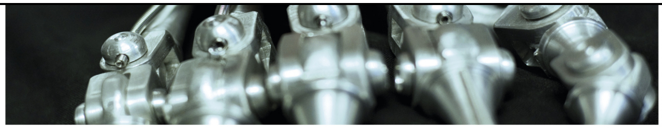

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INF3490 - Biologically inspired computing

Lecture 5: 21 September 2016

Intro to machine learning and
single-layer neural networks

Jim Tørresen

This Lecture

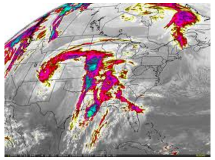
1. Introduction to learning/classification
2. Biological neuron
3. Perceptron and artificial neural networks

Things You Might Be Interested In

Learning from Data

The world is driven by data.

- Germany's climate research centre generates 10 petabytes per year
- Google processes 24 petabytes per day (2009, 1000 Terabytes)
- The Large Hadron Collider produces 60 gigabytes per minute (~12 DVDs)
- There are over 50m credit card transactions a day in the US alone.

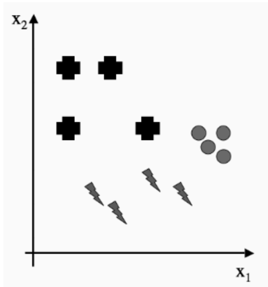


Big Data: If Data Had Mass, the Earth Would Be A Black Hole

- Around the world, computers capture and store terabytes of data everyday.
- Science has also taken advantage of the ability of computers to store massive amount of data.
- The **size and complexity** of these data sets means that humans are unable to extract useful information from them.

High-dimensional data

x_1	x_2	Class
0.1	1	1
0.15	0.2	2
0.48	0.6	3
0.1	0.6	1
0.2	0.15	2
0.5	0.55	3
0.2	1	1
0.3	0.25	2
0.52	0.6	3
0.3	0.6	1
0.4	0.2	2
0.52	0.5	3



A set of data points as numerical values as points plotted on a graph. It is easier for us to **visualize** data than to see it in a table, but if the data has ***more than three dimensions***, we can't view it all at once.

High-dimensional data

Two views of the same two wind turbines (Te Apiti wind farm, Ashhurst, New Zealand) taken at an angle of about 30^0 to each other. The **two-dimensional projections** of three-dimensional objects **hide information**.

Machine Learning

- Ever since computers were invented, we have wondered whether they might be made to learn.
- The ability of a program to **learn from experience** — that is, to modify its execution on the basis of newly acquired information.
- Machine learning is about automatically **extracting relevant information** from data and **applying it to analyze new data.**

Idea Behind

- Humans can:
  - **sense**: see, hear, feel, ++
  - **reason**: think, ***learn***, understand language, ++
  - **respond**: move, speak, act ++
- Artificial Intelligence aims to reproduce these capabilities.
- Machine Learning is ***one*** part of Artificial Intelligence.

Characteristics of ML

- Typically used for **classification tasks**
- **Learning from examples** to analyze new data
- **Generalization:** Provide sensible outputs for inputs not encountered during training
- Iterative learning process
- Learning from scratch or **adapt** a previously learned system

What is Learning?

- "Learning is any process by which a system improves performance from experience."
- Humans and other animals can display behaviours that we label as ***intelligent*** by ***learning from experience***.
  - Learning a set of new facts
  - Learning HOW to do something
  - Improving ability of something already learned

Ways humans learn things

- …talking, walking, running…
  - Learning by mimicking, reading or being told facts
- Tutoring
  - Being informed when one is correct
- Experience
  - Feedback from the environment
- Analogy
  - Comparing certain features of existing knowledge to new problems
- Self-reflection
  - Thinking things in ones own mind, deduction, discovery

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When to Use Learning?

- Human expertise does not exist (navigating on Mars).
- Humans are unable to explain their expertise (speech recognition).
- Solution changes in time (routing on a computer network).
- Solution needs to be adapted to particular cases (user biometrics)
- Interfacing computers with the real world (noisy data)
- Dealing with large amounts of (complex) data

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Why Machine Learning?

- **Extract knowledge**/information from **past experience/data**
- Use this knowledge/information to **analyze new experiences**/data
- Designing rules to deal with new data by hand can be difficult
  - How to write a program to detect a cat in an image?
- Collecting data can be easier
  - Find images with cats, and ones without them
- Use machine learning to automatically find such rules.

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What is the Learning Problem?

- Learning = Improving with experience at some task
  - Improve over task *T*
  - with respect to performance measure *P*
  - based on experience *E*

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Defining the Learning Task

(Improve on task, T, with respect to performance metric, P, based on experience, E)

T: Playing checkers

P: Percentage of games won against an arbitrary opponent

E: Playing practice games against itself

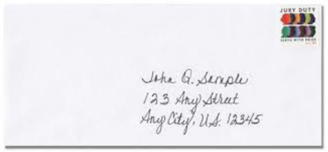
Defining the Learning Task

(Improve on task, T, with respect to performance metric, P, based on experience, E)

T: Recognizing hand-written words

P: Percentage of words correctly classified

E: Database of human-labeled images of handwritten words



Defining the Learning Task

(Improve on task, T, with respect to performance metric, P, based on experience, E)

T: Driving on four-lane highways using vision sensors

P: Average distance traveled before a human-judged error

E: A sequence of images and steering commands recorded while observing a human driver.

Types of Machine Learning

- ML can be loosely defined as **getting better at some task through practice**.
- This leads to a couple of vital questions:
  - How does the computer know whether it is getting better or not?
  - How does it know how to improve?

There are several different possible answers to these questions, and they produce different types of ML.

Types of ML

- **Supervised learning:** Training data ***includes desired outputs***. Based on this training set, the algorithm generalises to respond correctly to all possible inputs.
- **Unsupervised learning:** Training data ***does not include desired outputs***, instead the algorithm tries to identify similarities between the inputs that have something in common are categorised together.

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Types of ML

- **Reinforcement learning:** The algorithm is told when the answer is wrong, but ***does not get told how to correct it***. Algorithm must balance exploration of the unknown environment with exploitation of immediate rewards to maximize long-term rewards.
- **Evolutionary learning:** Biological organisms adapt to improve their survival rates and chance of having offspring in their environment, using the idea of ***fitness (how good the current solution is)***.

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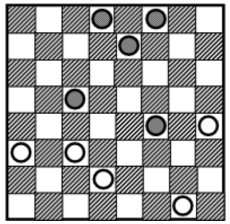
A Bit of History

- Arthur Samuel (1959) wrote a program that learned to play draughts ("checkers" if you're American).



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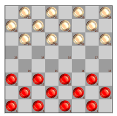
1940s
Human reasoning / logic first studied as a formal subject within mathematics (Claude Shannon, Kurt Godel et al).

1950s
The "Turing Test" is proposed: a test for true machine intelligence, expected to be passed by year 2000. Various game-playing programs built. 1956 "Dartmouth conference" coins the phrase "artificial intelligence".

1960s
A.I. funding increased (mainly military).
Neural networks: Perceptron
Minsky and Papert prove limitations of Perceptron

Ax. 1. $P(\varphi) \wedge \Box\, \forall x[\varphi(x) \to \psi(x)] \to P(\psi)$
Ax. 2. $P(\neg\varphi) \leftrightarrow \neg P(\varphi)$
Th. 1. $P(\varphi) \to \Diamond\, \exists x\, [\varphi(x)]$
Df. 1. $G(x) \iff \forall\varphi[P(\varphi) \to \varphi(x)]$
Ax. 3. $P(G)$
Th. 2. $\Diamond\, \exists x\, G(x)$
Df. 2. $\varphi \text{ ess } x \iff \varphi(x) \wedge \forall\psi\{\psi(x) \to \Box\, \forall x[\varphi(x) \to \psi(x)]\}$
Ax. 4. $P(\varphi) \to \Box\, P(\varphi)$
Th. 3. $G(x) \to G \text{ ess } x$
Df. 3. $E(x) \iff \forall\varphi[\varphi \text{ ess } x \to \Box\, \exists x\, \varphi(x)]$
Ax. 5. $P(E)$
Th. 4. $\Box\, \exists x\, G(x)$



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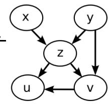
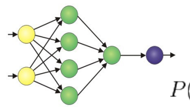
1970s
A.I. "winter". Funding dries up as people realise it's hard.
Limited computing power and dead-end frameworks.

1980s
Revival through bio-inspired algorithms: Neural networks (*connectionism, backpropagation*), Genetic Algorithms.
A.I. promises the world – lots of commercial investment – mostly fails.
Rule based "expert systems" used in medical / legal professions.
Another AI winter.

1990s
AI diverges into separate fields: Computer Vision, Automated Reasoning, Planning systems, Natural Language processing, **Machine Learning**…

…Machine Learning begins to overlap with statistics / probability theory.





$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}.$$

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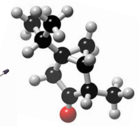
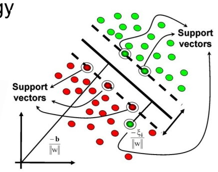
2000s

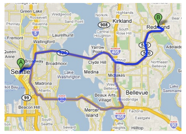
ML merging with statistics continues. Other subfields continue in parallel.

First commercial-strength applications: Google, Amazon, computer games, route-finding, credit card fraud detection, etc…

Tools adopted as standard by other fields e.g. biology





2010s…. ??????

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Supervised learning

- Training data provided as pairs:

$$\{(\mathbf{x}_1, \mathbf{f}(\mathbf{x}_1)), (\mathbf{x}_2, \mathbf{f}(\mathbf{x}_2)), \ldots, (\mathbf{x}_P, \mathbf{f}(\mathbf{x}_P))\}$$

- The goal is to predict an “output” *y* from an “input *x*”:

$$\mathbf{y} = \mathbf{f}(\mathbf{x})$$

- Output *y* for each input *x* is the “supervision” that is given to the learning algorithm.
  - Often obtained by manual annotation
  - Can be costly to do
- Most common examples
  - Classification
  - Regression

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Classification

- Training data consists of “inputs”, denoted *x*, and corresponding output “class labels”, denoted as *y*.
- Goal is to correctly predict for a test data input the corresponding class label.
- Learn a “classifier” *f(x)* from the input data that **outputs the class label or a probability over the class labels**.
- Example:
  - Input: image
  - Output: category label, eg “cat” vs. “no cat”

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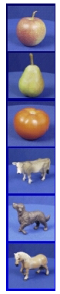
Example of classification

Given: training images and their categories



What are the categories of these test images?

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Classification

- Two main phases:
  - ***Training**:* Learn the classification model from labeled data.
  - ***Prediction**:* Use the pre-built model to classify new instances.
- Classification can be binary (two classes), or over a larger number of classes (multi-class).
  - In binary classification we often refer to one class as "positive", and the other as "negative"
- Binary classifier creates a boundaries in the input space between areas assigned to each class

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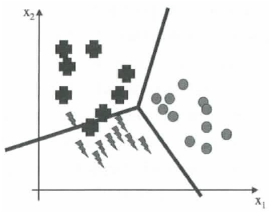
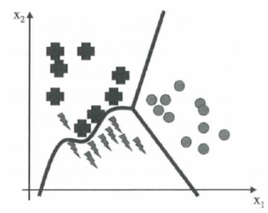
Classification using Decision Boundaries



A set of straight line decision boundaries for a classification problem.



An alternative set of decision boundaries that separate the plusses from lightening strikes better, but it requires a line that isn't straight.

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Regression

- Regression analysis is used to predict the value of one variable (the ***dependent variable***) on the basis of other variables (the ***independent variables***).
- Learn a continuous function.
- Given, the following data, can we find the value of the output when x = 0.44?
- Goal is to predict for input x an output $f(x)$ that is close to the true y.

x	t
0	0
0.5236	1.5
1.0472	-2.5981
1.5708	3.0
2.0944	-2.5981
2.6180	1.5
3.1416	0

- It is generally a problem of **function approximation**, or **interpolation**, working out the value between values that we know.

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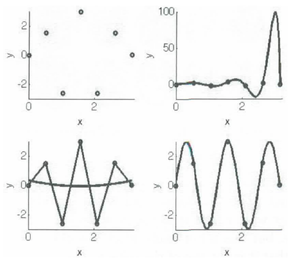
Which line has the best "fit" to the data?



- Top left: A few data points from a sample problem. Bottom left: Two possible ways to predict the values between the known data points: connecting the points with straight lines, or using a cubic approximation (which in this case misses all of the points). Top and bottom right: Two more complex approximators that passes through the points, although the lower one is rather better than the top.

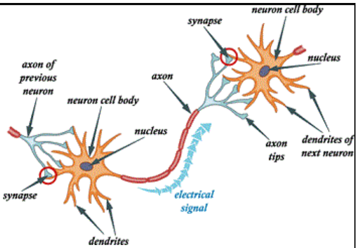
The Machine Learning Process

1. Data Collection and Preparation
2. Feature Selection and Extraction
3. Algorithm Choice
4. Parameters and Model Selection
5. Training
6. Evaluation

Neural Networks

- We are born with about 100 billion neurons
- A neuron may connect to as many as 10,000 other neurons
- Much parallel computation

Neural Networks



- Neuron:
  - many-inputs
  - one-output unit.
- Neurons are connected by **synapses**
- Signals “move” via **electrochemical signals** on a synapse
- The synapses release a chemical transmitter, enough of which can cause a neuron threshold to be reached, causing the neuron to “**fire**”
- Synapses can be **inhibitory** or **excitatory**
- Learning: Modification in the synapses

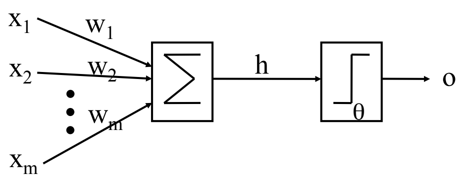
Hebb’s Rule

- Strength of a synaptic connection is proportional to the correlation of two connected neurons.
- If two neurons consistently fire simultaneously, synaptic connection is increased (if firing at different time, strength is reduced).
- “Cells that fire together, wire together.”

McCulloch and Pitts Neurons

- McCulloch & Pitts (1943) are generally recognised as the designers of the first artificial neural network.
- Many of their ideas still used today (e.g. many simple units combine to give increased computational power and the idea of a threshold).

McCulloch and Pitts Neurons



- Greatly simplified biological neurons.
- Sum the weighted inputs
  - If total is greater than some threshold, neuron "fires"
  - Otherwise does not

McCulloch and Pitts Neurons

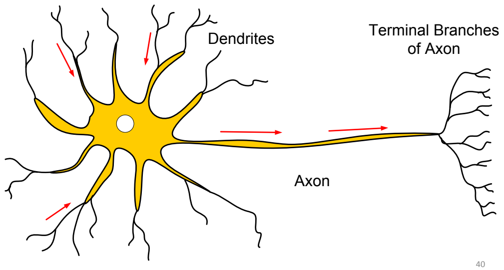
$$h = \sum_{i=1}^{m} x_i w_i \qquad o = \begin{cases} 1 & h \geq \theta \\ 0 & h < \theta \end{cases}$$

for some threshold θ

- The weight w_j can be positive or negative
  - Inhibitory or exitatory.
- Use only a linear sum of inputs.
- Synchronous processing.
- No resting state following excitation.
- Scalar output instead of a pulse (spike train).

Biologically Inspired

- Electro-chemical signals.
- Threshold output firing.



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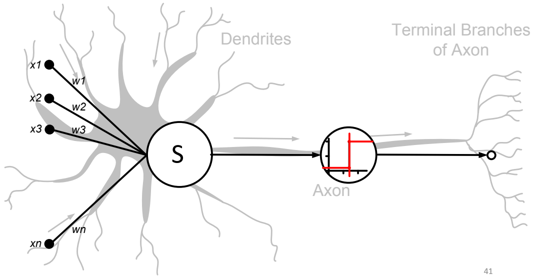
The Perceptron

- Binary classifier function.
- Threshold activation function.



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Limitations *(McCulloch and Pitts Neurons Model)*

- How realistic is this model?
- Not Very.
  - Real neurons are much more complicated.
  - Inputs to a real neuron are not necessary summed linearly.
  - ***Real neuron do not output a single output response, but a SPIKE TRAIN.***
  - Weights w_i can be positive or negative, whereas in biology connections are either excitatory OR inhibitory.

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Neural Networks

- Can put lots of McCulloch & Pitts neurons together.
- Connect them up in any way we like.
- In fact, assemblies of the neurons are capable of ***universal computation***.
  - Can perform any computation that a normal computer can.
  - Just have to solve for all the weights w_{ij}

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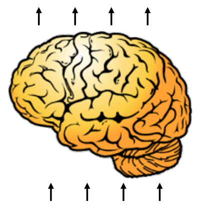
Neural Networks

Biological

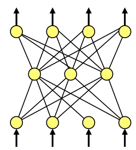
Output



Input

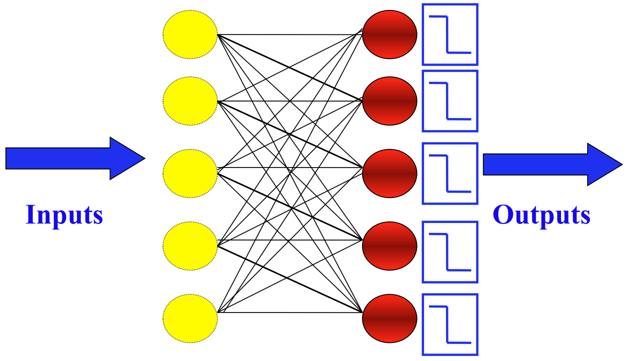
Artificial Neural Network (ANN)

Output



Input

The Perceptron Network

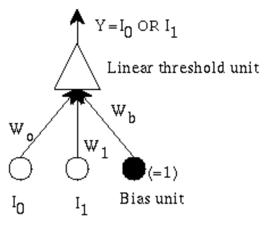


Training Neurons

- Adapting the weights is learning
  - How does the network know it is right?
  - How do we adapt the weights to make the network right more often?
- Training set with target outputs (supervised learning).
- Learning rule.

A Simple Perceptron

- One unit (the loneliest network)
- *Change the weights by an amount proportional to the difference between the desired output and the actual output.*



$$\mathbf{w_{ij} \leftarrow w_{ij} + \Delta w_{ij}}$$

Updating the Weights

$$w_{ij} \leftarrow w_{ij} + \Delta w_{ij}$$

- Aim: minimize the ***error*** at the output
- If *E* = *t*-*y*, want *E* to be 0
- Use:

$$\Delta w_{ij} = \eta \cdot (t_j - y_j) \cdot x_i$$

Learning rate *Input* *Desired output* *Actual output* *Error*

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The Learning Rate η

- η controls the size of the weight changes.
- Why not η = 1 ?
  - Weight change a lot, whenever the answer is wrong.
  - Makes the network unstable.
- Small η
  - *Weights need to see the inputs more often before they change significantly.*
  - *Network takes longer to learn.*
  - *But, more stable network.*

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Bias Input

- *What happens when all the inputs to a neuron are zero?*
  - It doesn't matter what the weights are,
  - The only way that we can control whether neuron fires or not is *through the threshold*.
- That's why threshold should be ***adjustable***.
  - Changing the threshold requires an extra parameter that we need to write code for.
- We add to each neuron an extra input ***with a fixed value***.

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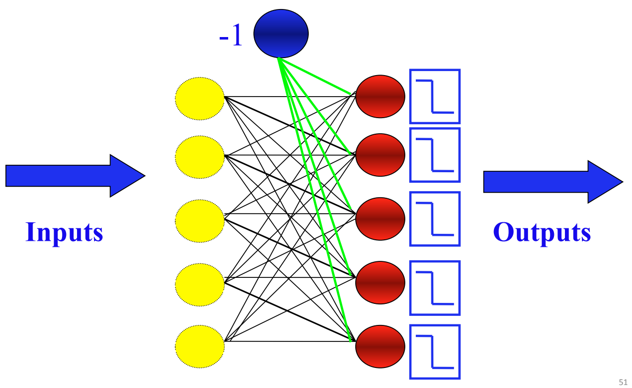
Biases Replace Thresholds



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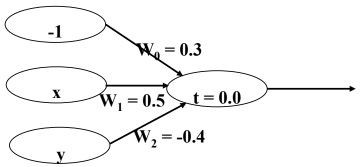
Training a Perceptron

Aim (Boolean AND)

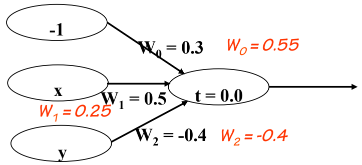
Input 1	Input 2	Output
0	0	0
0	1	0
1	0	0
1	1	1

Training a Perceptron



I_1	I_2	I_3	Summation	Output
-1	0	0	(-1*0.3) + (0*0.5) + (0*-0.4) = -0.3	0
-1	0	1	(-1*0.3) + (0*0.5) + (1*-0.4) = -0.7	0
-1	1	0	(-1*0.3) + (1*0.5) + (0*-0.4) = 0.2	~~1~~
-1	1	1	(-1*0.3) + (1*0.5) + (1*-0.4) = -0.2	~~0~~

Training a Perceptron

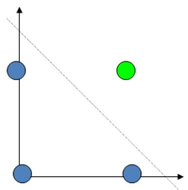


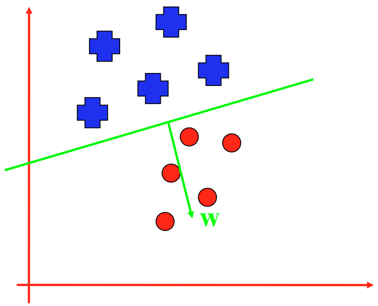
$\eta = 0.25$

$W_0 = 0.3 + 0.25 * (0-1) * -1 = 0.55$
$W_1 = 0.5 + 0.25 * (0-1) * 1 = 0.25$
$W_2 = -0.4 + 0.25 * (0-1) * 0 = -0.4$

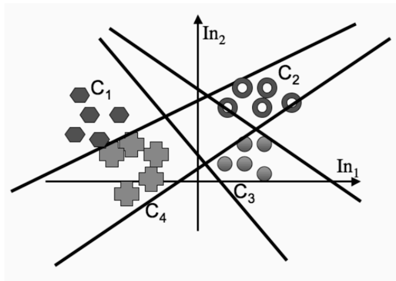
I_1	I_2	I_3	Summation	Output
-1	1	0	(-1*0.55) + (1*0.25) + (0*-0.4) = -0.3	~~1~~ <u>0</u>

Linear Separability





More Than One Neuron



- The weights for each neuron separately describe a straight line.

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Perceptron Limitations

- A single layer perceptron can only learn ***linearly separable*** problems.
  - Boolean AND function is linearly separable, whereas Boolean XOR function (and the parity problem in general) ***is not***.

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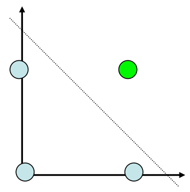
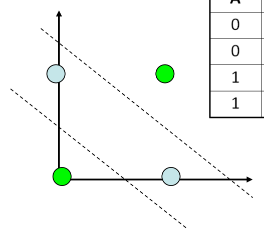
Linear Separability



Boolean AND

✓



Boolean XOR

✗

A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

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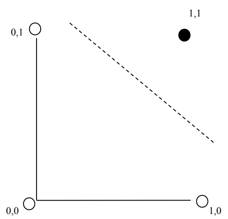
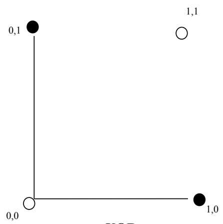
What Can Perceptrons Represent?



AND

✓



XOR

✗

- **Only linearly separable functions can be represented by a perceptron**

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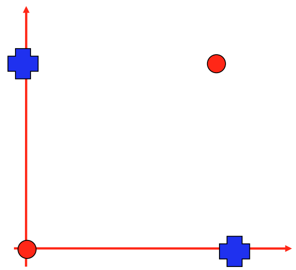
Limitations of the Perceptron

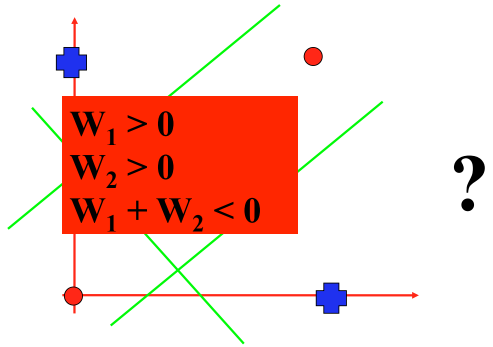
Linear Separability



The Exclusive Or (XOR) function

A	B	Out
0	0	0
0	1	1
1	0	1
1	1	0

Limitations of the Perceptron

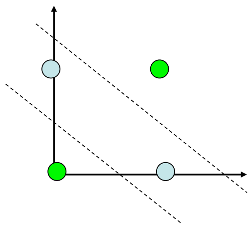


Perceptron Limitations

- **Multi-layer perceptron** can solve this problem
- More than one layer of perceptrons (with a hardlimiting activation function) can learn any Boolean function
- A learning algorithm for multi-layer perceptrons was not developed until much later
  - backpropagation algorithm (replacing the hardlimiter with a sigmoid activation function)

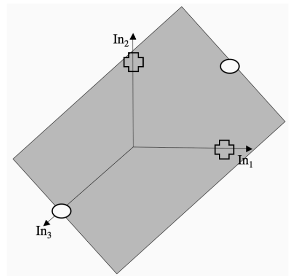
Perceptron Limitations

- XOR problem: What if we use ***more layers of neurons*** in a perceptron?
  - Each neuron implementing one decision boundary and the next layer combining the two?



Perceptron Limitations

In_1	In_2	In_3	Output
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	1



A decision boundary (the shaded plane) solving the XOR problem in 3D with the crosses below the surface and the circles above it.

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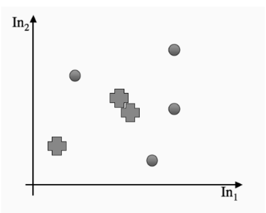
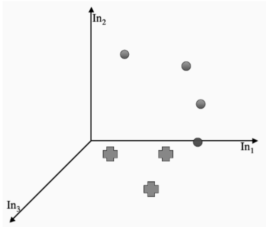
Perceptron Limitations





Left: Non-separable 2D dataset. Right: The same dataset with third coordinate x_1 x x_2, which makes it separable.

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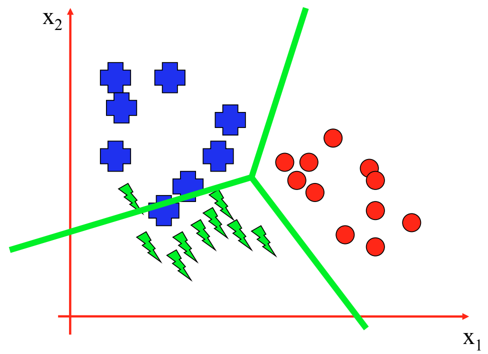
Decision Boundaries



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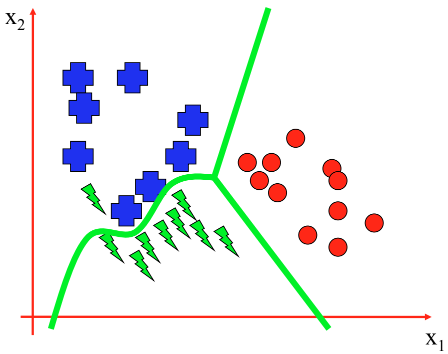
Decision Boundaries



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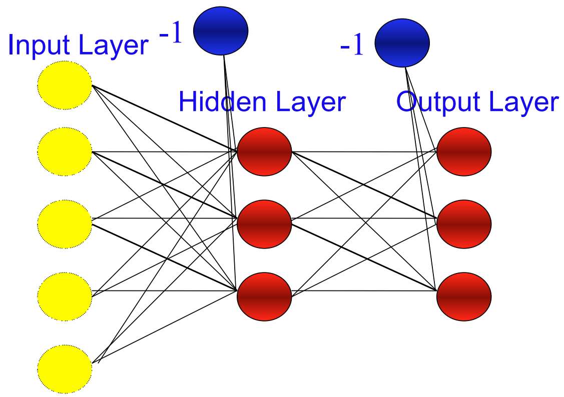
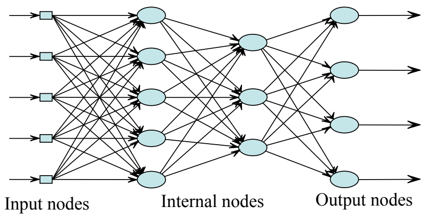
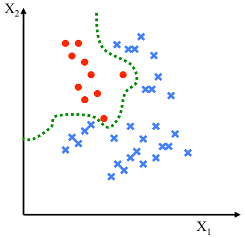
The Multi-Layer Perceptron



MLP Decision Boundary – Nonlinear Problems, Solved!

In contrast to perceptrons, multilayer networks can learn not only multiple decision boundaries, but the boundaries may be nonlinear.





And Finally….

"If the brain were so simple that we could understand it then we'd be so simple that we couldn't"

-- Lyall Watson