UiO : Department of Informatics
University of Oslo

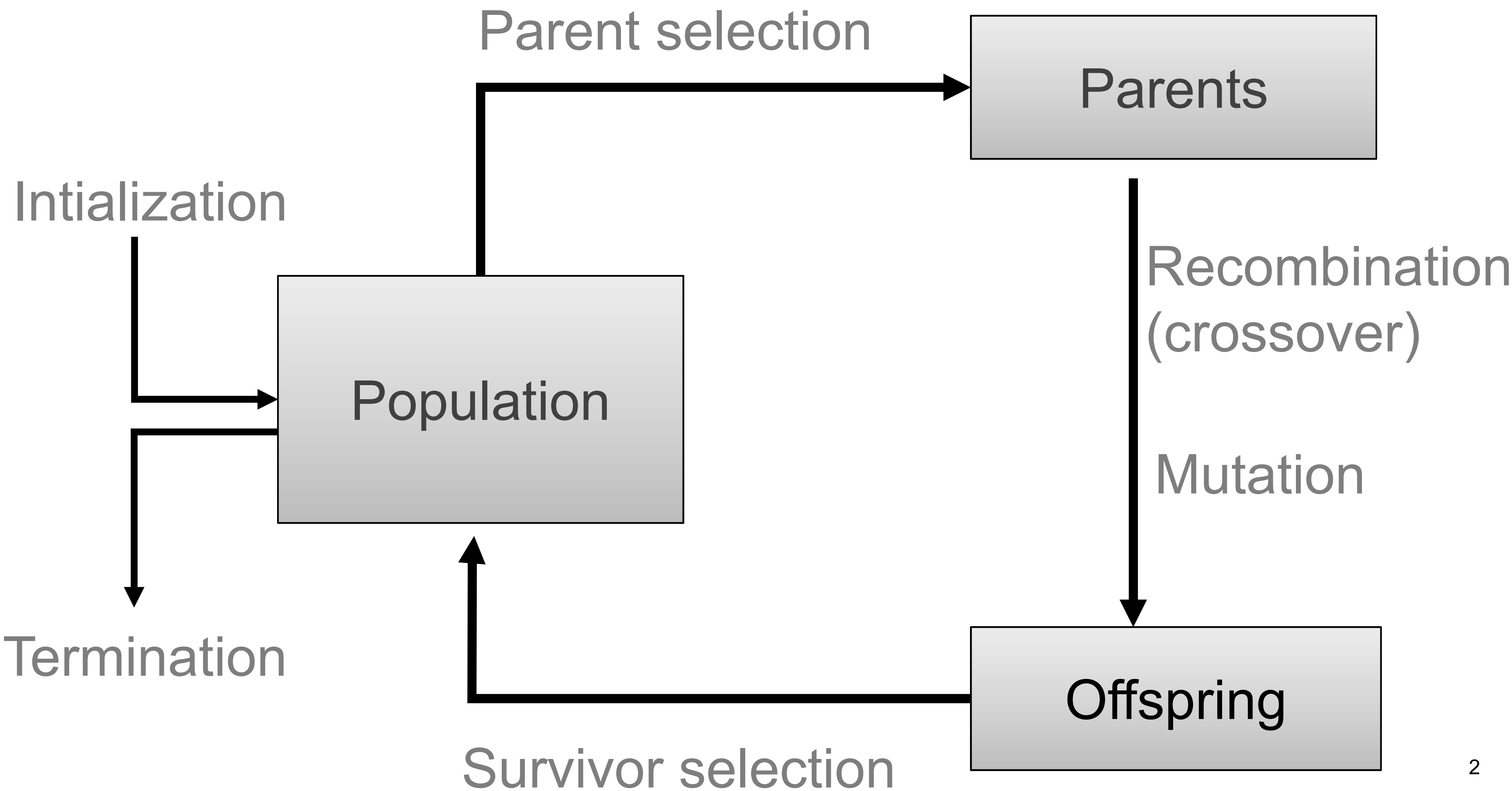
# INF3490 - Biologically inspired computing

Lecture 3: Eiben and Smith, chapter 5-6

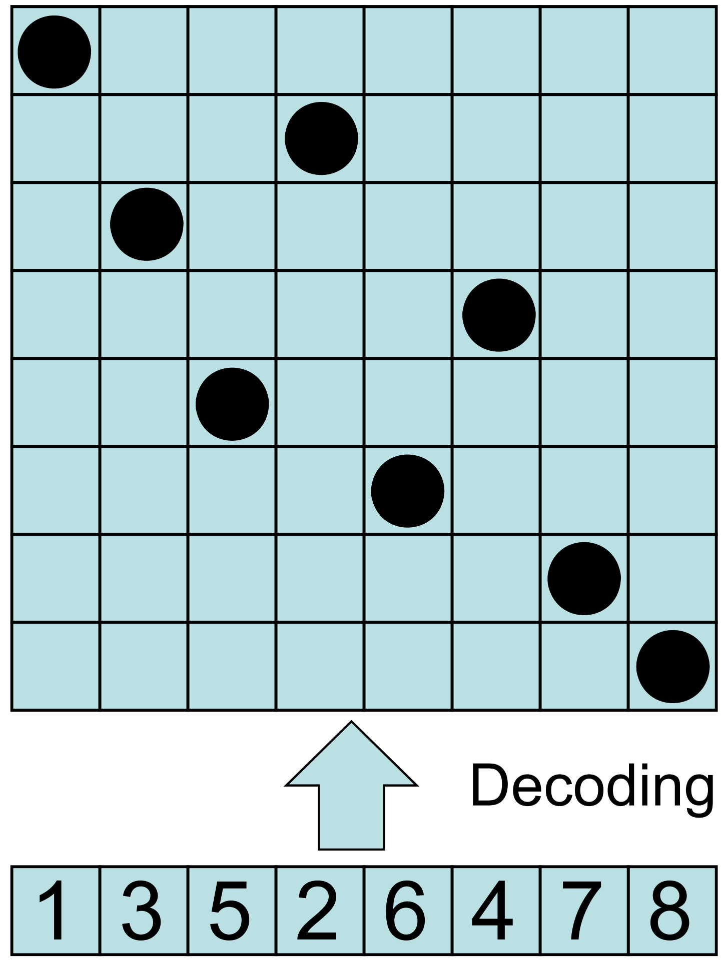
# Evolutionary Algorithms - Population management and popular algorithms

Kai Olav Ellefsen

# Repetition:
# General scheme of EAs



# Repetition: Genotype & Phenotype

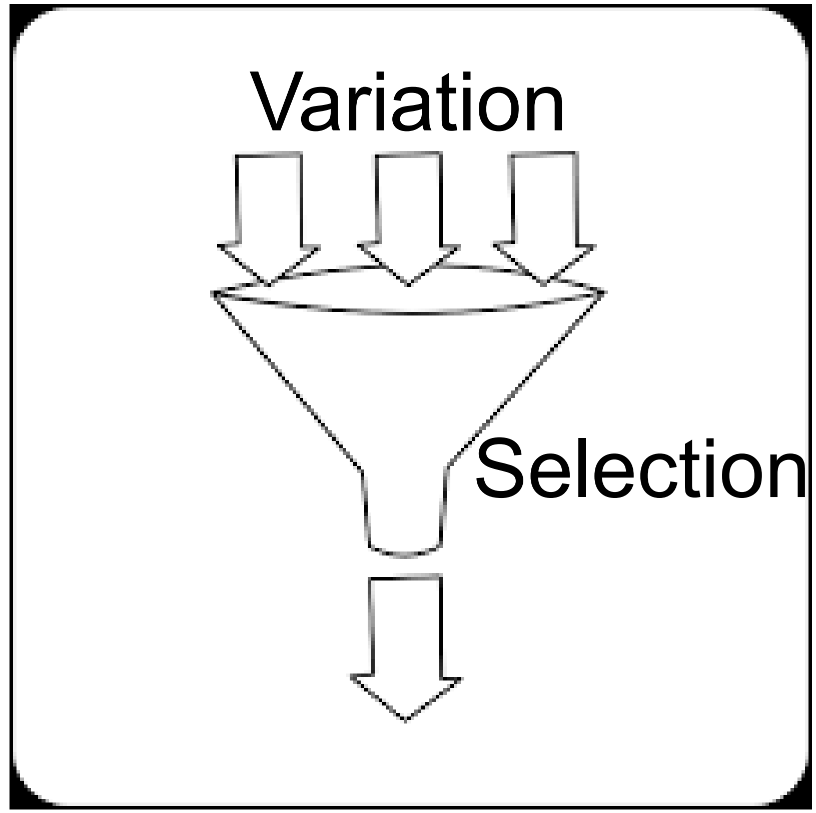



Phenotype:
A solution representation we can **evaluate**

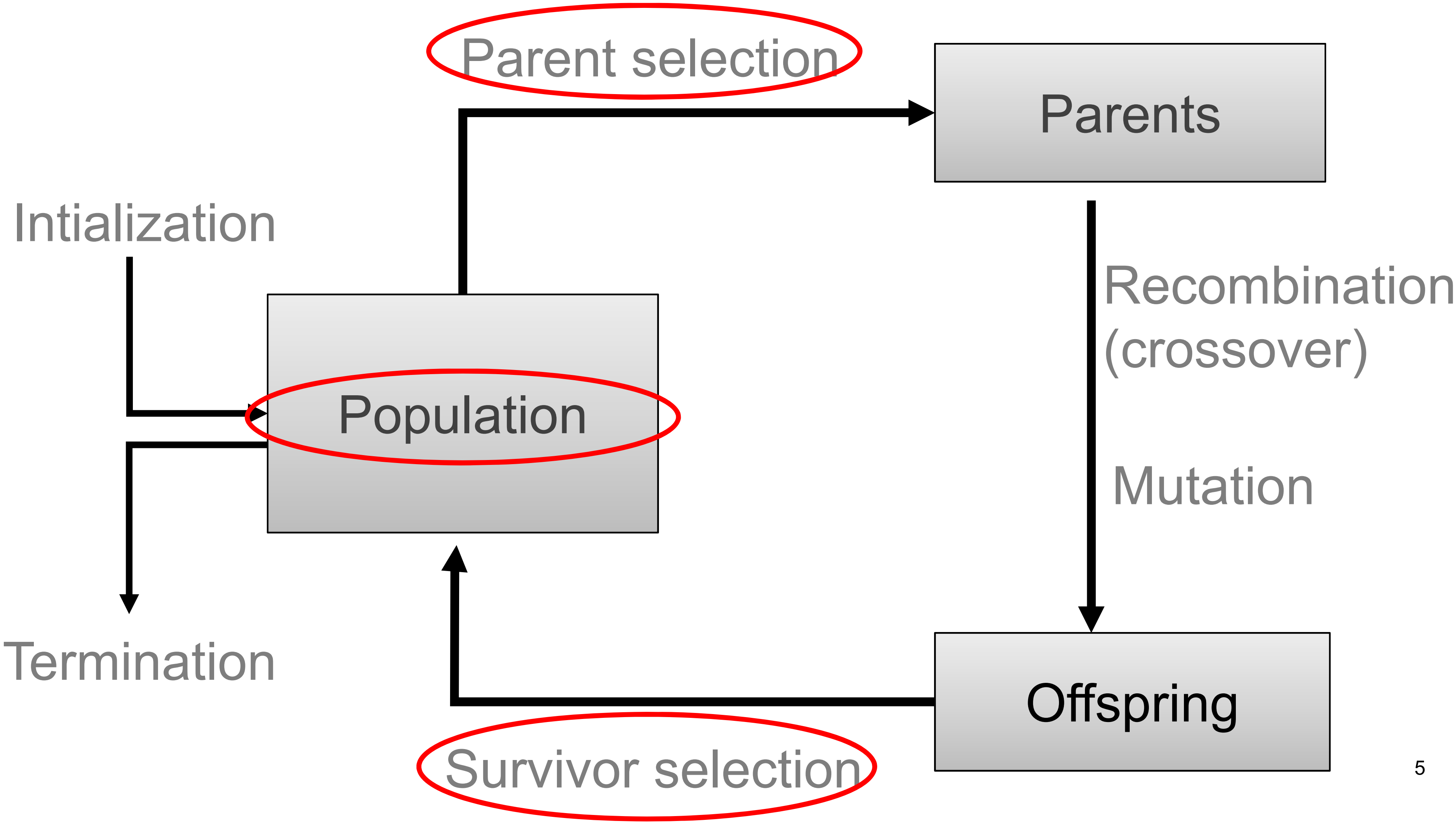
Genotype:
A solution representation applicable to **variation**

# Chapter 5: Fitness, Selection and Population Management

- **Selection** is second fundamental force for evolutionary systems
- Components exist of:
  - Population management models
  - Selection operators
  - Preserving diversity



# Scheme of an EA:
# General scheme of EAs



# Population Management Models: Introduction

- Two different population management models exist:
  - **Generational model**
    - each individual survives for exactly one generation
    - the entire set of parents is replaced by the offspring
  - **Steady-state model**
    - one offspring is generated per generation
    - one member of population replaced
- Generation Gap
  - The proportion of the population replaced
  - Parameter = 1.0 for G-GA, = 1/pop_size for SS-GA

# Population Management Models: Fitness based competition

- Selection can occur in two places:
  - **Parent selection** (selects mating pairs)
  - **Survivor selection** (replaces population)
- Selection works on the population

  -> selection operators are representation-independent !
- **Selection pressure**: As selection pressure increases, fitter solutions are more likely to survive, or be chosen as parents

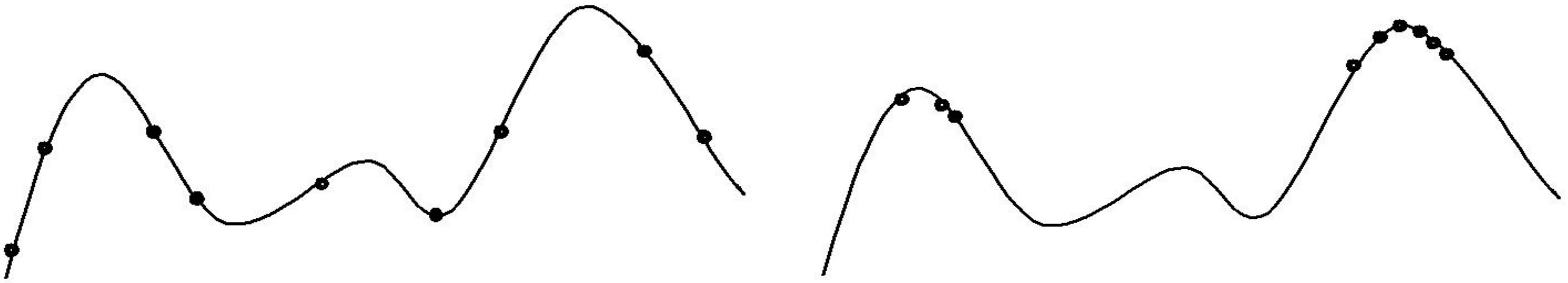
# Effect of Selection Pressure

- Low Pressure
- High Pressure

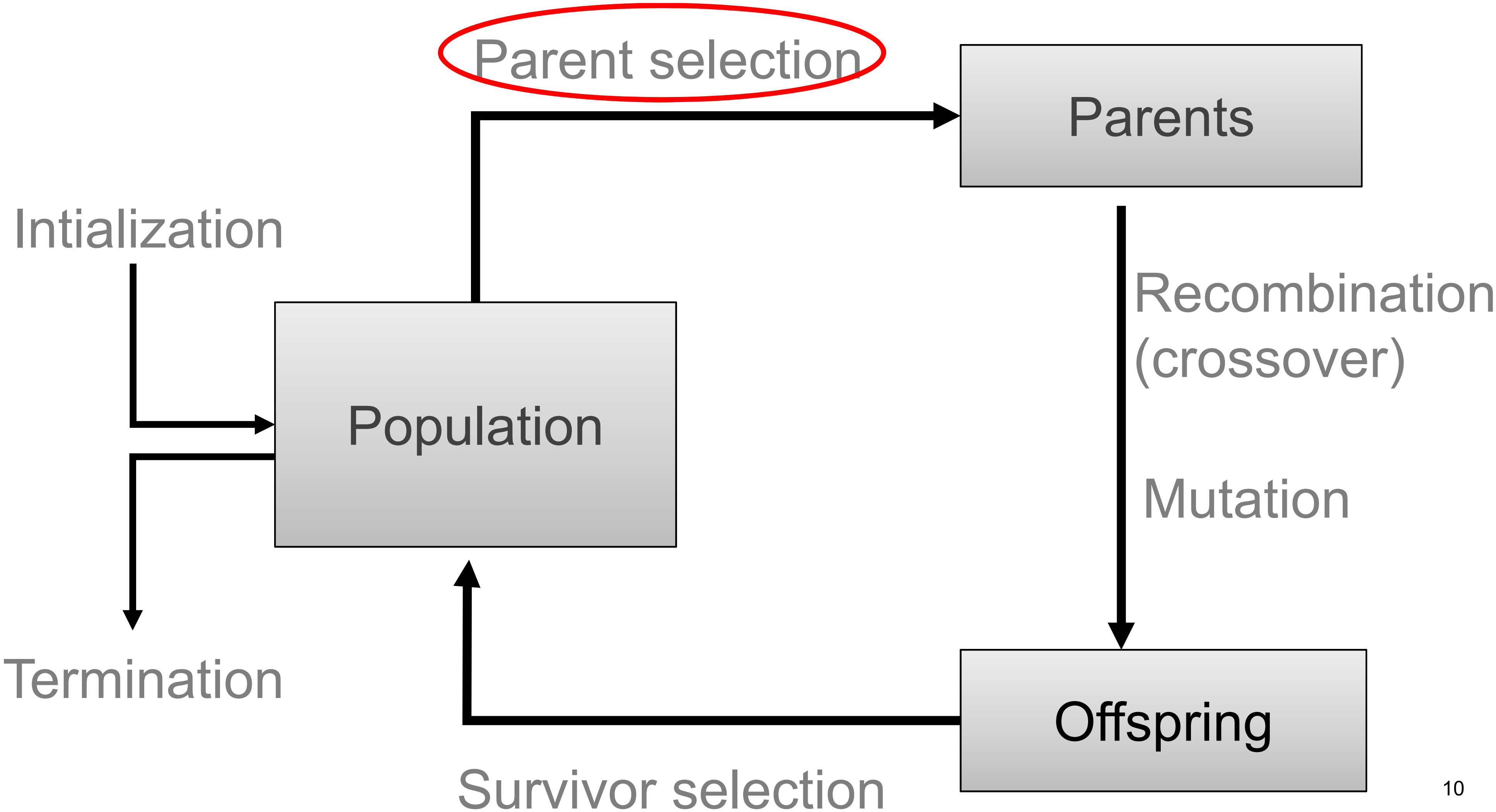
# Why Not Always High Selection Pressure?

Exploration

Exploitation



# Scheme of an EA:
# General scheme of EAs



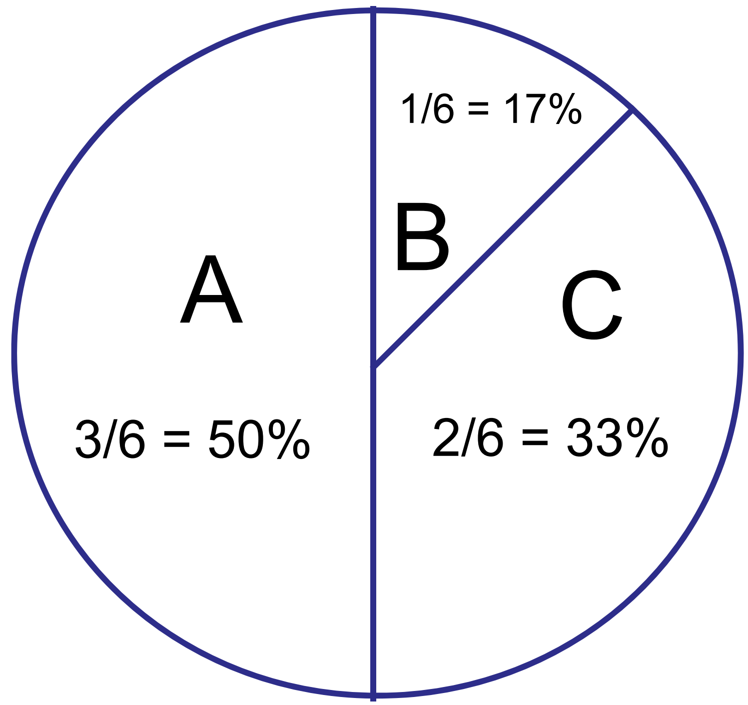
# Parent Selection: Fitness-Proportionate Selection

Example: roulette wheel selection

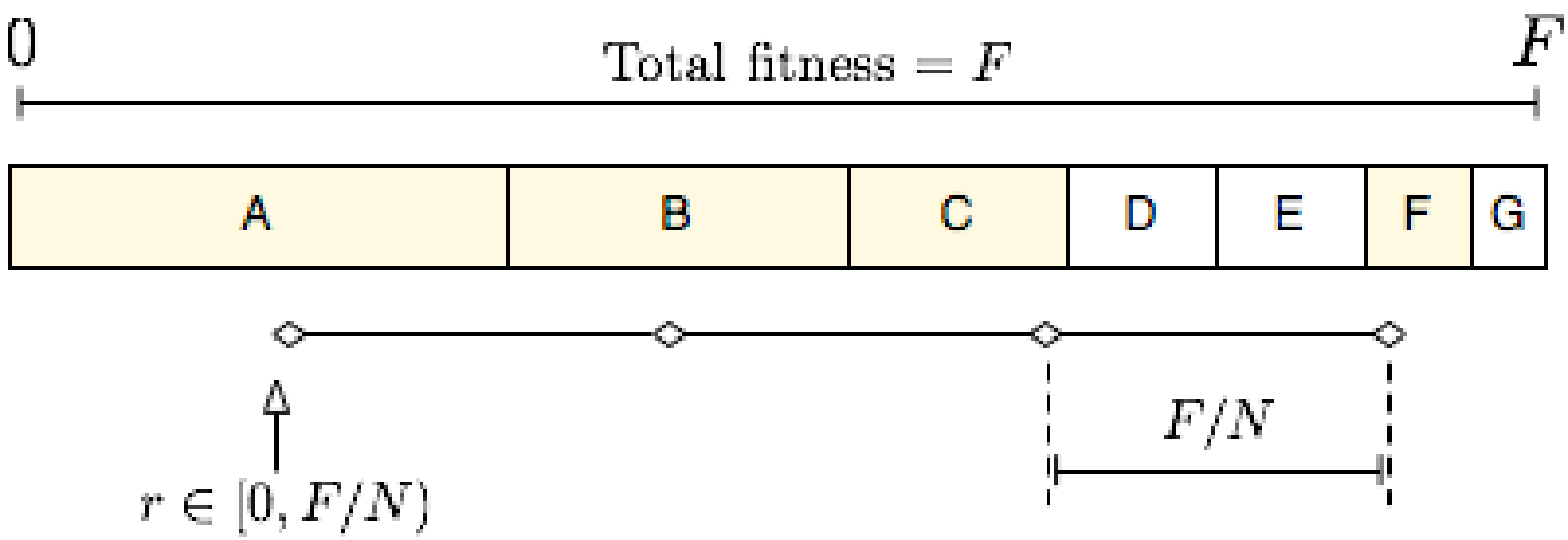
fitness(A) = 3

fitness(B) = 1

fitness(C) = 2



# Stochastic Universal Sampling




**Stochastic universal sampling (SUS)**
Select multiple individuals by making **one** spin of the wheel with **a number of equally spaced arms**

# Parent Selection: Fitness-Proportionate Selection (FPS)

- Probability for individual *i* to be selected for mating in a population size *μ* with FPS is

$$P_{FPS}(i) = f_i \Big/ \sum_{j=1}^{\mu} f_j$$

- Problems include
  - One highly fit member can rapidly take over if rest of population is much less fit: **Premature Convergence**
  - At end of runs when finesses are similar, loss of selection pressure
- **Scaling** can fix the last problem by:
  - **Windowing**: $f'(i) = f(i) - \beta^t$

  where $\beta$ is worst fitness in this (last n) generations
  - **Sigma Scaling**: $f'(i) = \max(f(i) - (\bar{f} - c \bullet \sigma_f), 0)$

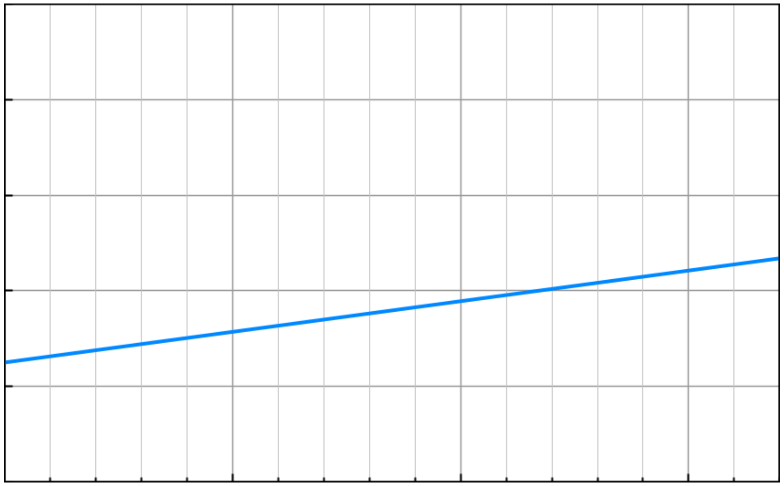
  where *c* is a constant, usually 2.0

# Parent Selection: Rank-based Selection

- Attempt to remove problems of FPS by basing selection probabilities on ***relative* rather than *absolute* fitness**
- **Rank population** according to fitness and then base selection probabilities on rank (fittest has rank $\mu$-*1* and worst rank 0)
- This imposes a sorting overhead on the algorithm

# Rank-based Selection: Linear Ranking

$$P_{lin-rank}(i) = \frac{(2-s)}{\mu} + \frac{2i(s-1)}{\mu(\mu-1)}$$

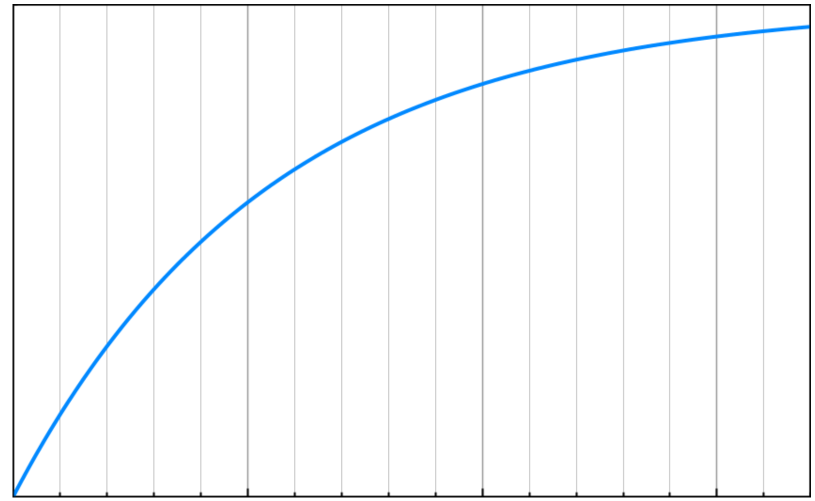


- Parameterised by factor *s:* $1 < s \leq 2$
  - Tunes selection pressure
- Simple 3 member example

| Individual | Fitness | Rank | $P_{selFP}$ | $P_{selLR}$ $(s = 2)$ | $P_{selLR}$ $(s = 1.5)$ |
|---|---|---|---|---|---|
| A | 1 | 0 | 0.1 | 0 | 0.167 |
| B | 4 | 1 | 0.4 | 0.33 | 0.33 |
| C | 5 | 2 | 0.5 | 0.67 | 0.5 |
| Sum | 10 | | 1.0 | 1.0 | 1.0 |

# Rank-based selection: Exponential Ranking

$$P_{\exp-rank}(i) = \frac{1-e^{-i}}{c}$$



- Linear Ranking is limited in selection pressure
- Exponential Ranking can allocate more than 2 copies to fittest individual
- Normalise constant factor *c* according to population size

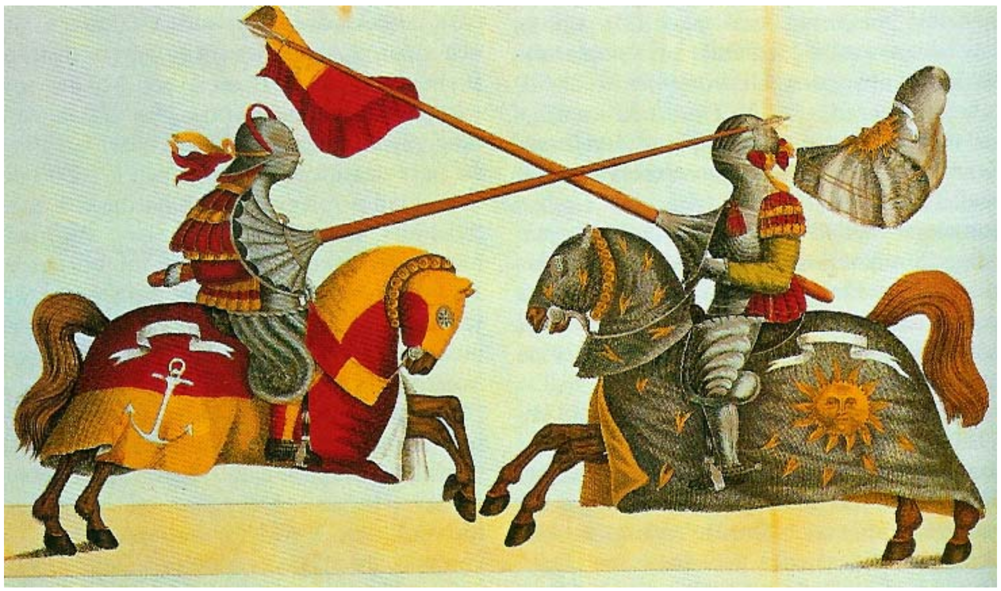
# Parent Selection: Tournament Selection (1/3)

- All methods above rely on global population statistics
  - Could be a bottleneck esp. on parallel machines, very large population
  - Relies on presence of external fitness function which might not exist: e.g. evolving game players

# Parent Selection: Tournament Selection (2/3)

Idea for a procedure using only local fitness information:

- Pick *k* members at random then select the best of these
- Repeat to select more individuals



# Parent Selection: Tournament Selection (3/3)

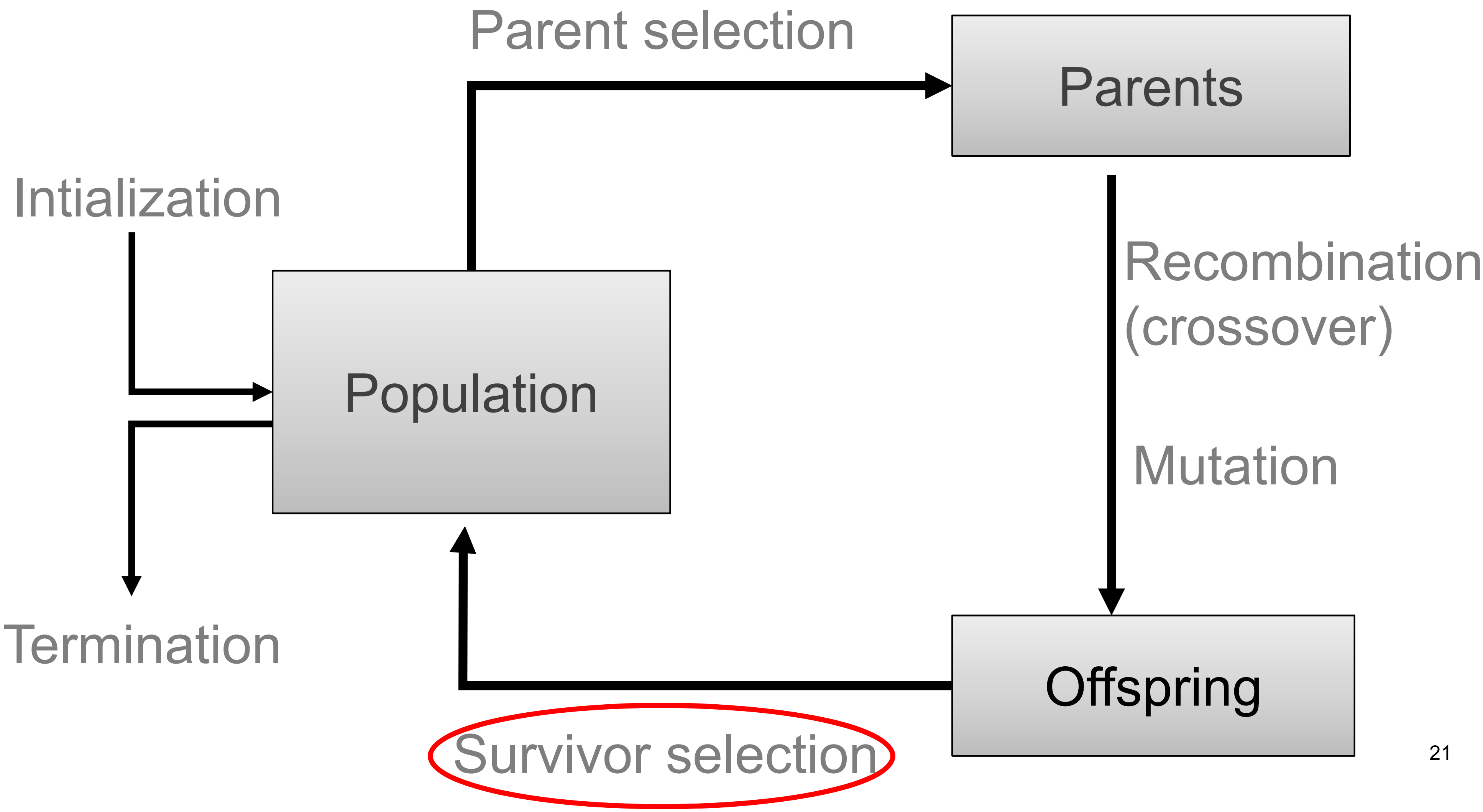
- Probability of selecting *i* will depend on:
  - Rank of *i*
  - Size of sample *k*
    - higher *k* increases selection pressure
  - Whether contestants are picked with replacement
    - Picking without replacement increases selection pressure
  - Whether fittest contestant always wins (deterministic) or this happens with probability *p*

# Parent Selection: Uniform

$$P_{uniform}(i) = \frac{1}{\mu}$$

- Parents are selected by uniform random distribution whenever an operator needs one/some
- Uniform parent selection is unbiased - every individual has the **same probability** to be selected

# Scheme of an EA: General scheme of EAs



# Survivor Selection (Replacement)

- From a set of μ parents and λ offspring: Select a set of μ individuals **forming the next generation**
- Survivor selection can be divided into two approaches:
  - **Age-Based Replacement**
    - Fitness is not taken into account
    - In SS-GA can implement as “delete-random” (not recommended) or as first-in-first-out (a.k.a. delete-oldest)
  - **Fitness-Based Replacement**

# Fitness-based replacement (1/2)

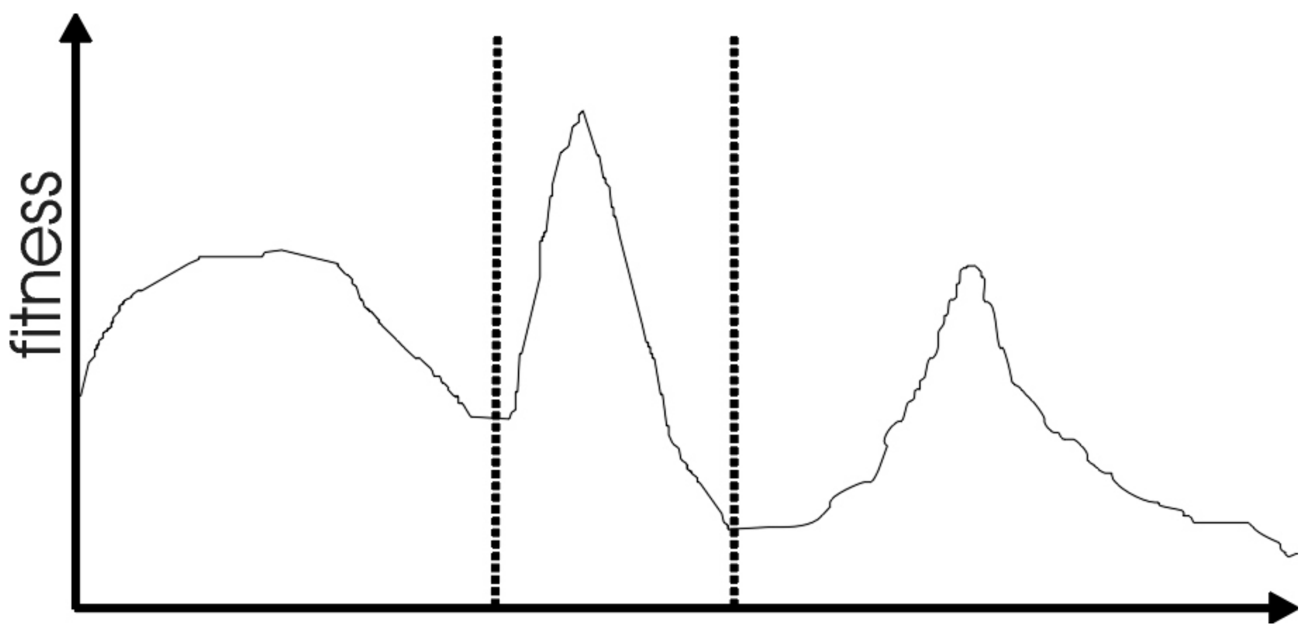
- Elitism
  - Always **keep** at least one copy of **the fittest solution** so far
  - Widely used in both population models (GGA, SSGA)
- Delete Worst
  - The worst $\lambda$ individuals are replaced
- Round-robin tournament (from EP)
  - Pairwise competitions in round-robin format:
    - Each individual x is **evaluated against q other** randomly chosen individuals in 1-on-1 tournaments
    - For each comparison, a "win" is assigned if x is better than its opponent
    - The $\mu$ solutions with the greatest number of wins are the winners of the tournament
  - Parameter q allows tuning selection pressure
  - Typically $q$ = 10

# Fitness-based replacement (2/2) (from ES)

- **($\mu$,$\lambda$)-selection** (best candidates can be lost)
  - based on the set of **children only** ($\lambda > \mu$)
  - choose the **best** $\mu$ offspring for next generation
- **($\mu$+$\lambda$)-selection** (elitist strategy)
  - based on the set of **parents and children**
  - choose the **best** $\mu$ offspring for next generation
- Often ($\mu$,$\lambda$)-selection is preferred because it is better in leaving local optima

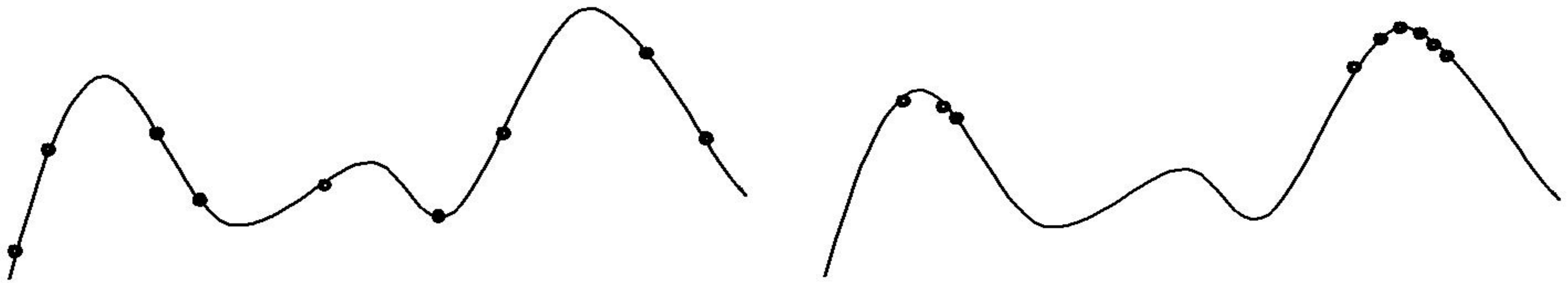
# Multimodality

Most interesting problems have more than one locally optimal solution.



# Multimodality

- Often might want to identify several possible peaks
- Different peaks may be different good ways to solve the problem.
- We therefore need methods to **preserve diversity** (instead of converging to one peak)



# Approaches for Preserving Diversity: Introduction

- Explicit vs implicit
- Implicit approaches:
  - Impose an equivalent of geographical separation
  - Impose an equivalent of speciation
- Explicit approaches
  - Make similar individuals compete for resources (fitness)
  - Make similar individuals compete with each other for survival

# Explicit Approaches for Preserving Diversity: Fitness Sharing (1/2)

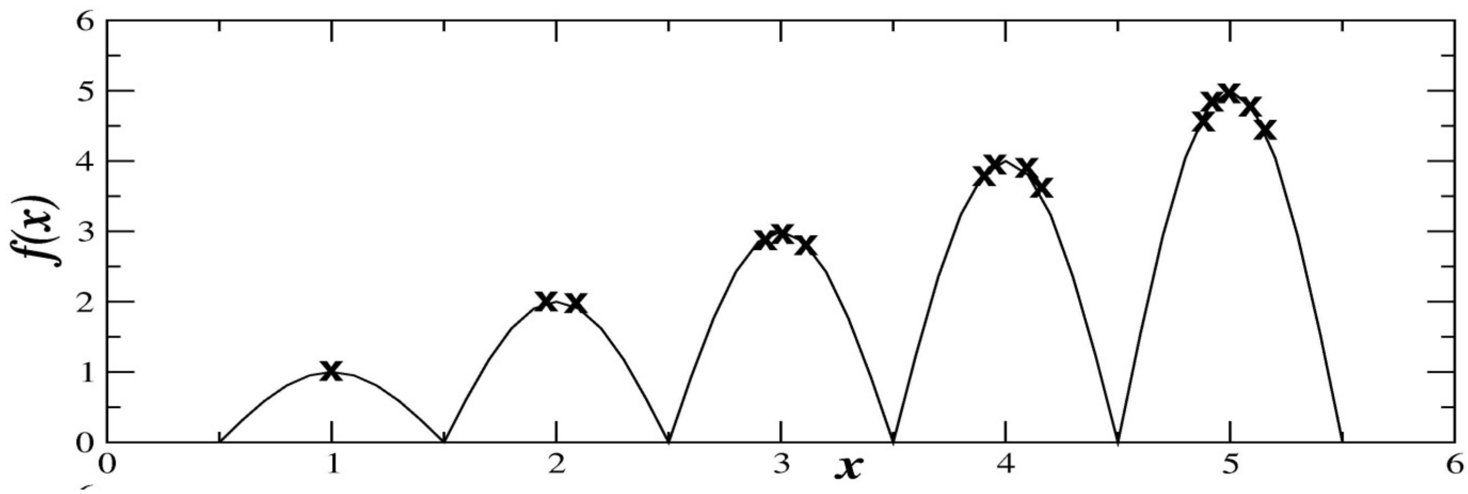
- Restricts the number of individuals within a given niche by "sharing" their fitness
- Need to set the size of the niche $\sigma_{share}$ in either genotype or phenotype space
- run EA as normal but after each generation set

$$f'(i) = \frac{f(i)}{\sum_{j=1}^{\mu} sh(d(i,j))} \qquad sh(d) = \begin{cases} 1 - d/\sigma & d \le \sigma \\ 0 & otherwise \end{cases}$$

# Explicit Approaches for Preserving Diversity: Fitness Sharing (2/2)

$$f'(i) = \frac{f(i)}{\sum_{j=1}^{\mu} sh(d(i,j))} \qquad sh(d) = \begin{cases} 1 - d/\sigma & d \le \sigma \\ 0 & otherwise \end{cases}$$

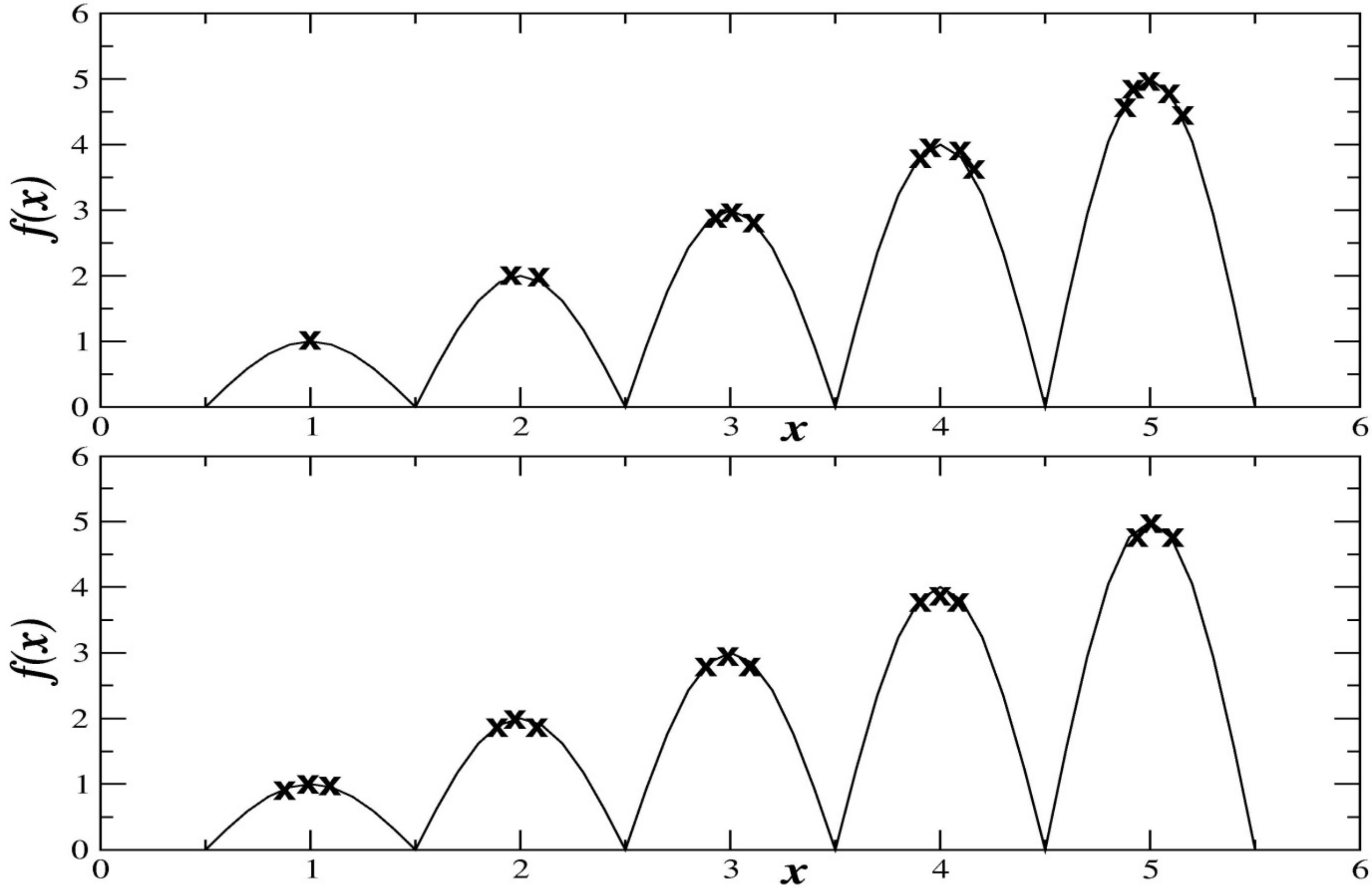




# Explicit Approaches for Preserving Diversity: Crowding

- Idea: New individuals replace *similar* individuals
- Randomly shuffle and pair parents, produce 2 offspring
- Each offspring competes with their **nearest** parent for survival (using a distance measure)
- Result: Even distribution among niches.

# Explicit Approaches for Preserving Diversity: Crowding vs Fitness sharing



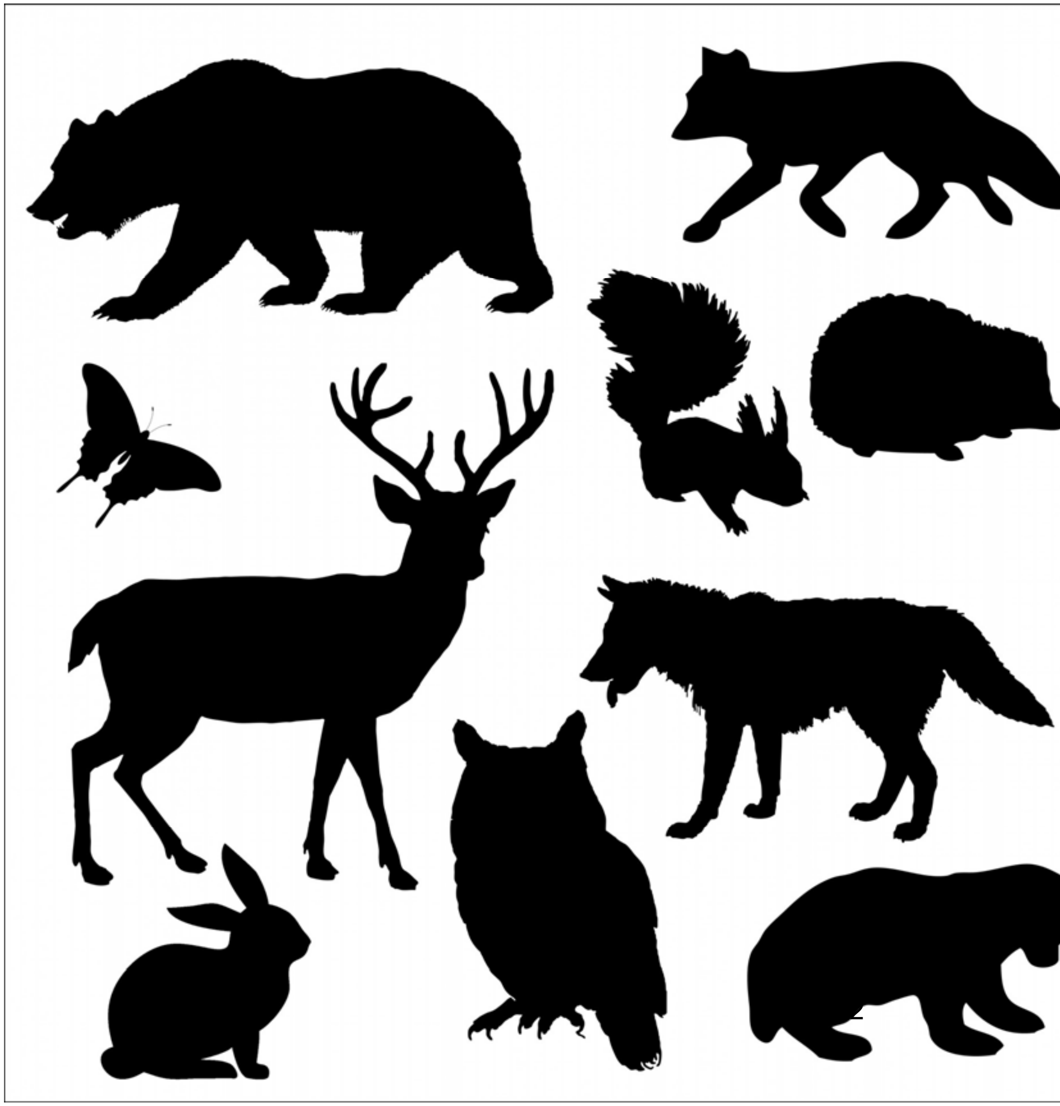

Fitness Sharing

Crowding

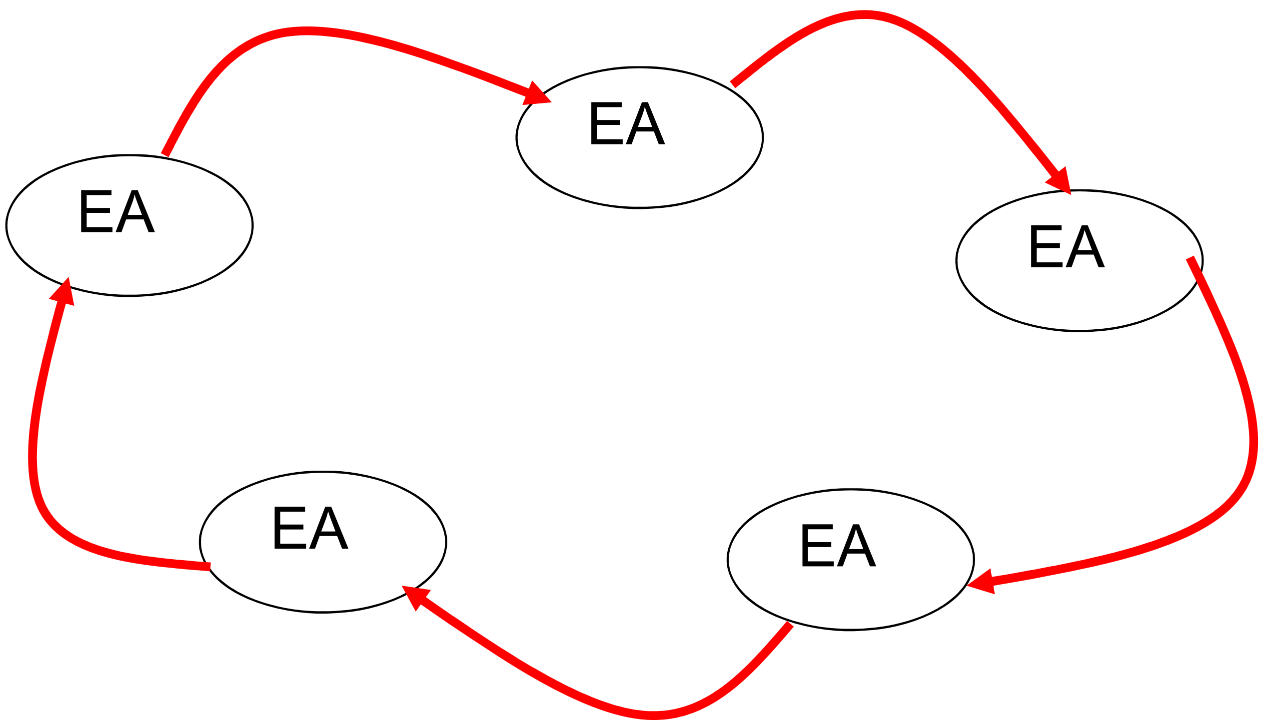
Observe the number of individuals per niche

# Implicit Approaches for Preserving Diversity: Automatic Speciation

- Either only mate with genotypically / phenotypically similar members or
- Add species-tags to genotype
  - initially randomly set
  - when selecting partner for recombination, only pick members with a good match



# Implicit Approaches for Preserving Diversity: “Island” Model Parallel EAs




Periodic migration of individual solutions between populations

# Implicit Approaches for Preserving Diversity: “Island” Model Parallel EAs

- Run multiple populations in parallel
- After a (usually fixed) number of generations (an ***Epoch***), exchange individuals with neighbours
- Repeat until ending criteria met
- Partially inspired by parallel/clustered systems

# Chapter 6:
# Popular Evolutionary Algorithm Variants

Historical EA variants:

- Genetic Algorithms
- Evolution Strategies
- Evolutionary Programming
- Genetic Programming

| **Algorithm** | **Chromosome Representation** | **Crossover** | **Mutation** |
|---|---|---|---|
| Genetic Algorithm (GA) | Array | X | X |
| Genetic Programming (GP) | Tree | X | X |
| Evolution Strategies (ES) | Array | (X) | X |
| Evolutionary Programming (EP) | No constraints | - | X |

# Genetic Algorithms: Overview Simple GA

- Developed: USA in the 1960's
- Early names: Holland, DeJong, Goldberg
- Typically applied to:
  - discrete function optimization
  - benchmark for comparison with other algorithms
  - straightforward problems with binary representation
- Features:
  - not too fast
  - missing new variants (elitism, sus)
  - often modelled by theorists

# Genetic Algorithms: Overview Simple GA (2/2)

- Holland's original GA is now known as the simple genetic algorithm (SGA)
- Other GAs use different:
  - Representations
  - Mutations
  - Crossovers
  - Selection mechanisms

# Genetic Algorithms: SGA reproduction cycle

- **Select parents** for the mating pool
  (size of mating pool = population size)
- Shuffle the mating pool
- **Apply crossover** for each consecutive pair with probability $p_c$, otherwise copy parents
- **Apply mutation** for each offspring (bit-flip with probability $p_m$ independently for each bit)
- **Replace the whole population** with the resulting offspring

# Genetic Algorithms: An example after Goldberg ’89

- Simple problem: max $x^2$ over {0,1,…,31}
- GA approach:
  - Representation: binary code, e.g., 01101 $\leftrightarrow$ 13
  - Population size: 4
  - 1-point x-over, bitwise mutation
  - Roulette wheel selection
  - Random initialisation
- We show one generational cycle done by hand

# $X^2$ example: Selection

| String no. | Initial population | $x$ Value | Fitness $f(x) = x^2$ | $Prob_i$ | Expected count | Actual count |
|---|---|---|---|---|---|---|
| 1 | 0 1 1 0 1 | 13 | 169 | 0.14 | 0.58 | 1 |
| 2 | 1 1 0 0 0 | 24 | 576 | 0.49 | 1.97 | 2 |
| 3 | 0 1 0 0 0 | 8 | 64 | 0.06 | 0.22 | 0 |
| 4 | 1 0 0 1 1 | 19 | 361 | 0.31 | 1.23 | 1 |
| Sum | | | 1170 | 1.00 | 4.00 | 4 |
| Average | | | 293 | 0.25 | 1.00 | 1 |
| Max | | | 576 | 0.49 | 1.97 | 2 |

# $X^2$ example: Crossover

| String no. | Mating pool | Crossover point | Offspring after xover | $x$ Value | Fitness $f(x) = x^2$ |
|---|---|---|---|---|---|
| 1 | 0 1 1 0 \| 1 | 4 | 0 1 1 0 0 | 12 | 144 |
| 2 | 1 1 0 0 \| 0 | 4 | 1 1 0 0 1 | 25 | 625 |
| 2 | 1 1 \| 0 0 0 | 2 | 1 1 0 1 1 | 27 | 729 |
| 4 | 1 0 \| 0 1 1 | 2 | 1 0 0 0 0 | 16 | 256 |
| Sum | | | | | 1754 |
| Average | | | | | 439 |
| Max | | | | | 729 |

# $X^2$ example: Mutation

| String no. | Offspring after xover | Offspring after mutation | $x$ Value | Fitness $f(x) = x^2$ |
|---|---|---|---|---|
| 1 | 0 1 1 0 0 | $\boxed{1}$ 1 1 0 0 | 26 | 676 |
| 2 | 1 1 0 0 1 | 1 1 0 0 1 | 25 | 625 |
| 2 | 1 1 0 1 1 | 1 1 0 1 1 | 27 | 729 |
| 4 | 1 0 0 0 0 | 1 0 $\boxed{1}$ 0 0 | 18 | 324 |
| Sum | | | | 2354 |
| Average | | | | 588.5 |
| Max | | | | 729 |

# Genetic Algorithms: The simple GA

- Has been subject of many (early) studies
  - still often used as benchmark for novel GAs
- Shows many shortcomings, e.g.,
  - Representation is too restrictive
  - Mutation & crossover operators only applicable for bit-string & integer representations
  - Selection mechanism sensitive for converging populations with close fitness values
  - Generational population model can be improved with explicit survivor selection

# Genetic Algorithms: Simple GA (SGA) summary

| | |
|---|---|
| Representation | Bit-strings |
| Recombination | 1-Point crossover |
| Mutation | Bit flip |
| Parent selection | Fitness proportional – implemented by Roulette Wheel |
| Survivor selection | Generational |

# Evolution Strategies: Quick overview

- Developed: Germany in the 1960's by Rechenberg and Schwefel
- Typically applied to numerical optimisation
- Attributed features:
  - fast
  - good optimizer for real-valued optimisation
  - relatively much theory
- Special:
  - self-adaptation of (mutation) parameters standard

# Evolution Strategies: Example (1+1) ES

- Task: minimise $f : R^n \rightarrow R$
- Algorithm: "two-membered ES" using
  - Vectors from $R^n$ directly as chromosomes
  - Population size 1
  - Only mutation creating one child
  - Greedy selection

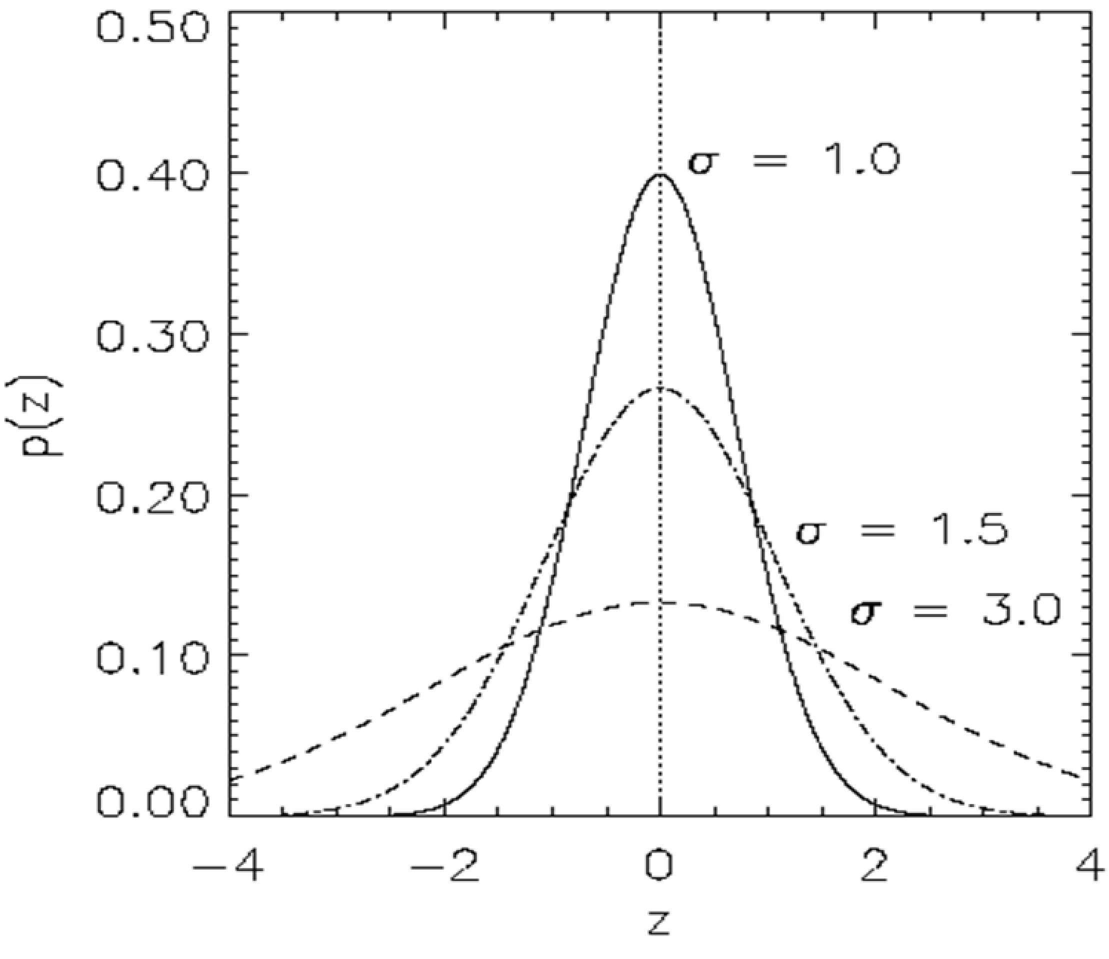
# Evolution Strategies: Representation

- Chromosomes consist of two parts:
  - Object variables: $x_1,\ldots,x_n$
  - Strategy parameters (mutation rate, etc): $p_1,\ldots,p_m$

- Full size: $\langle x_1,\ldots,x_n, p_1,\ldots,p_n \rangle$

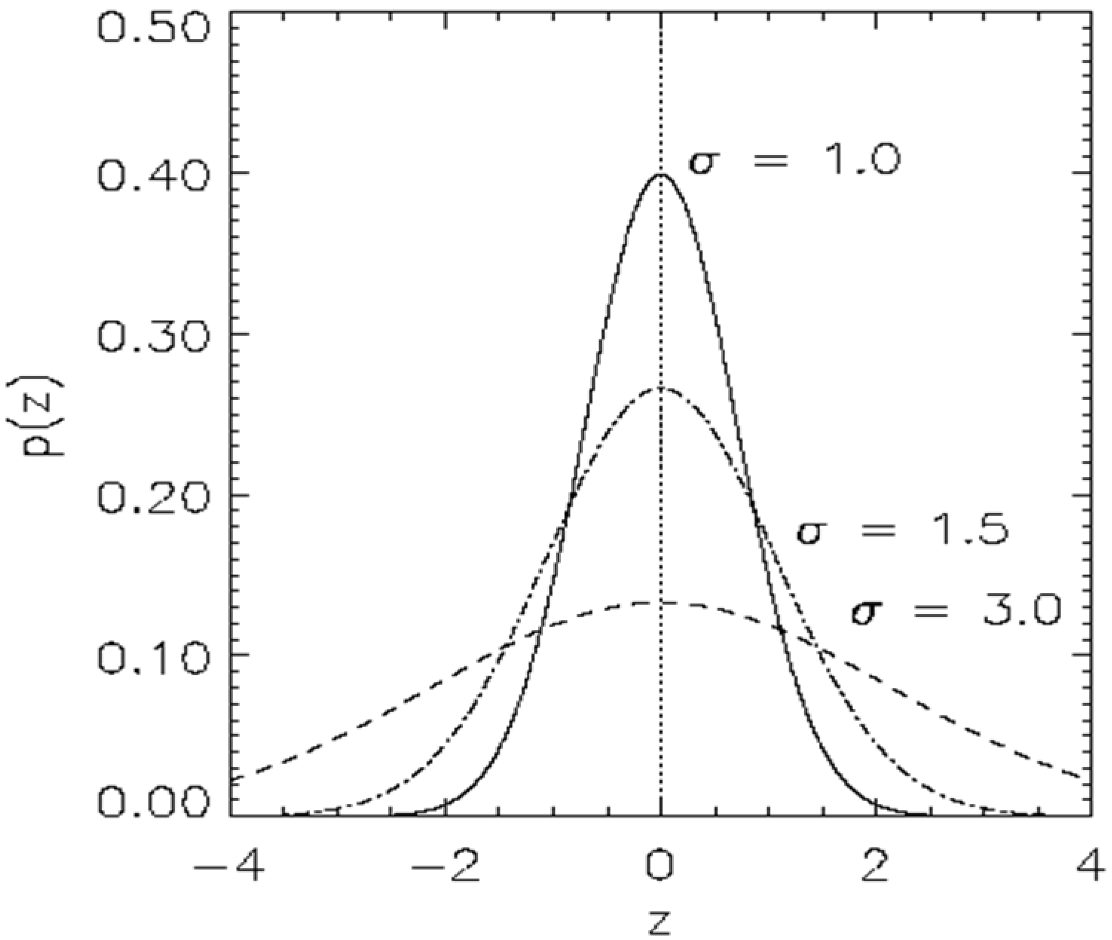
# Evolution Strategies: Adaptive Mutation

- z values drawn from normal distribution N(ξ,σ)
  - mean ξ is set to 0
  - variation σ is called **mutation step size**
- σ is varied on the fly by the “1/5 success rule”



# The "1/5 success rule"

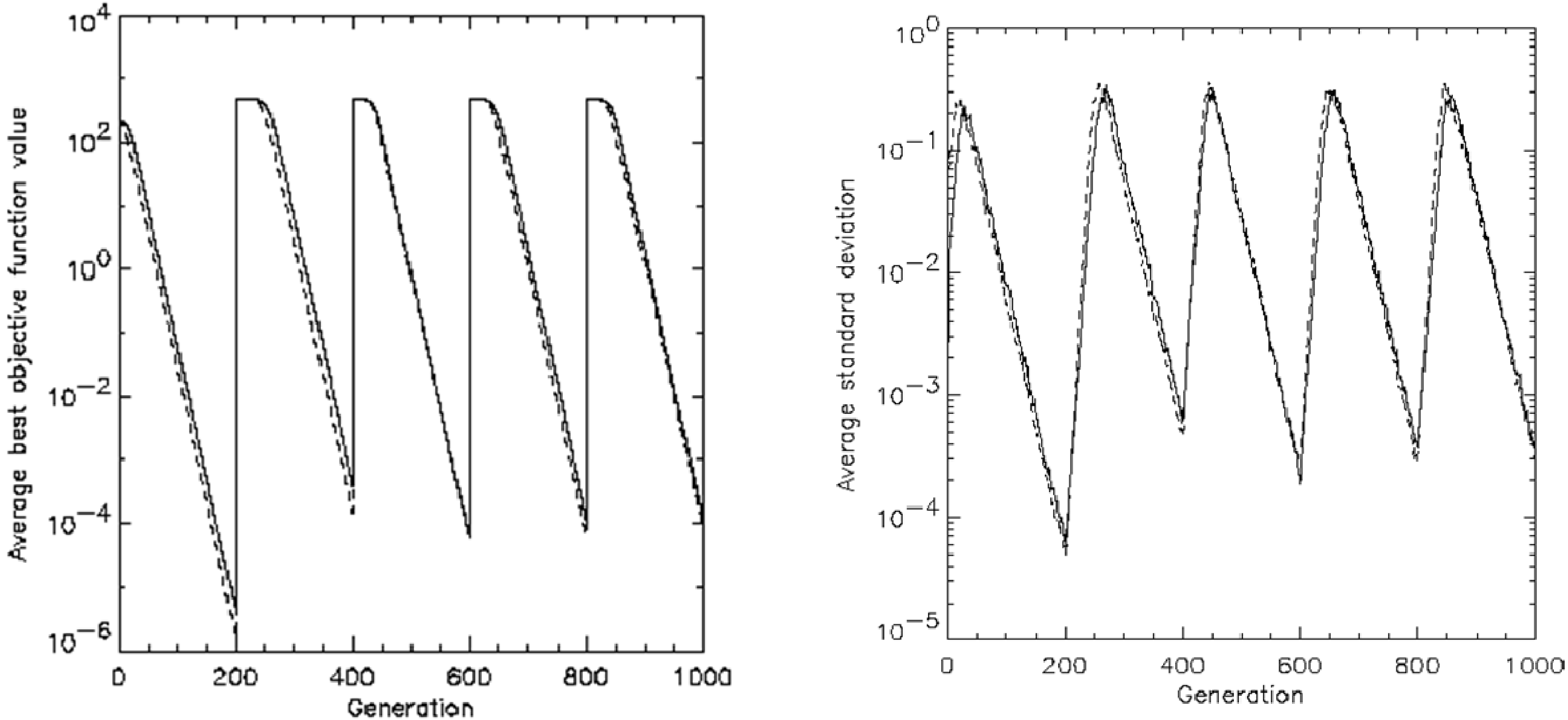
- Goal: Balance exploration and exploitation
- Resets $\sigma$ after every k iterations by
  - $\sigma = \sigma / c$ if $p_s > 1/5$
  - $\sigma = \sigma \cdot c$ if $p_s < 1/5$
  - $\sigma = \sigma$ if $p_s = 1/5$
- where $p_s$ is the % of successful mutations, $0.8 \leq c \leq 1$



# Evolution Strategies: Self-adaptation illustrated (1/2)

- Given a dynamically changing fitness landscape (optimum location shifted every 200 generations)
- Self-adaptive ES is able to
  - follow the optimum and
  - adjust the mutation step size after every shift !

# Evolution Strategies: Self-adaptation illustrated cont'd (2/2)




Changes in the fitness values (left) and the mutation step sizes (right)

# Evolution Strategies: Parent selection

- Parents are selected by **uniform random distribution** whenever an operator needs one/some
- Thus: ES parent selection is unbiased - every individual has the **same probability** to be selected

# Evolution Strategies: Recombination

- Two parents create one child
- Acts per variable / position by either
  - Averaging parental values, or
  - Selecting one of the parental values
- From two or more parents by either:
  - Local recombination: Two parents make a child
  - Global recombination: Selecting two parents randomly for each gene

# Evolution Strategies: Names of recombinations

| | Two fixed parents | Two parents selected for each i |
|---|---|---|
| $z_i = (x_i + y_i)/2$ | Local intermediary | Global intermediary |
| $z_i$ is $x_i$ or $y_i$ chosen randomly | Local discrete | Global discrete |

# Evolution Strategies: ES summary

| | |
|---|---|
| Representation | Real-valued vectors |
| Recombination | Discrete or intermediary |
| Mutation | Gaussian perturbation |
| Parent selection | Uniform random |
| Survivor selection | $(\mu,\lambda)$ or $(\mu+\lambda)$ |

# Evolutionary Programming: Quick overview

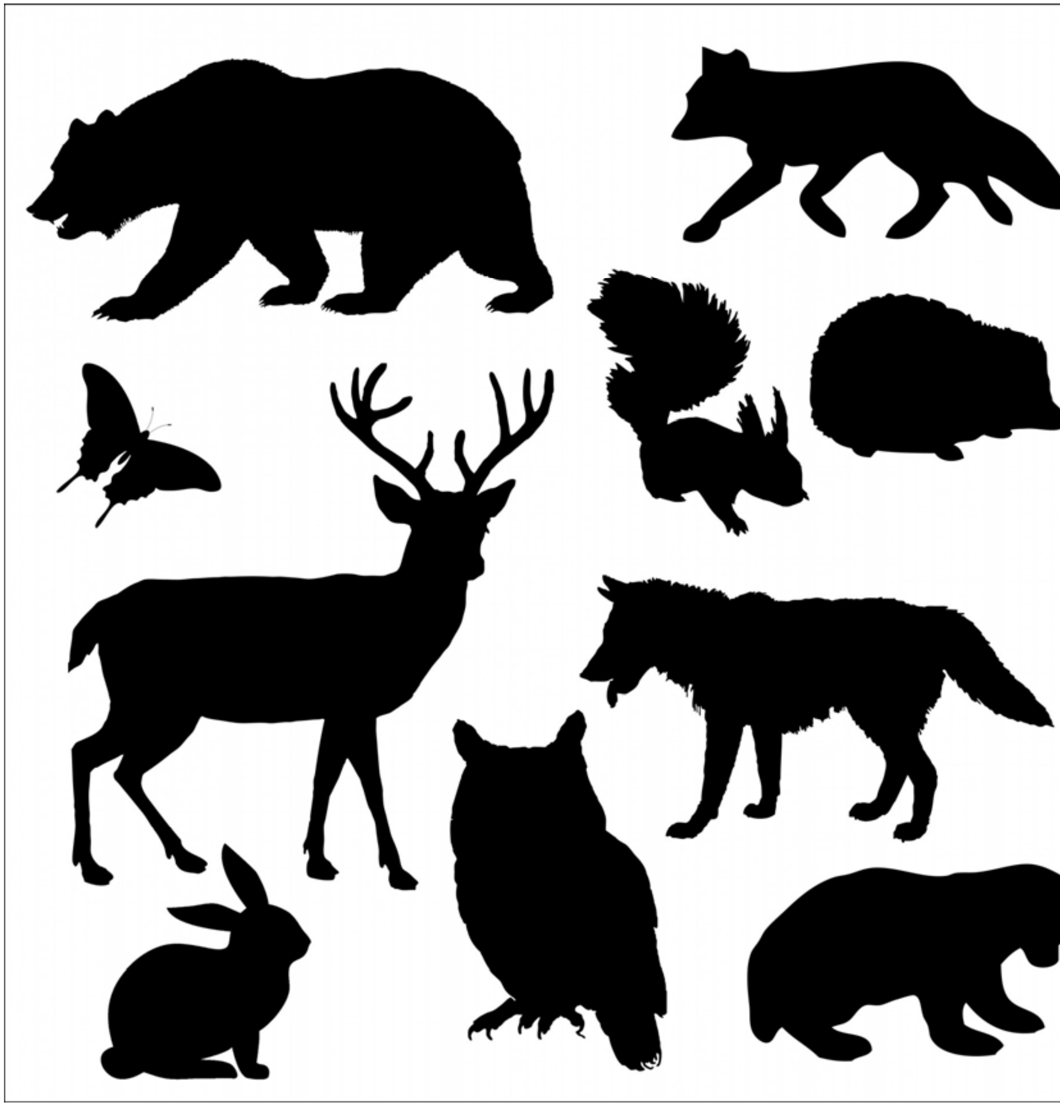
- Developed: USA in the 1960's by Fogel et al.
- Typically applied to:
  - traditional EP: prediction by finite state machines
  - contemporary EP: (numerical) optimization
- Attributed features:
  - very open framework: any representation and mutation op's OK
  - Contemporary EP has almost merged with ES
- Special:
  - **no recombination**
  - self-adaptation of parameters standard (contemporary EP)

# Evolutionary Programming: Representation

- For continuous parameter optimisation
- Chromosomes consist of two parts:
  - Object variables: $x_1,\ldots,x_n$
  - Mutation step sizes: $\sigma_1,\ldots,\sigma_n$
- Full size: $\langle x_1,\ldots,x_n, \sigma_1,\ldots,\sigma_n \rangle$

# Evolutionary Programming: Recombination

- None
- Rationale: one point in the search space stands for a species, not for an individual and there can be no crossover between species



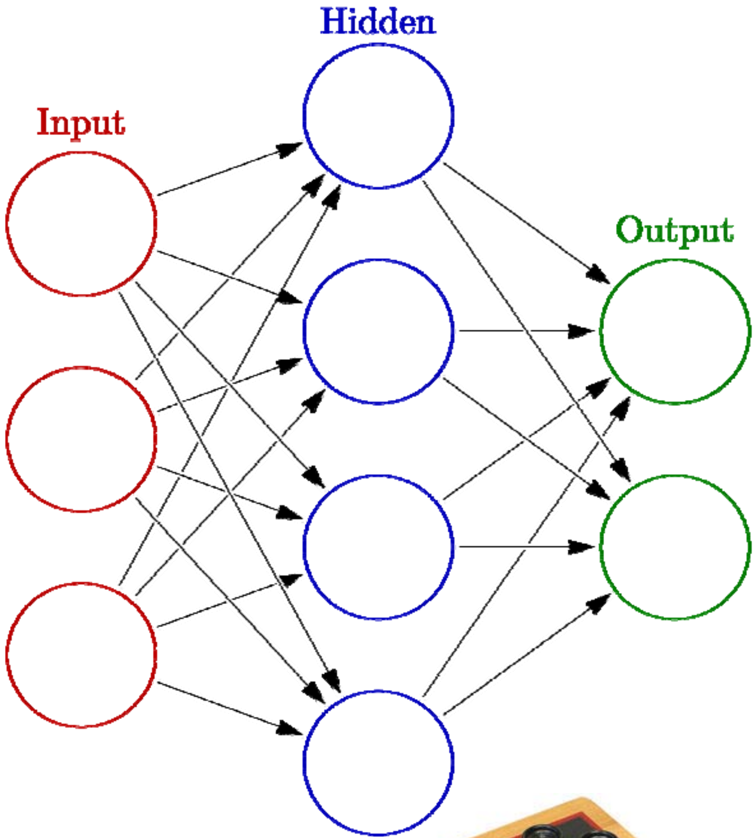
# Evolutionary Programming: Selection

- Each individual creates one child by mutation
  - Deterministic
  - Not biased by fitness

- Parents and offspring compete for survival in round-robin tournaments.

# Evolutionary Programming: Evolving checkers player (Fogel'02) (1/2)

- Neural nets for evaluating future values of moves are evolved
- NNs have fixed structure with 5046 weights, these are evolved
- Representation:
  – vector of 5046 real numbers for NN weights
- Population size 15




# Evolutionary Programming: Evolving checkers player (Fogel'02) (2/2)

- Tournament size q = 5
- Programs (with NN inside) play against other programs, no human trainer
- After 840 generation (6 months!) best strategy was tested against humans
- Program earned "expert class" ranking outperforming 99.61% of all rated players

# Evolutionary Programming: Summary

| | |
|---|---|
| Representation | Real-valued vectors |
| Recombination | None |
| Mutation | Gaussian perturbation |
| Parent selection | Deterministic (each parent one offspring) |
| Survivor selection | Probabilistic ($\mu+\lambda$) |

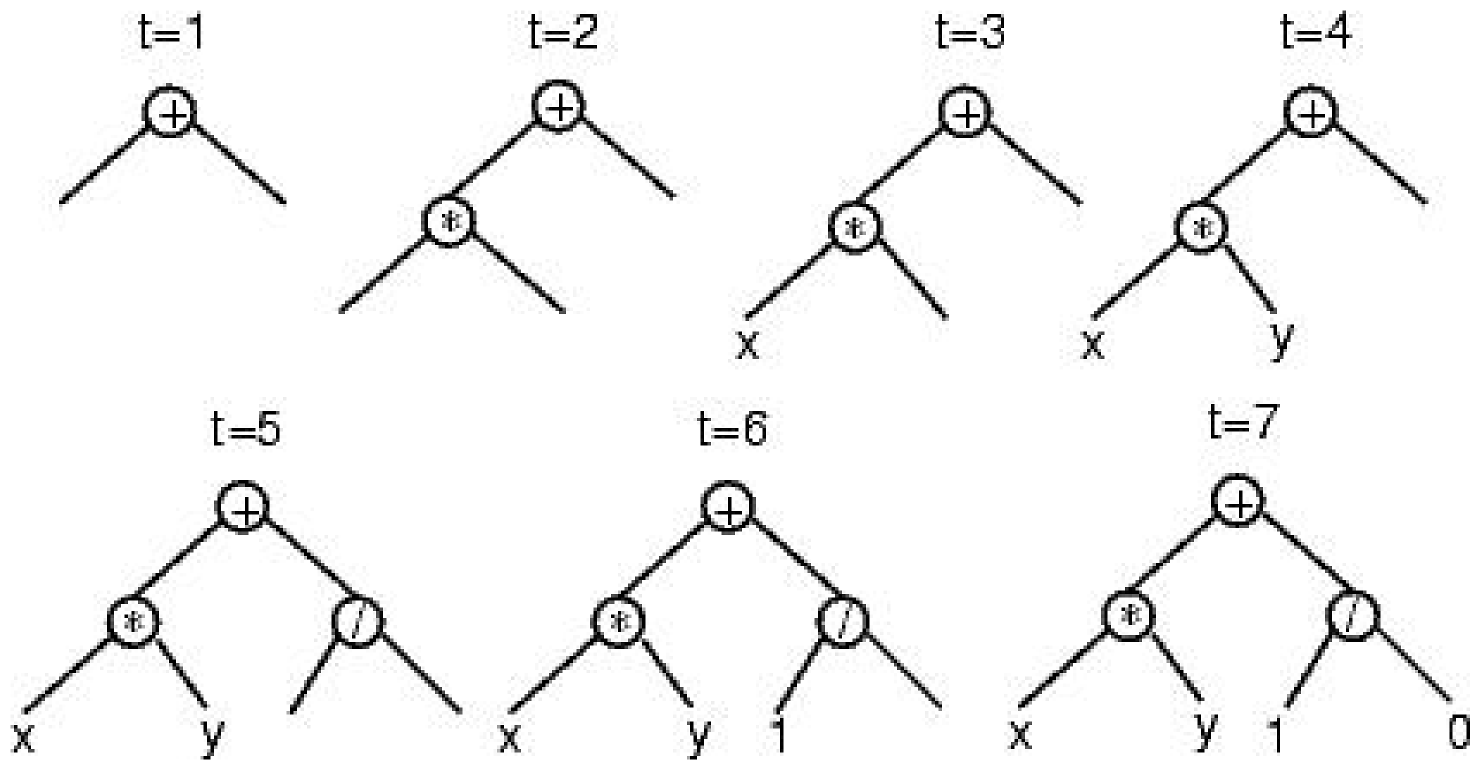
# Genetic Programming: Quick overview

- Developed: USA in the 1990's by Koza
- Typically applied to:
  - machine learning tasks (prediction, classification…)
- Attributed features:
  - "automatic evolution of computer programs"
  - needs huge populations (thousands)
  - slow
- Special:
  - non-linear chromosomes: trees
  - mutation possible but not necessary

# Genetic Programming: Initialisation

- Maximum initial depth of trees $D_{max}$ is set
- Full method (each branch has depth = $D_{max}$):
  - nodes at depth $d < D_{max}$ randomly chosen from **function set F (IF, AND, =, >, *, etc.)**
  - nodes at depth $d = D_{max}$ randomly chosen from **terminal set T (x,y,5000,NOC, etc.)**
- Grow method (each branch has depth $\leq D_{max}$):
  - nodes at depth $d < D_{max}$ randomly chosen from $F \cup T$
  - nodes at depth $d = D_{max}$ randomly chosen from T
- Common GP initialisation: ramped half-and-half, where grow & full method each deliver half of initial population

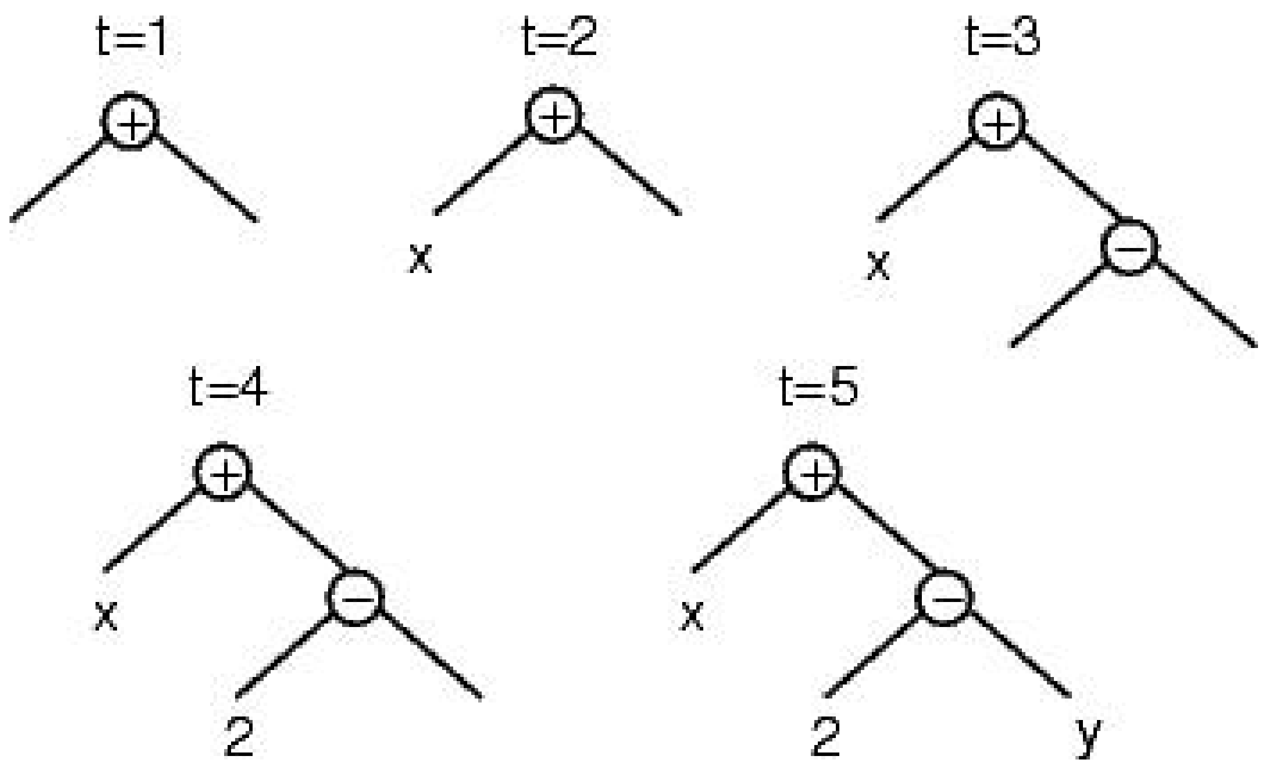
# Genetic Programming: Full Initialisation to depth 2






Source: www.gp-field-guide.org.uk

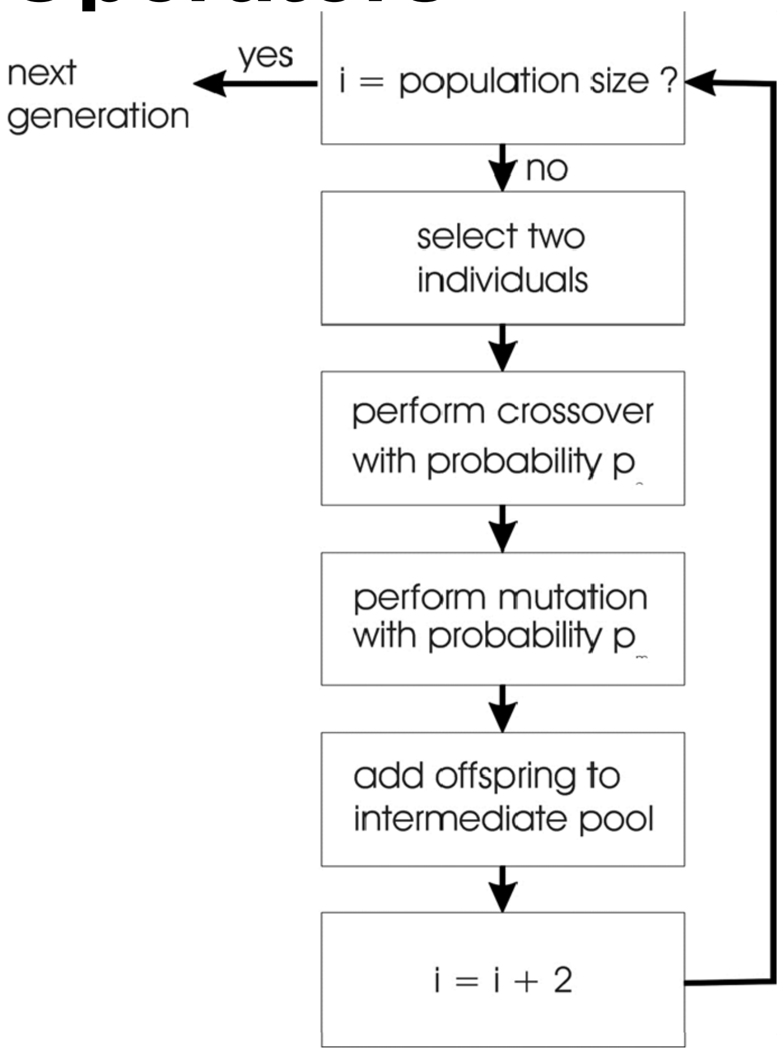
# Genetic Programming: Grow Initialisation to depth 2



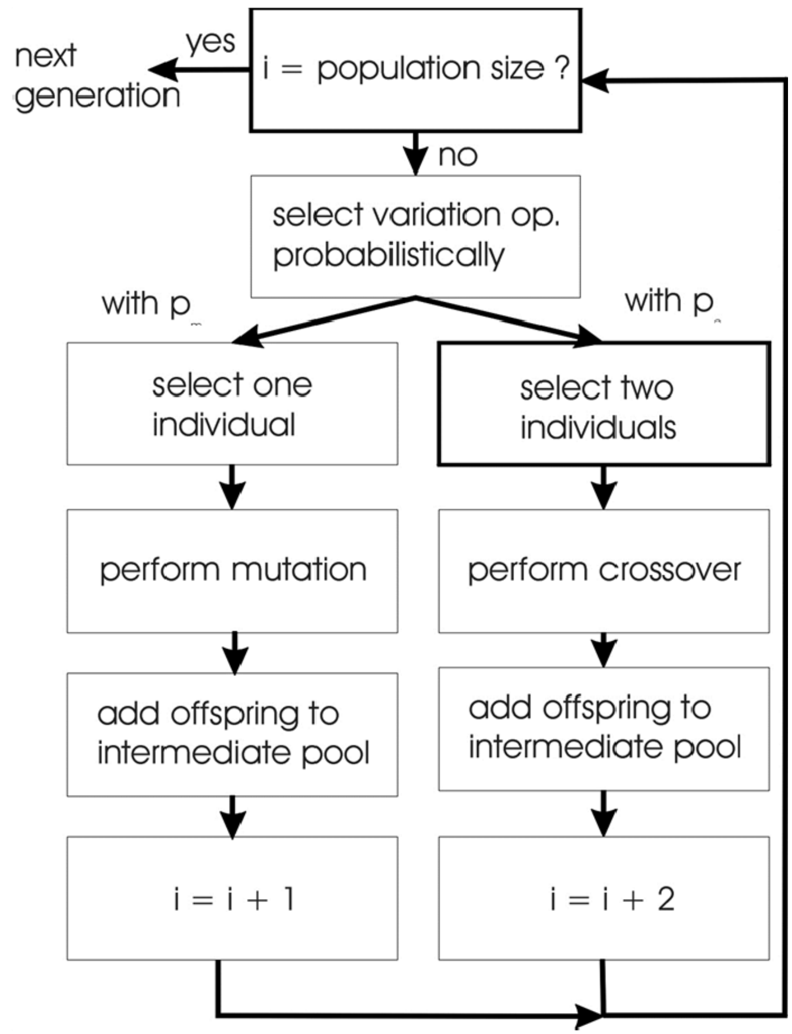



[Source: www.gp-field-guide.org.uk](http://www.gp-field-guide.org.uk)

# Genetic Programming: Variation Operators



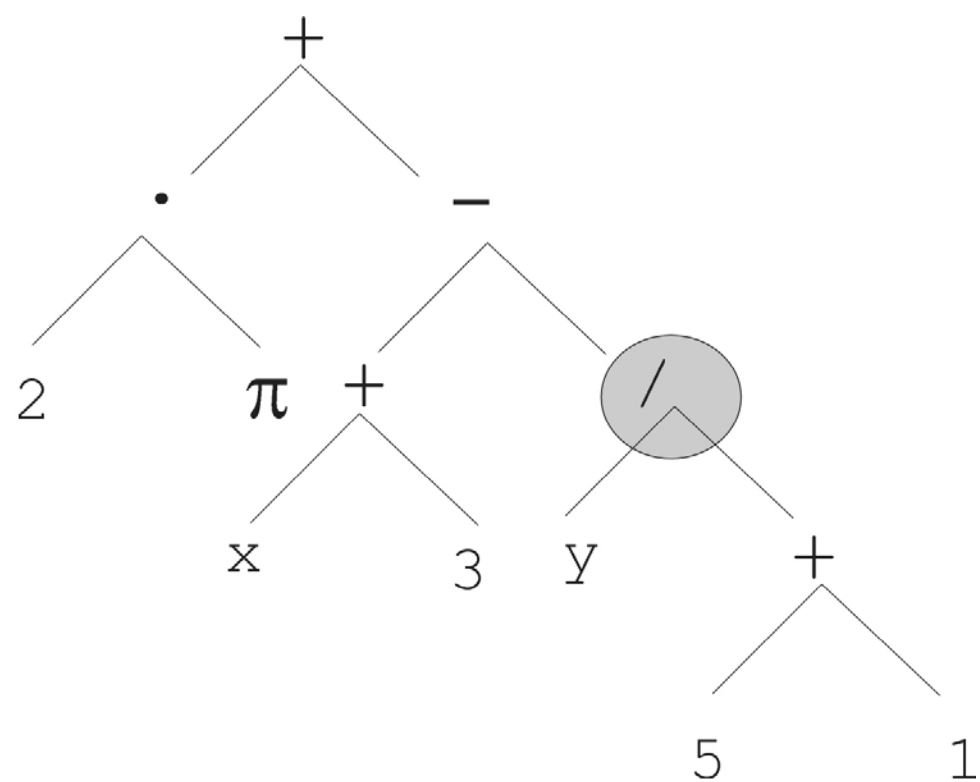

GA flowchart

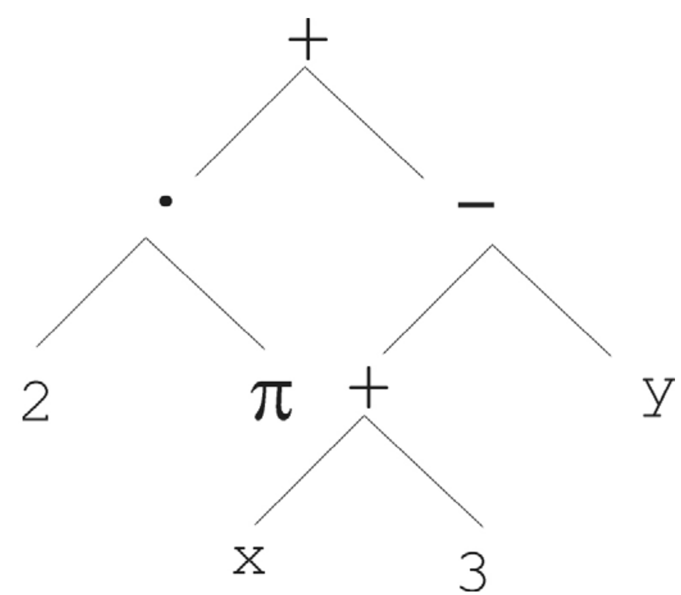



GP flowchart

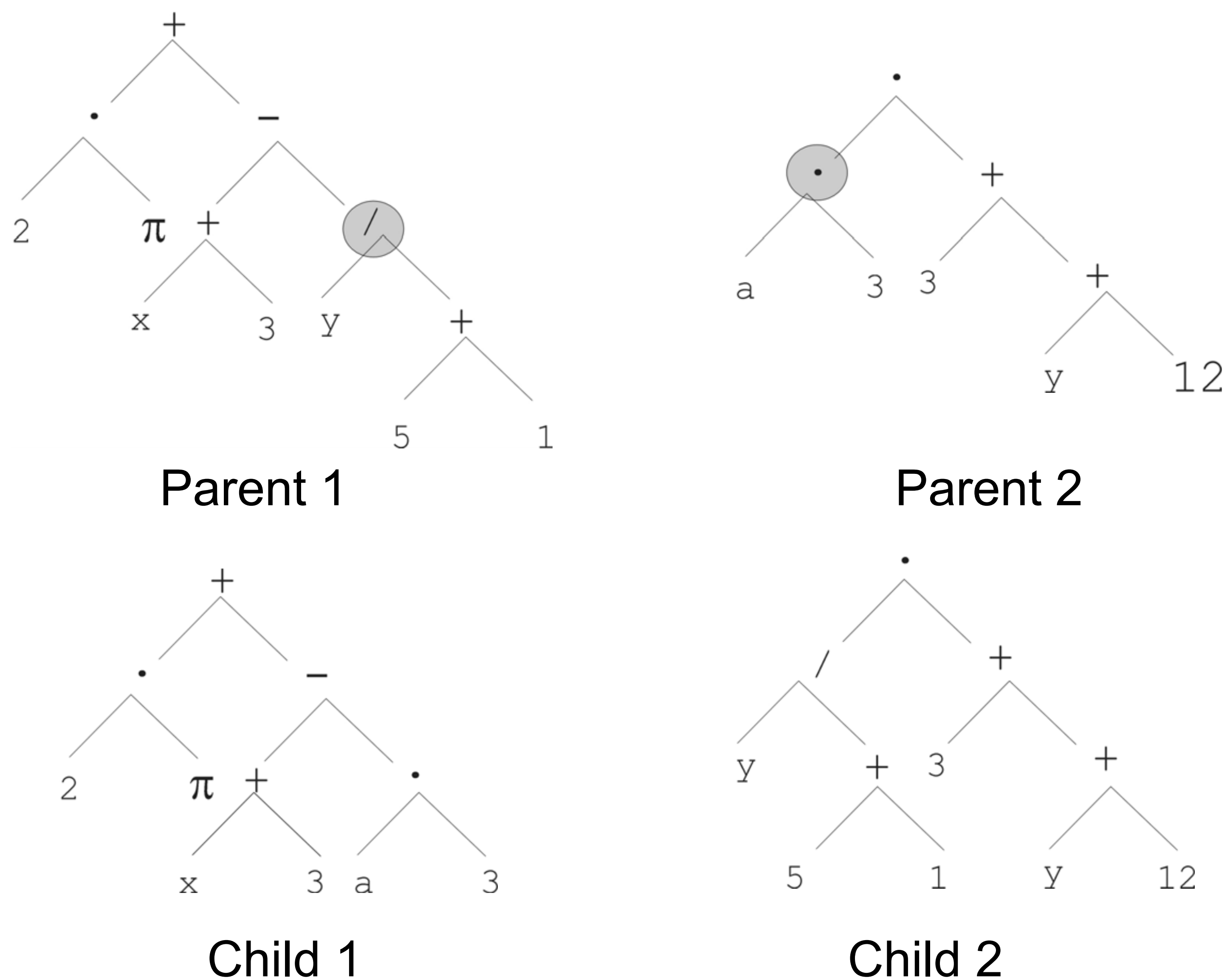
# Genetic Programming: Mutation

- Most common mutation: replace randomly chosen subtree by randomly generated tree



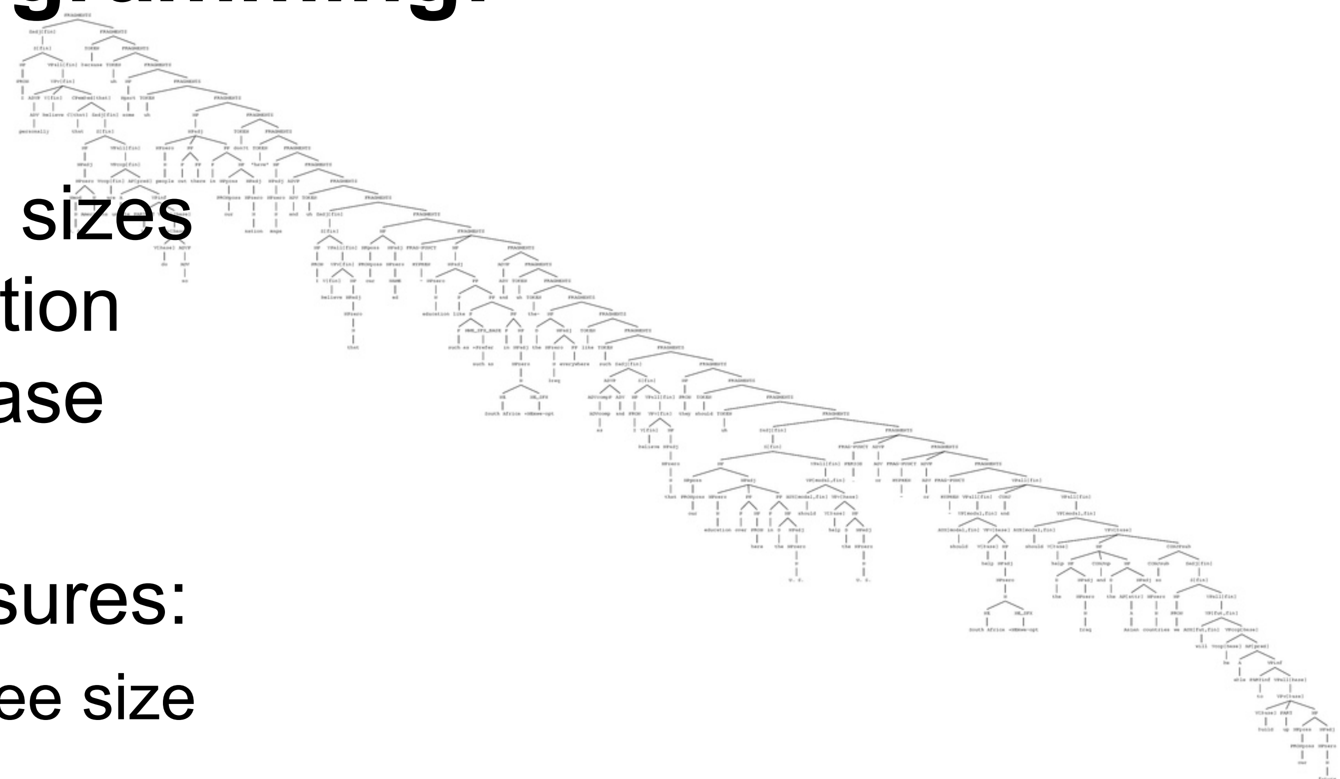



# Genetic Programming: Recombination



# Genetic Programming: Bloat

- Average tree sizes in the population tend to increase over time
- Countermeasures:
  - Maximum tree size
  - Parsimony pressure: penalty for being oversized



# Genetic Programming: Summary

| | |
|---|---|
| Representation | Tree structures |
| Recombination | Exchange of subtrees |
| Mutation | Random change in trees |
| Parent selection | Fitness proportional |
| Survivor selection | Generational replacement |

# Summary: The standard EA variants

| Name | Representation | Crossover | Mutation | Parent selection | Survivor selection | Specialty |
|---|---|---|---|---|---|---|
| Genetic Algorithm | Usually fixed-length vector | Any or none | Any | Any | Any | None |
| Evolution Strategies | Real-valued vector | Discrete or intermediate recombination | Gaussian | Random draw | Best N | Strategy parameters |
| Evolutionary Programming | Real-valued vector | None | Gaussian | One child each | Tournament | Strategy parameters |
| Genetic Programming | Tree | Swap sub-tree | Replace sub-tree | Usually fitness proportional | Generational replacement | None |

# Particle Swarm Optimisation: Quick overview

- Developed: in 1995 by Kennedy and Eberhart
- Inspired by social behavior of bird flocking/fish schooling

# Particle Swarm Optimisation: Representation (1/2)

- Every population member can be considered as a pair $\langle \bar{x}, \bar{p} \rangle$ where the first vector is candidate solution and the second one a perturbation vector in $IR^n$
- The perturbation vector determines how the solution vector is changed to produce a new one: $\bar{x}' = \bar{x} + \bar{p}'$, where $\bar{p}'$ is calculated from $\bar{p}$ and some additional information

# Particle Swarm Optimisation: Representation (2/2)

- A member is a point in space with a position and a velocity
- The perturbation is defined as the weighted sum of three components:
  - Current perturbation vector
  - Vector difference current position to best position of member so far
  - Vector difference from current position to best position of population so far



Image source: http://wirelesstechthoughts.blogspot.no/

# A Reminder about Search Landscapes

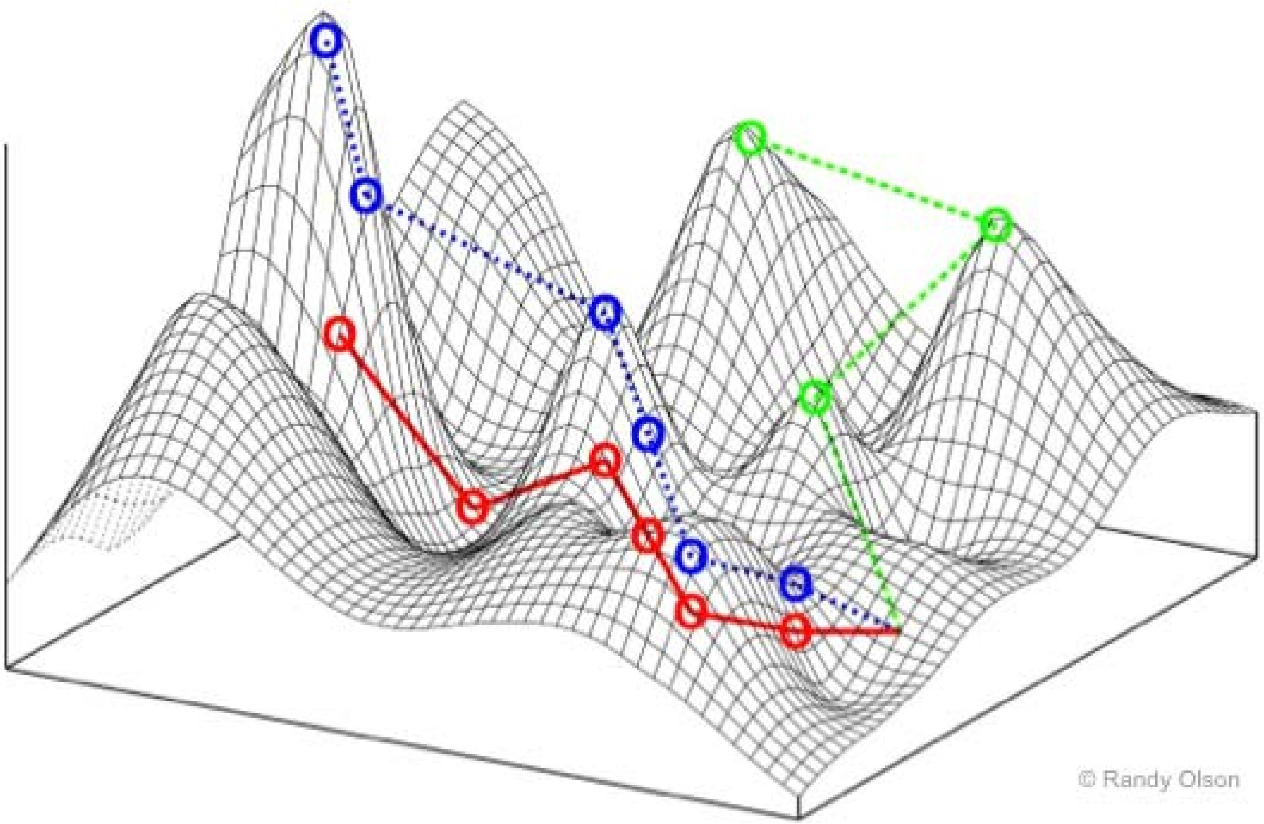





Image source: Randy Olson, wikipedia

# PSO: Velocity and Position Update

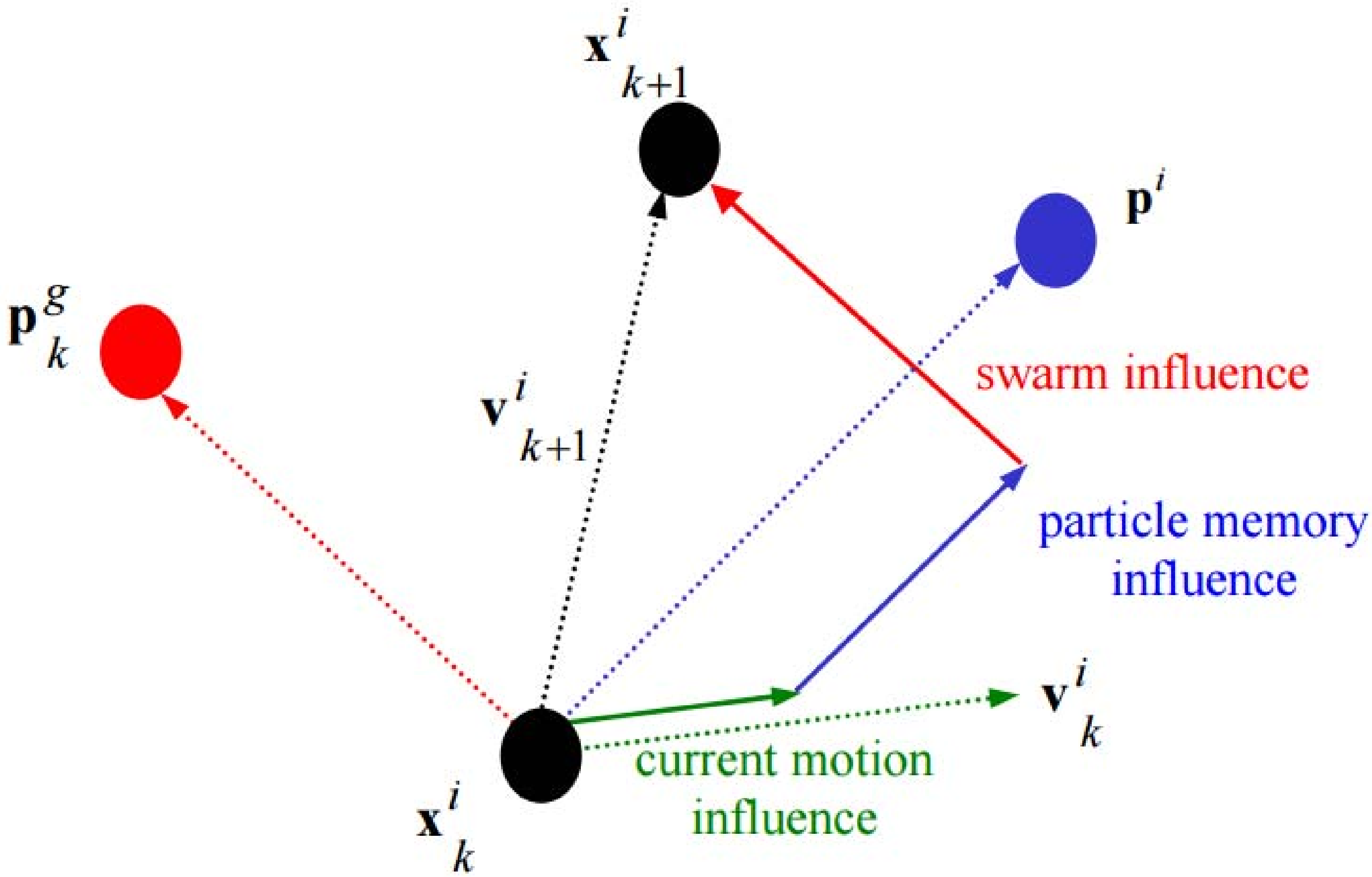
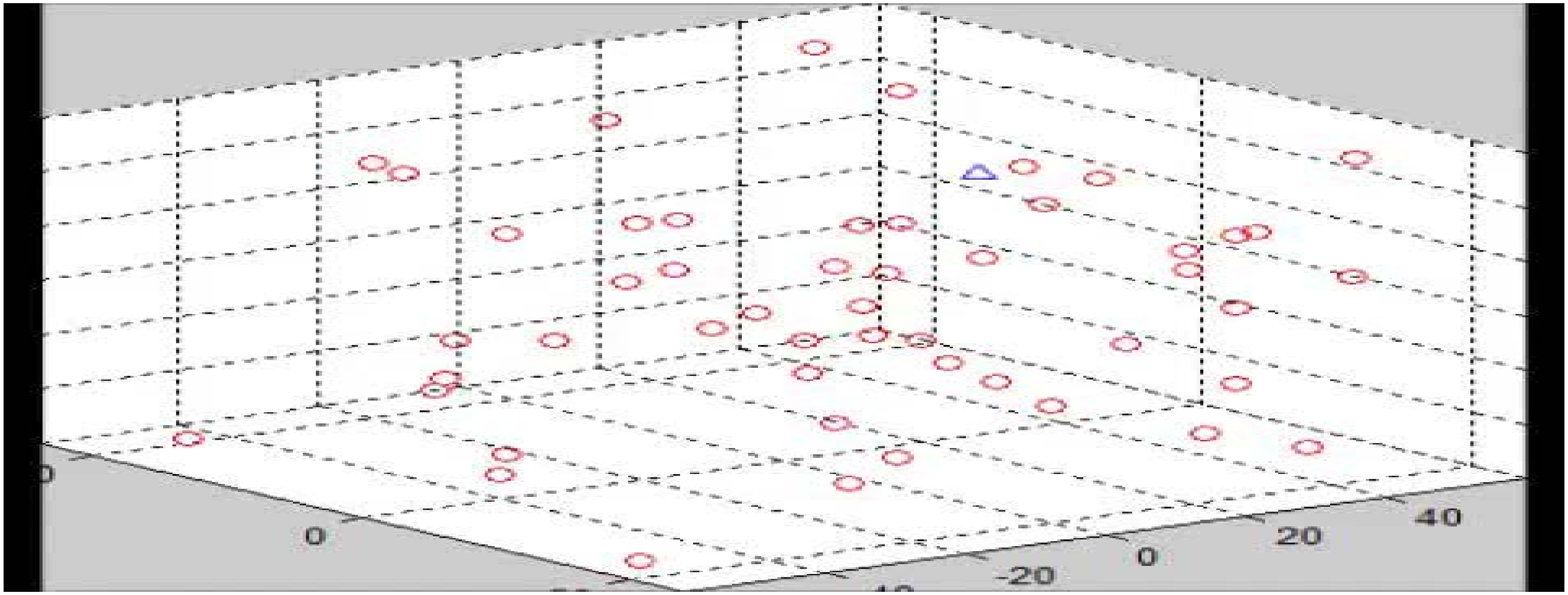





Image source: Hassan et al. 2004: A COPMARISON OF PARTICLE SWARM OPTIMIZATION AND THE GENETIC ALGORITHM

# Particle Swarm Optimisation: Example moving target

- Optimum moves randomly
- Particles do not know position of optimum but do know which particle is closest and are attracted to that one



# Particle Swarm Optimisation: Summary

| | |
|---|---|
| Representation | Real-valued vectors |
| Recombination | None |
| Mutation | Adding velocity vector |
| Parent selection | Deterministic (each parent creates one offspring via mutation) |
| Survivor selection | Generational (offspring replaces parents) |