5 Example - Robot modeling

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Assuming

$\cancel{L_2 = \bar{L}_2 + \Delta L_2(L_2)}$ Static $L_2 \Rightarrow \dot{L}_2$

$s_1 \approx \theta_1$ $\cancel{\text{ha } \alpha_1} =$ $s_1 + \theta_1 - \theta_1$ Linearization

$$mL_2^2\ddot{\theta}_1 + 2mL_2\underset{0}{\underset{\shortparallel}{\dot{L}_2}}\dot{\theta}_1 - mgL_2(s_1 + \theta_1 - \theta_1) = \tau_1$$

$$\underbrace{mL_2^2\ddot{\theta}_1 - mgL_2\theta_1}_{\text{Linear model}} + \underbrace{mgL_2(\theta_1 - s_1)}_{\text{Disturbance}} = \tau_1$$

$$J\cancel{B}\ddot{\theta}_1 - G\theta_1 + D = \tau_1$$

(book uses D)

9 Laplace transform

Definition

$$x(s) = \int_0^\infty e^{-st} x(t)\, dt \qquad (1)$$

Example:

Laplace transform of $\frac{dx}{dt}$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \int_0^\infty e^{-st} \frac{dx}{dt}\, dt$$

$$\int u'v = uv - \int uv'$$

$$u' = \frac{dx}{dt} \qquad u = x(t)$$

$$v = e^{-st} \qquad v' = -s e^{-st}$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = e^{-st} x(t)\Big|_0^\infty - \int_0^\infty -s e^{-st} x(t)\, dt$$

$$= \underbrace{e^{-s\infty}}_{0} x(\infty) - \underbrace{e^{-s\cdot 0}}_{1} x(0)$$

$$+ s \underbrace{\int_0^\infty e^{-st} x(t)\, dt}_{= x(s) \text{ see } (1)}$$

$$= s\, x(s) - x(0)$$

10 $m\ddot{x}(t) + b\dot{x}(t) + kx(t) = f(t)$ 3

$$\downarrow \mathcal{L}$$

$$m\mathcal{L}\{\ddot{x}(t)\} + b\mathcal{L}\{\dot{x}(t)\} + k\mathcal{L}\{x(t)\} = \mathcal{L}\{f(t)\}$$

$$m(s^2X(s) - s\,x(0) - \dot{x}(0))$$
$$+ b(sX(s) - x(0)) + kX(s) = F(s)$$

$$(ms^2 + bs + k)X(s) = F(s) + m\dot{x}(0) + (ms+c)\,x(0)$$

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$$J\ddot{\theta} - G\theta_1 + D = \tau_1$$

$$Js^2\theta_1 - G\theta_1 + D = \tau_1$$

$$Js^2\theta_1 - G\theta_1 = \tau_1 - D$$

$$\cancel{\theta_1 = \frac{\tau_1}{Js^2 -}} \qquad (Js^2 - 1)\theta_1 = \tau - D$$

$$\theta_1 = \frac{\tau}{Js^2 - 1} - \frac{D}{Js^2 - 1}$$

$$\theta_1 = H\tau - HD$$

$$H = \frac{1}{Js^2 - 1}$$

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Example mass-spring-damper system

$$\sum F = m a = F - b\dot{x} - kx$$

$$m\ddot{x} + b\dot{x} + kx = F$$

$$m s^2 X + b s X + k X = F$$

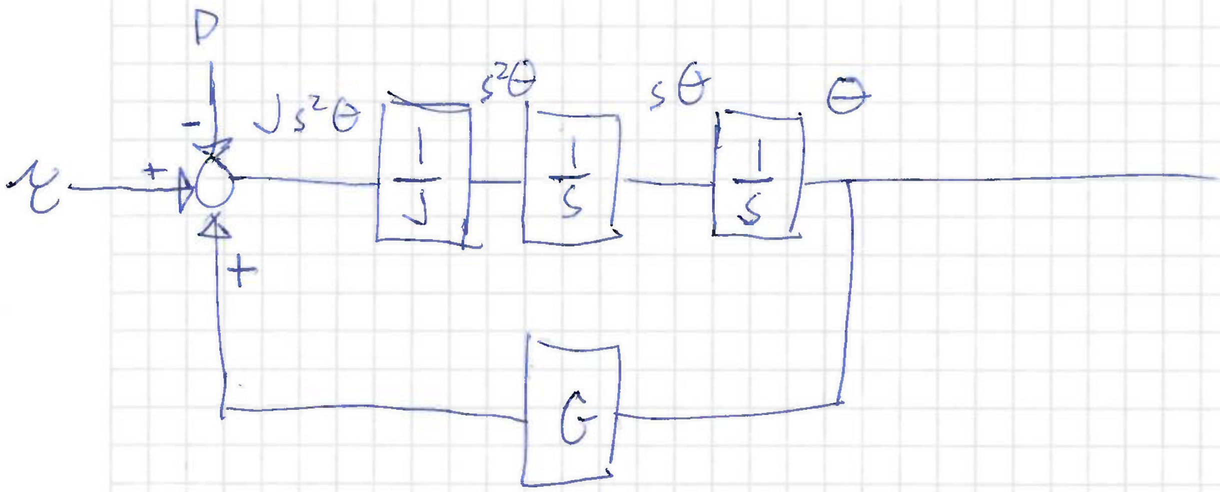
$$(m s^2 + b s + k) X = F$$

$$\frac{X}{F} = \frac{1}{m s^2 + b s + k}$$

21 Block diagram Robot

$$Js^2\Theta - G\Theta + D = \tau_1$$

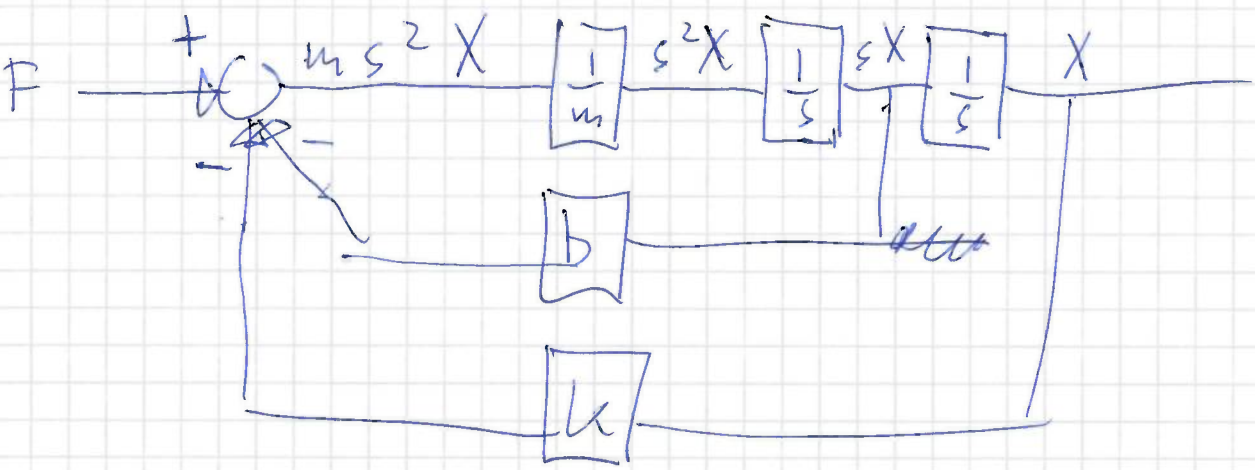
$$Js^2\Theta = \tau + G\Theta - D$$



22 Block diagram mass-spring-damper

$$ms^2 X + bsX + kX = F$$

$$ms^2 X = F - bsX - kX$$



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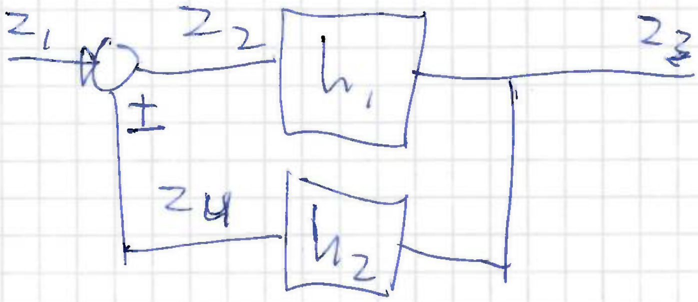
Manipulation of block diagrams



$$
\begin{aligned}
z_3 &= z_2 h_1 \\
&= (z_1 \pm z_4) h_1 \\
&= (z_1 \pm z_3 h_2) h_1
\end{aligned}
$$

$$z_3 \mp z_3 h_2 h_1 = z_1 h_1$$

$$z_3 = \frac{h_1}{1 \mp h_1 h_2} z_1$$

Control theory Intro

$$m x s^2 + f x s + k x = u$$

$$x(m s^2 + f s + k) = u$$

$$\frac{x}{u} = \frac{1}{m s^2 + f s + k}$$

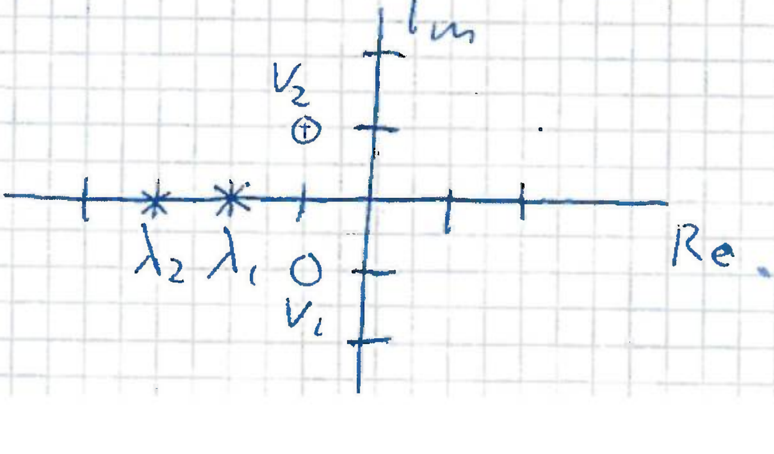
28 **Example 1: poles and zeros**

$$H(s) = \frac{2s^2 + 4s + 4}{s^2 + 5s + 6}$$

$$= 2\,\frac{(s+1+j)(s+1-j)}{(s+2)(s+3)}$$

$$v_1 = -1 - j \qquad v_2 = -1 + j$$

$$\lambda_1 = -2 \qquad \lambda_2 = -3$$



Example 2: Mass-spring-damper system

$$h(s) = \frac{1}{ms^2 + fs + k}$$

$$= \frac{\frac{1}{m}}{s^2 + \frac{f}{m} s + \frac{k}{m}}$$

$$= \frac{\frac{1}{m}}{(s-\lambda_1)(s-\lambda_2)}$$

$$\lambda_{1,2} = -\frac{f}{2m}\left(1 \pm \sqrt{1 - 4\frac{km}{f^2}}\right)$$

Three cases

$\zeta > 1$ I $\quad 4\frac{km}{f^2} < 1 \quad$ Two distinct real roots

$\zeta = 1$ II $\quad 4\frac{km}{f^2} = 1 \quad$ Two real roots of same value

$\zeta < 1$ III $\quad 4\frac{km}{f^2} > 1 \quad$ Two complex conjugate roots

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Root locus plots (Balchen 4.7.4)

Example:

$$h(s) = \frac{1}{\frac{T_1}{k}(s-\lambda_1)(s-\lambda_2)}$$

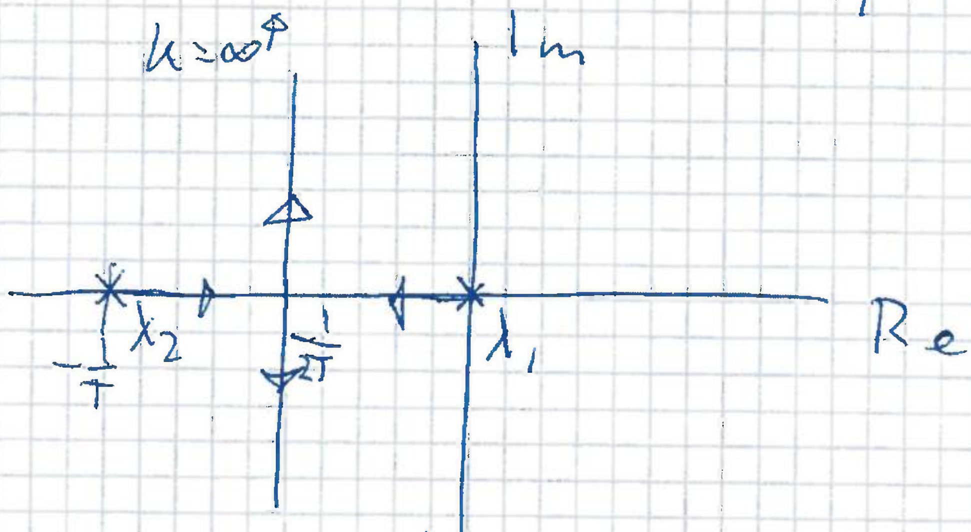
$$= \frac{1}{\frac{T_1}{k}(s+\alpha+j\beta)(s+\alpha-j\beta)}$$

$$\lambda_{1,2} = -\frac{1}{2T_1}\left(1 \pm \sqrt{1-4T_1 k}\right) \quad \forall k \le \frac{1}{4T_1}$$

$$\alpha = \frac{1}{2T_1} \qquad \beta = \frac{1}{2T_1}\sqrt{4T_1 k - 1} \quad \forall k > \frac{1}{4T}$$

Starting with $k = 0$

$\lambda_1 = 0 \qquad \lambda_2 = -\frac{1}{T_1}$



When $k = \frac{1}{4T}$ $\quad \lambda_1 = \lambda_2 \Rightarrow \lambda = -\frac{1}{2T}$

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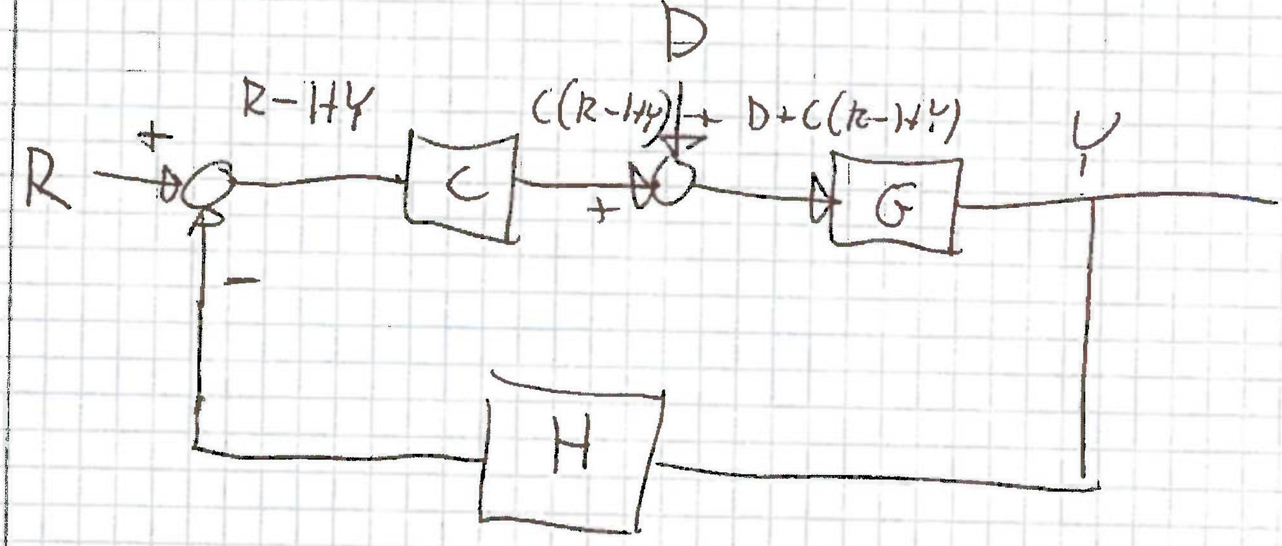
Feedback systems



$$Y = GD + GCR - GCHY$$

$$(1 + GCH)\,Y = GD + GCR$$

$$Y = \underbrace{\frac{GC}{1 + CGH}}_{W} R + \underbrace{\frac{G}{1 + CGH}}_{W_D} D$$