# INF3490 exercise answers - week 5 2015

## Problem 1
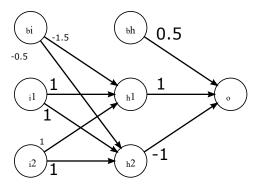
**a)**

| Bias $= -1.5$ | | | Bias $= -0.5$ | | |
|---|---|---|---|---|---|
| Input | Weighted sum | Output | Input | Weighted sum | Output |
| $(0,0)$ | $-1.5$ | $0$ | $(0,0)$ | $-0.5$ | $0$ |
| $(0,1)$ | $-0.5$ | $0$ | $(0,1)$ | $0.5$ | $1$ |
| $(1,0)$ | $-0.5$ | $0$ | $(1,0)$ | $0.5$ | $1$ |
| $(1,1)$ | $0.5$ | $1$ | $(1,1)$ | $1.5$ | $1$ |

**b and c)**  See the the file http://www.uio.no/studier/emner/matnat/ifi/INF3490/h15/exercises/bool_perceptron.py

## Problem 2

If we just want it to work in the binary case, we can simply switch signs on all the output weights to create the opposite function, as shown on the right.

If we want it to work for all integers, we have to change the weights in the first layer. First, note that the output layer encodes the function $h_2 - h_1 - 0.5 > 0$ which is equivalent to $h_2 \wedge \neg h_1$ since $h_1$ and $h_2$ are either 0 or 1. If we equate true with 1 and false with 0 we can write $h_1$: $i_1+i_2$-1.5>0 and $h_2$: $i_1+i_2$-0.5>0, so the output is



$o: i_1+i_2 > 0.5 \wedge i_1+i_2 \leq 1.5 \quad \Leftrightarrow \quad o: 0.5 < i_1+i_2 \leq 1.5$

Limiting $i_1$ and $i_2$ to integers gives us $o: i_1 + i_2 = 1$. So the output is 1 if and only if the sum of the inputs are 1. We can get the equivalence function by changing this to $o: i_1 - i_2 = 0$. Going back a step or two we see that we can achieve this by making the output

$$o: i_1 - i_2 > -0.5 \ \wedge \ i_1 - i_2 \leq 0.5$$

or something similar. This has the same logical form that we already have, so we don't need to change the weights of the output layer, but only those of the hidden layer.

Tracing back, we see that we would need $h_1 : i_1 - i_2 - 0.5 > 0$ and $h_2 : i_1 - i_2 + 0.5 > 0$ as the hidden layer outputs. This implies the following weights: