INF3480/INF4380 - Assignment 3(a)

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Due: 12. April 2018, 12:00 (24h)

Introduction

In this last assignment we will look more closely at control of a robot. We will continue to work with the CrustCrawler robot. The first part of the assignment will start with modelling the robot arm to describe the dynamic equations. In the second part of the assignment (handed out in one week) we will move on to test control theory (PID) in practice and move on generate the robot trajectory. In the end, you will test your work on the real CrustCrawler!

Important: *please note that this is just the first part of the assignment.* The second part, requiring significantly more work, will be handed out one week later.

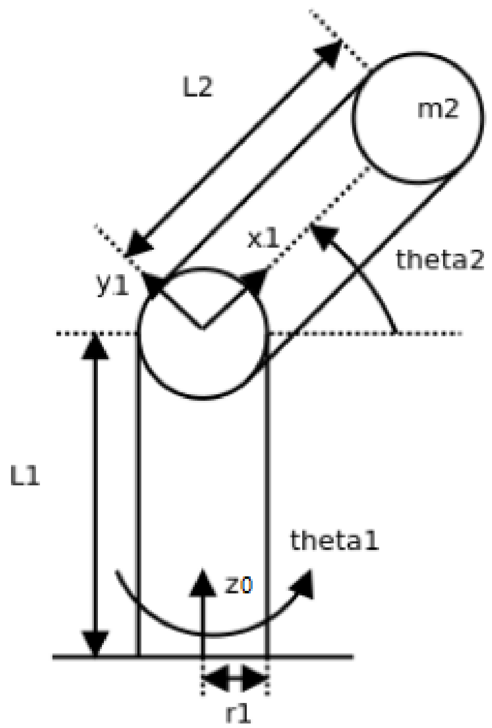

Figure 1: Two link simplified CrustCrawler

Task 1

In this exercise we will further develop the model for the CrustCrawler. We will use the ***Euler Lagrange Equations*** to derive the dynamic equations. This method derives the ***Lagrangian*** $\mathcal{L}$ with respect to each joint variable.

$\mathcal{L} = \mathcal{K} - \mathcal{P}$, and the general formula for potential and kinetic energy can be written as

$$P = mgh \tag{1}$$

where m is the mass, g is the gravitational constant and h is the height above the defined zero plane. In our robot, the base is the zero plane.

$$K = \frac{1}{2} m \boldsymbol{v}^T \boldsymbol{v} + \frac{1}{2} \boldsymbol{\omega}^T \mathcal{I} \boldsymbol{\omega} \tag{2}$$

where the velocity vector $\boldsymbol{v}$ (linear velocity) and $\boldsymbol{\omega}$ (angular velocity) is three dimensional vectors and can be expressed in any reference frame. The inner product of the velocities produce a scalar value. In this task the velocities must be relative to the base frame. The kinetic energy is given in equation 2, where $\mathcal{I}$ is the inertia tensor expressed relative to the base. For kinetic and potential energy of an n-Link robot, see equation (7.48) and (7.52) in the course book (Spong).

We will model the first motor, Link 1 and the second motor as a cylinder with radius r_1, height L_1 and an evenly distributed mass of m_1, see figure 1. In our case, rotating the cylinder around z_0 gives a moment of inertia about the z axis in the body attached frame (the link) given as

$$I_{1,zz} = \frac{m_1 r_1^2}{2} \tag{3}$$

The rest of the inertia tensor $\mathbf{I}_1$ is zero. For simplification, we set the inertia tensor $\mathbf{I}_2$ to zero, and model the mass of link two as a point mass at the end of the link, together with the mass of the third motor. We denote this combined mass m2. Similarly is the mass of link three modelled as a point mass at the tip of the link. We denote this as m3.

a) Derive the ***Lagrangian*** for the two link CrustCrawler. You can find the Jacobian in equation 4 for calculating velocities.

b) Derive the dynamic model for the two link CrustCrawler. Your answer should be two equations.

We will now expand the model to include three joints, see figure 2. We simplify by setting the inertia tensor $\mathbf{I}_3$ to zero.

$$\mathbf{J} = \begin{bmatrix} -s_1(L_2c_2 + L_3c_{23}) & -c_1(L_2s_2 + L_3s_{23}) & -c_1(L_3s_{23}) \\ c_1(L_2c_2 + L_3c_{23}) & -s_1(L_2s_2 + L_3s_{23}) & -s_1(L_3s_{23}) \\ 0 & (L_2c_2 + L_3c_{23}) & L_3c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix} \tag{4}$$

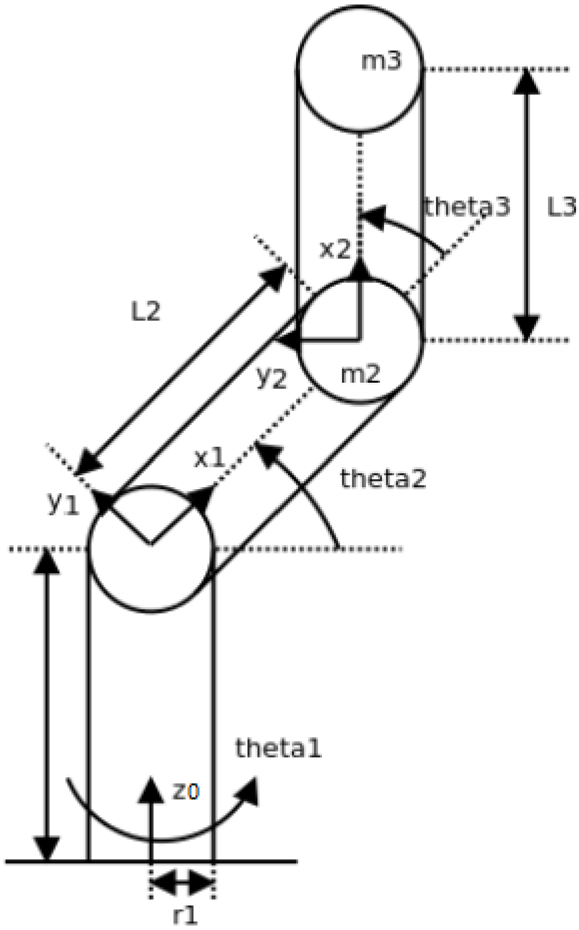

Figure 2: Three link simplified CrustCrawler

c) Derive the ***Lagrangian*** for the three link CrustCrawler.

d) Derive the dynamic model for the three link CrustCrawler. Did your dynamic equations for joint 1 and 2 match those from task b)?

e) Write the dynamic equations from d) in the following form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau$$

Requirements:
Each student must hand in their own assignment, and you are required to have read the following declaration to student submissions at the department of informatics: http://www.ifi.uio.no/studinf/skjemaer/declaration.pdf

IMPORTANT: Name the pdf file: "inf3480-oblig3-*your_username.pdf*". All deadline and devilry3 questions are to be directed to Nikolai (email below).

Submit your assignment at https://devilry3.ifi.uio.no.
Your submission must include:

- A pdf-document with answers to the questions.
- A README.txt containing a short reflection on the assignment; what was difficult, what was easy, was there anything you could have done better?

If you have used MATLAB, Sympy or other tools for computing an answer, your solution and approach must be illustrated and explained thoroughly in the pdf file. The files containing the code must also be named and delivered.

Deadline: 12. April 2018, 12:00 (24h)

You can use the slack channel *assignment 3* for general questions about the assignment, and the channel *dynamics* for discussion. Do not hesitate to contact us if you have any further questions.

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