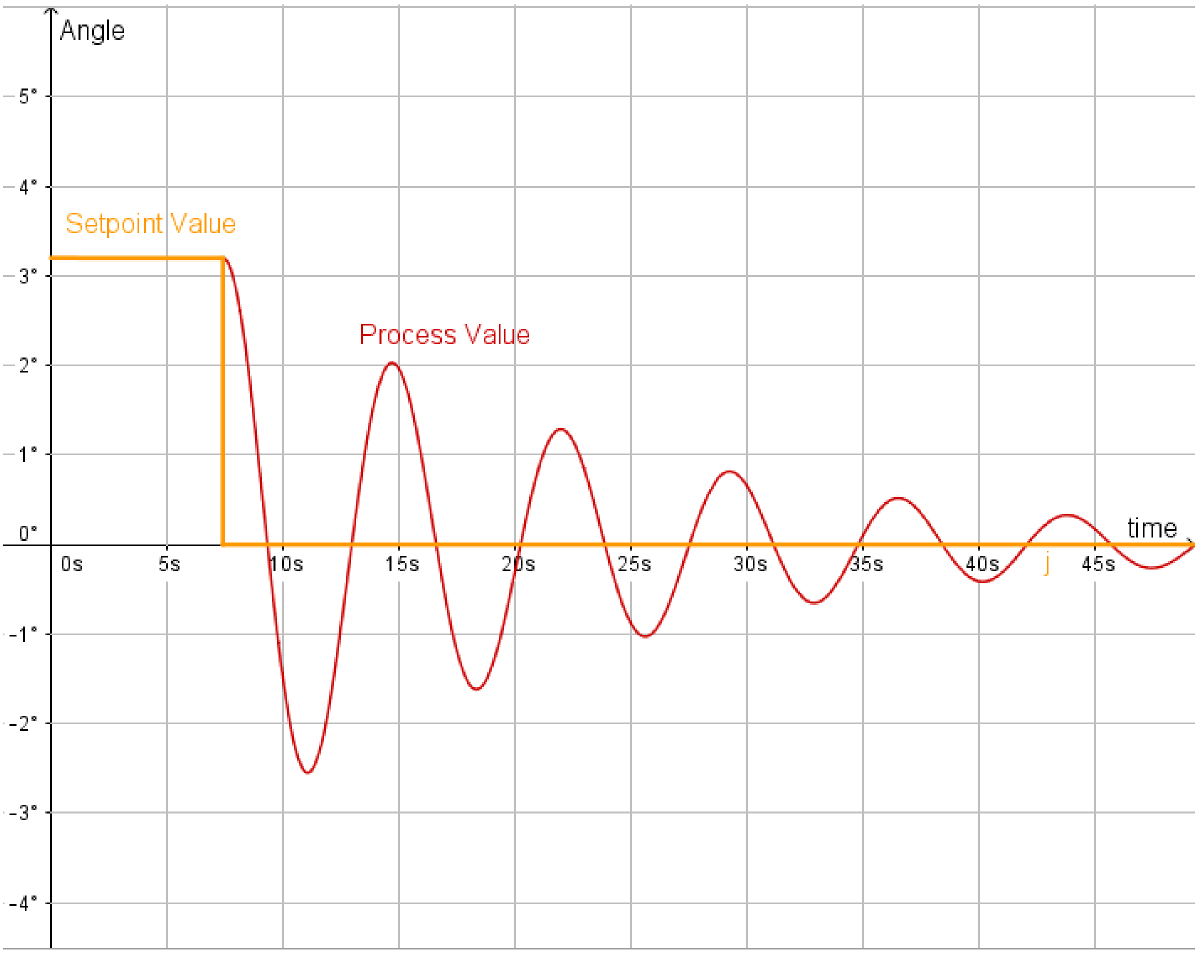

Figure 6: Under damped system

Here, the damping ratio $\zeta < 1$. Our process cannot tolerate oscillations, but we want the fastest response possible. What is our desired system called, and what will ζ be in that case?

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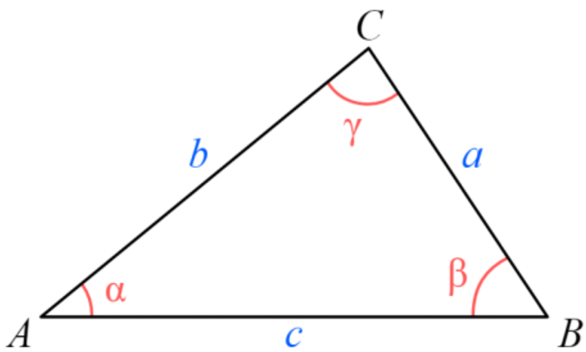
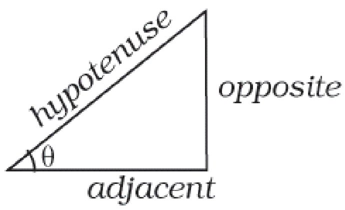


$$c^2 = a^2 + b^2 - 2ab\ \cos\gamma$$



$$\sin\theta = \frac{o}{h} \qquad \cos\theta = \frac{a}{h} \qquad \tan\theta = \frac{o}{a}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\text{radians} = \text{degrees} \times \frac{\pi}{180}$$

$$\cos^2\theta + \sin^2\theta = 1$$

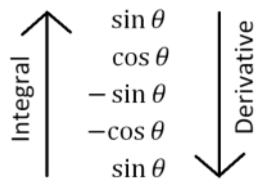
$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

$$\cos^2 u = \frac{1+\cos(2u)}{2}$$

$$\tan^2 u = \frac{1-\cos(2u)}{1+\cos(2u)}$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \\ &= \frac{1-\tan^2\theta}{1+\tan^2\theta}\end{aligned}$$



$$\sin(u+v) = \sin u\cos v + \cos u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u\tan v}$$

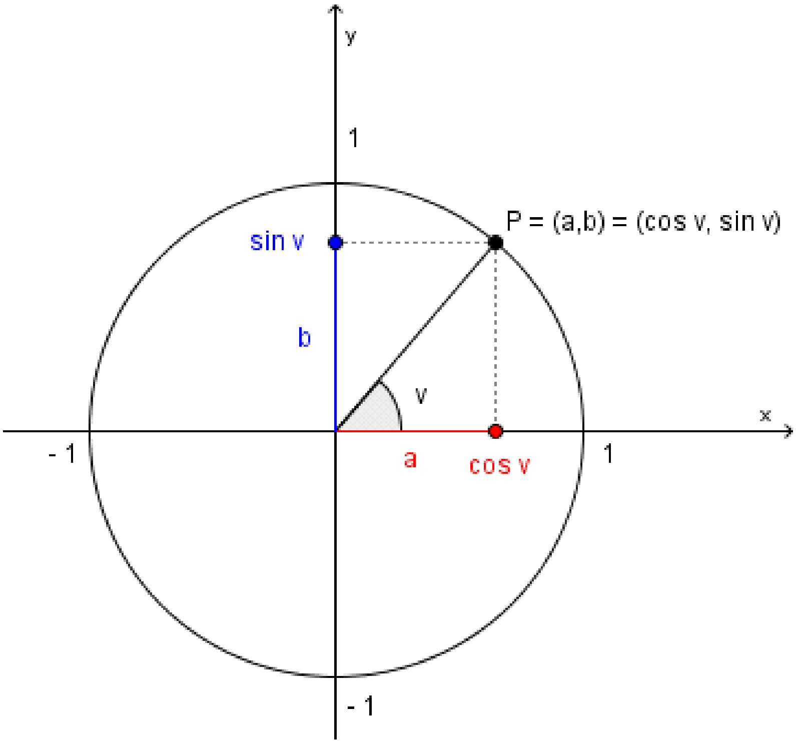
$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\sin(u-v) = \sin u\cos v - \cos u\sin v$$

$$\cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u\tan v}$$

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta \\ &= \frac{2\tan\theta}{1+\tan^2\theta}\end{aligned}$$



$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \ge 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

Deg	0	30	45	60	90
Rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	0
Tan	0	$\sqrt{3}^{-1}$	$\sqrt{3}^{0}$	$\sqrt{3}^{1}$	Not defined

$\mathbf{A} = [a, b, c]$ $\qquad$ $\mathbf{B} = [d, e, f]$

$$\begin{array}{ccccc} x & y & z & x & y \\ a & b & c & a & b \\ d & e & f & d & e \end{array}$$

$\mathbf{A} \times \mathbf{B} = [(bf - ce), (cd - af), (ae - bd)]$

Consider the matrices

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

Multiplying gives

$$AB = \begin{bmatrix} -1 & -2 \\ 11 & 4 \end{bmatrix}, \quad BA = \begin{bmatrix} 3 & 6 \\ -3 & 0 \end{bmatrix}$$

Thus, $AB \neq BA$.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \times \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix}$$

$$3 \times 2 \quad \times \quad 2 \times 5 \quad = \quad 3 \times 5$$

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \text{X} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}_{C}$$

A, B and C are square metrices of size N x N
a, b, c and d are submatrices of A, of size N/2 x N/2
e, f, g and h are submatrices of B, of size N/2 x N/2

Time domain	Laplace domain
$x(t)$	$x(s) = L\{x(t)\} = \int_0^\infty e^{-st} x(t) dt$
$\dot{x}(t)$	$sx(s) - x(0)$
$\ddot{x}(t)$	$s^2 x(s) - sx(0) - \dot{x}(0)$
Ct	$\frac{C}{s^2}$
step	$\frac{1}{s}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$

Time domain	Laplace domain
$x(t - \alpha) H(t - \alpha)$	$e^{-\alpha s} x(s)$
$e^{-at} x(t)$	$x(s + a)$
$x(at)$	$\frac{1}{a} x\left(\frac{s}{a}\right)$
$C\delta(t)$	C