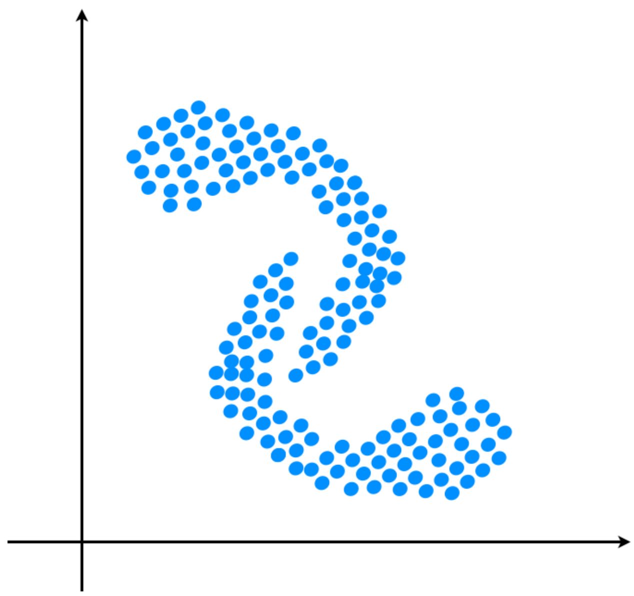
Problem 1

The following graph shows 2D data point. It is clear from the graphs that there are two clusters. Will k-means clustering be able to find these two, if we define $k = 2$? If not, why?



Problem 2

In SOMs, what role do the predefined topological (neighborhood) relationships between neurons in the map space play in the discovery of topological (neighborhood) relationships between input vectors in the data space?

Problem 3

In SOMs, a Gaussian function, as defined below, can be used to define the neighborhood relation

$$N(i,j) = e^{-\frac{\|i-j\|^2}{2\sigma^2(t)}}$$

where i and j are two neurons, whose position on the lattice is be given by (row, column), and $||i - j||$ is the Euclidean distance between them. So, if i's position is $(2,2)$, j's position is $(3,3)$, and $\sigma(t) = 1$, $N(i,j)$ will be e^{-1}.

As can also be seen, the further a neuron i is from the winning neuron j on the lattice, the smaller is $N(i,j)$. What would happen if $N(i,j)$ is set to and remains zero for all except the winning neuron? What would happen if $N(i,j)$ is set to and remains 1 for all neurons including the winning neuron? Why is it important to have $N(i,j)$ large for distant (on the lattice/map space) neurons in the beginning of the learning process i.e. j should have a larger neighborhood, and smaller as time goes by? This can be controlled by $\sigma(t) = \sigma_0 e^{-t/T}$, where σ_0 is the initial value of σ, and T is a constant.

Example Matlab script that can be run to see how the neighborhood function may change with time:

```
figure;
[X,Y] = meshgrid(-5:.1:5, -5:.1:5);
sigma0 = 5;
for i=1:100
    Z = (exp(-(X.^2 + Y.^2)/(2*sigma^2)));
    surf(X,Y,Z);
    sigma = sigma0 * exp(-i/10);
    surf(X, Y, Z);
    pause(0.01);
end
```

Describe one other neighborhood relationship function that can be used for SOMs, and explain in what way it might influence the learning process?