INF3510 Information Security
University of Oslo
Spring 2017

Lecture 4

Cryptography

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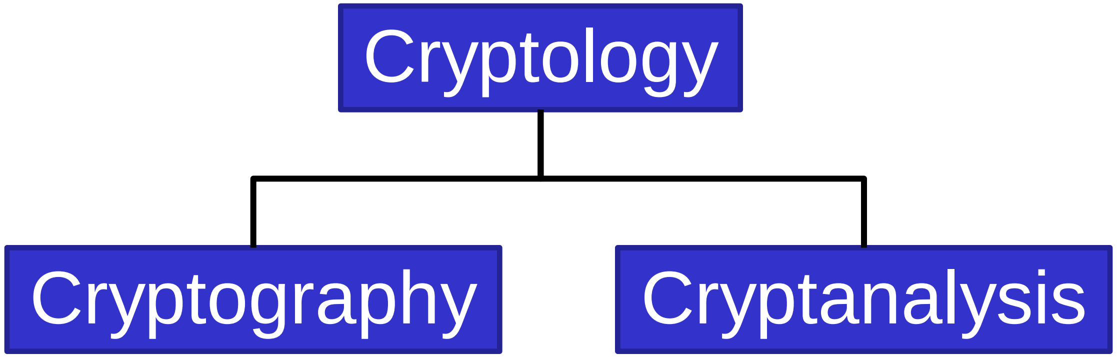
Outline

- What is cryptography?
- Brief crypto history
- Security issues
- Symmetric cryptography
  - Stream ciphers
  - Block ciphers
  - Hash functions
- Asymmetric cryptography
  - Factoring based mechanisms
  - Discrete Logarithms
  - Digital signatures
  - Quantum Resistant Crypto

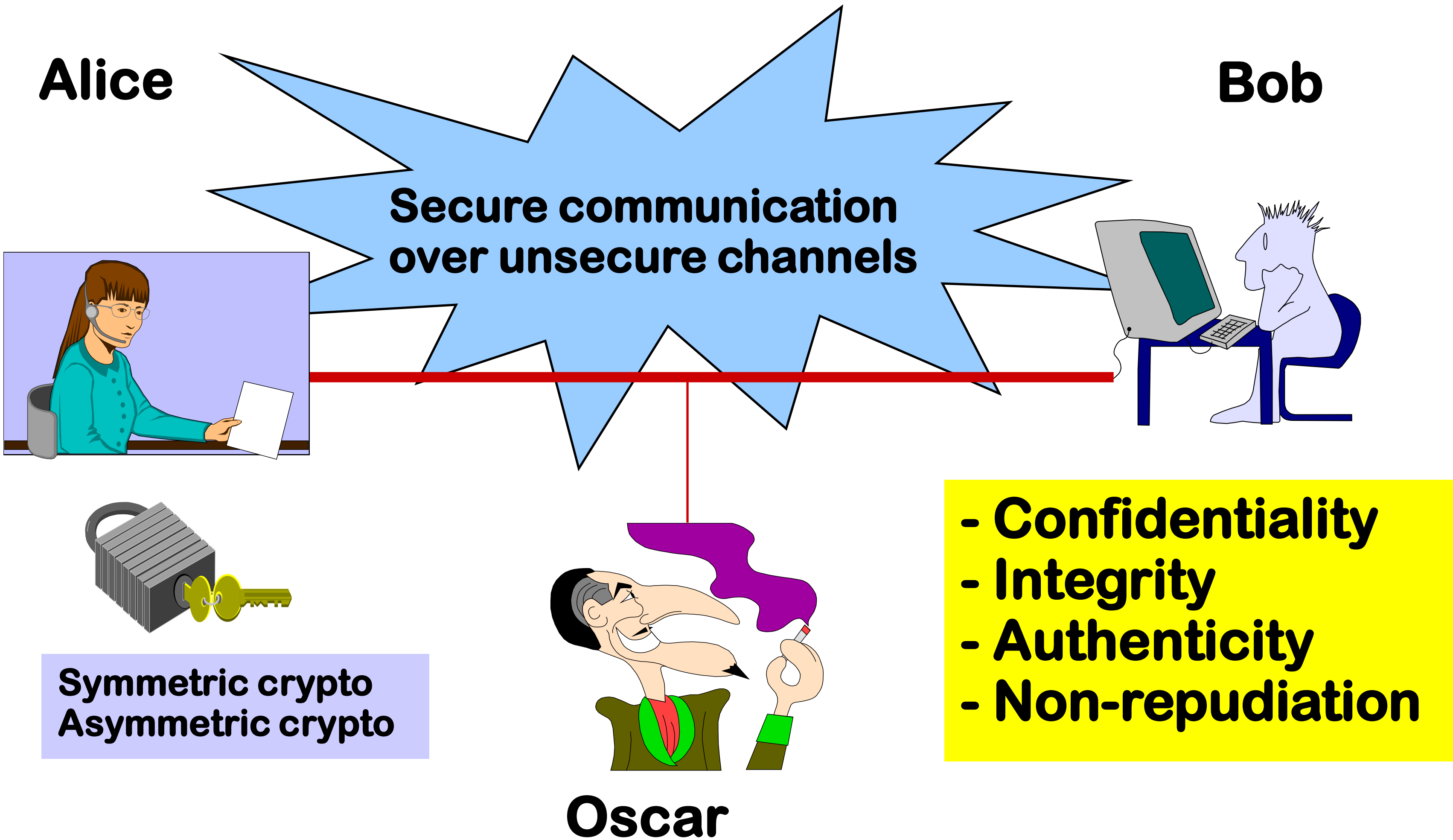
Want to learn more?
Look up UNIK 4220

Terminology

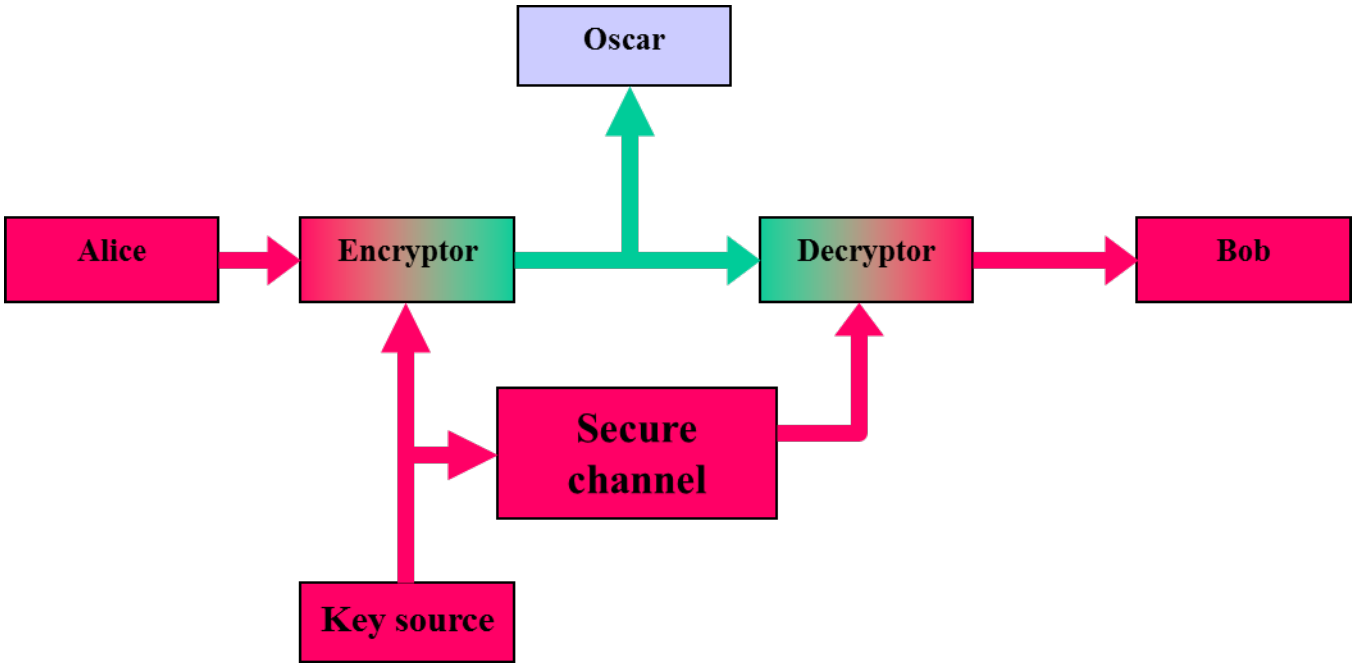


- **Cryptography** is the science of secret writing with the goal of hiding the meaning of a message.
- **Cryptanalysis** is the science and sometimes art of *breaking* cryptosystems.

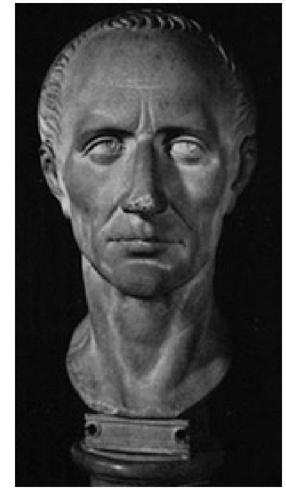
What is cryptology?



Model of symmetric cryptosystem



Caesar cipher



Example: Caesar cipher

🏱 = {abcdefghijklmnopqrstuvwxyz}

👍 = {DEFGHIJKLMNOPQRSTUVWXYZABC}

Plaintext: `kryptologi er et spennende fag`

Chiphertext: `NUBSWRORJL HU HT VSHQQHQGH IDJ`

Note: Caesar chipher in this form does not include a variable key, but is an instance of a “shift-cipher” using key $K = 3$.

Numerical encoding of the alphabet

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

p	q	r	s	t	u	v	w	x	y	z	æ	ø	å	
14	16	17	18	19	20	21	22	23	24	25	26	27	28	

Using this encoding many classical crypto systems can be expressed as algebraic functions over Z_{26} (English alphabet) or Z_{29} (Norwegian alphabet)

Shift cipher

Let $\mathcal{P} = \mathcal{C} = Z_{29}$. For $0 \leq K \leq 28$, we define

$$E(x, K) = x + K \pmod{29}$$

and

$$D(y, K) = y - K \pmod{29}$$

$(x, y \in Z_{29})$

Question: What is the size of the key space?

Puzzle: ct =

LAHYCXPAJYQHRBWNNMNMOXABNLDANLXVVDWRLJCRXWB

Find the plaintext!

Exhaustive search

For[i=0, i<26, i++, Print["Key = ", i, " Plain = ", decrypt[ct,1,i]]]

Key = 0 Plain = LAHYCXPAJYQHRBWNNMNMOXABNLDANLXVVDWRLJCRXWB

Key = 1 Plain = KZGXBWOZIXPGQAVMMLMLNWZAMKCZMKWUUCVQKIBQWVA

Key = 2 Plain = JYFWAVNYHWOFPZULLKLKMVYZLJBYLJVTTBUPJHAPVUZ

Key = 3 Plain = IXEVZUMXGVNEOYTKKJKJLUXYKIAXKIUSSATOIGZOUTY

Key = 4 Plain = HWDUYTLWFUMDNXSJJIJIKTWXJHZWJHTRRZSNHFYNTSX

Key = 5 Plain = GVCTXSKVETLCMWRIIHIHJSVWIGYVIGSQQYRMGEXMSRW

Key = 6 Plain = FUBSWRJUDSKBLVQHHGHGIRUVHFXUHFRPPXQLFDWLRQV

Key = 7 Plain = ETARVQITCRJAKUPGGFGFHQTUGEWTGEQOOWPKECVKQPU

Key = 8 Plain = DSZQUPHSBQIZJTOFFEFEGPSTFDVSFDPNNVOJDBUJPOT

Key = 9 Plain = CRYPTOGRAPHYISNEEDEDFORSECURECOMMUNICATIONS

Key = 10 Plain = BQXOSNFQZOGXHRMDDCDCENQRDBTQDBNLLTMHBZSHNMR

Key = 11 Plain = APWNRMEPYNFWGQLCCBCBDMPQCASPCAMKKSLGAYRGMLQ

Key = 12 Plain = ZOVMQLDOXMEVFPKBBABACLOPBZROBZLJJRKFZXQFLKP

- .
-

Substitution cipher - example

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
U	D	M	I	P	Y	Æ	K	O	X	S	N	Å	F	A

p	q	r	s	t	u	v	w	x	y	z	æ	ø	å	
E	R	T	Z	B	Ø	C	Q	G	W	H	L	V	J	

Plaintext: fermatssisteteorem
Ciphertext: YPTÅUBZZOZBPBPATPÅ

What is the size of the key space?

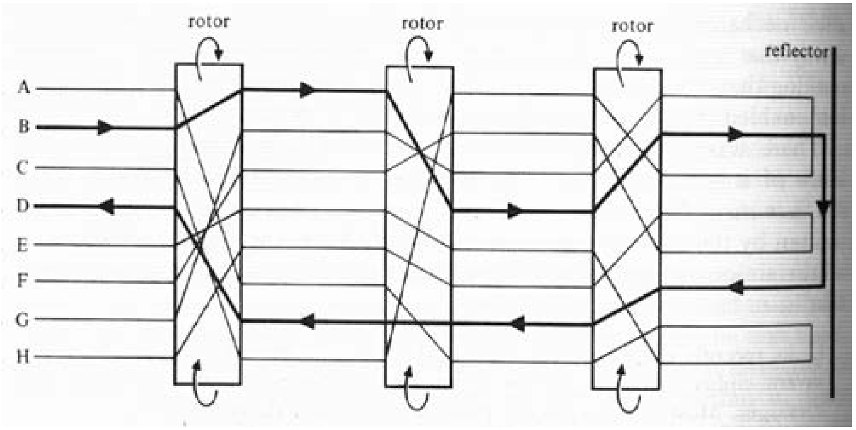
8841761993739701954543616000000 🕔 2^{103}

Lessons learned

- A cipher with a small keyspace can easily be attacked by *exhaustive search*
- A *large keyspace* is necessary for a secure cipher, but it is by itself not suffcient
- Monoalphabetical substitution ciphers can easily be broken

Enigma

- **German WW II crypto machine**
- **Many different variants**
- **Polyalphabetical substitution**
- **Analysed by Polish and English mathematicians**



(c) 1995, Morton Swimmer

Enigma key list

Geheim! **Sonder - Maschinenschlüssel BGT**

Datum	Walzenlage	Ringstellung	Steckerverbindungen	Grundstellung
31.	IV II I	F T R	HR AT IW SK UY DF GV LJ BO MX	vyj
30.	III V II	Y V P	OR KI JV OE ZK MU BF YC DS GP	cqr
29.	V IV I	O H R	UX JC PB BK TA ED ST DS LU MI	vhf

Practical complexity for attacking Enigma

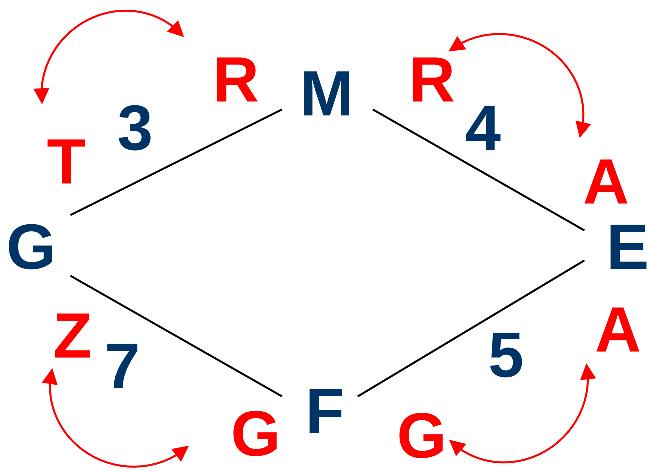
Cryptoanalytical assumptions during WW II:

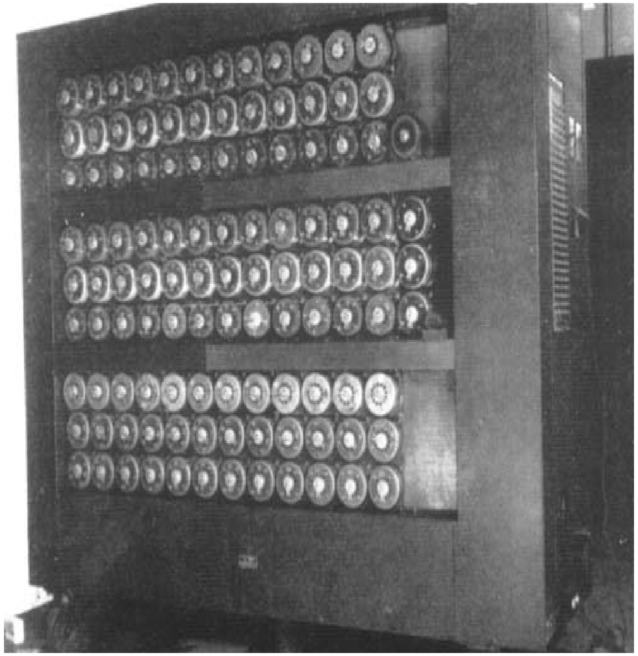
- 3 out of 5 rotors with known wiring
- 10 stecker couplings
- Known reflector

N = 150 738 274 937 250 · 60 · 17 576 · 676 =
107458687327250619360000 (77 bits)

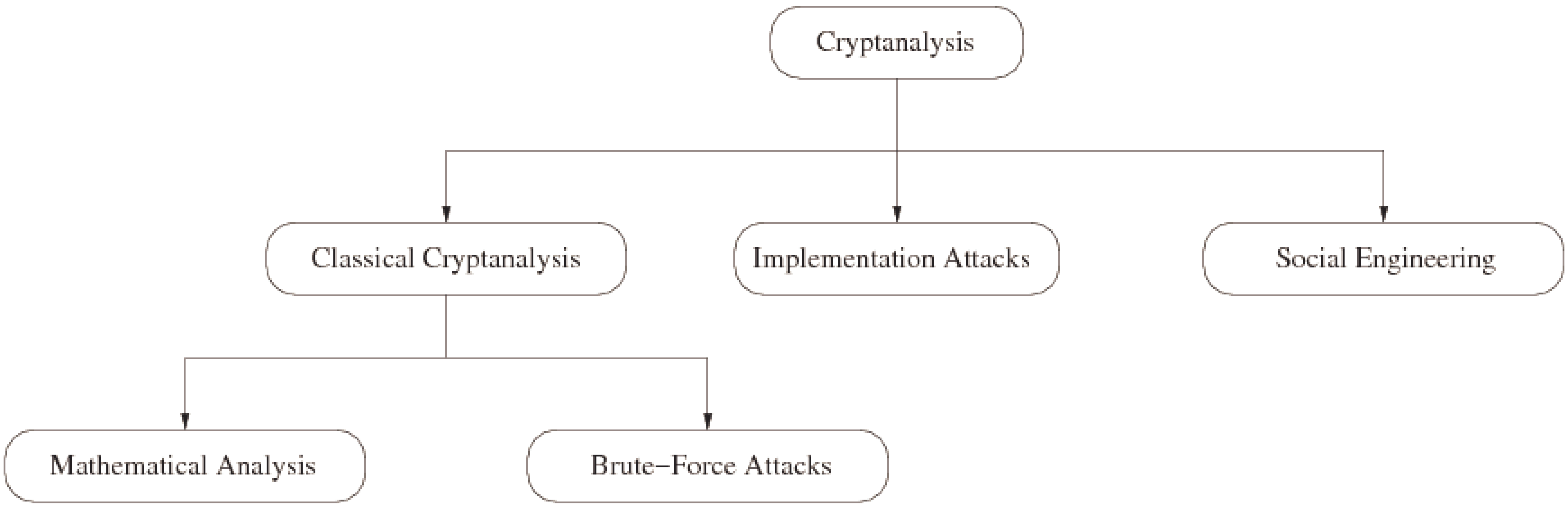
Attacking ENIGMA

Posisjon:	1	2	3	4	5	6	7
Chiffertekst:	J	T	G	E	F	P	G
Crib:	R	O	M	M	E	L	F





Cryptanalysis: Attacking Cryptosystems



- **Classical Attacks**
  - Mathematical Analysis
  - Brute-Force Attack
- **Implementation Attack**: Try to extract the key through reverse engineering or power measurement, e.g., for a banking smart card.
- **Social Engineering**: E.g., trick a user into giving up her password

Brute-Force Attack (or Exhaustive Key Search)

- Treats the cipher as a black box
- Requires (at least) 1 plaintext-ciphertext pair (x_0, y_0)
- Check all possible keys until condition is fulfilled:

$$d_K(y_0) = x_0$$

- How many keys to we need ?

Key length in bit	Key space	Security life time (assuming brute-force as best possible attack)
64	2^{64}	**Short term** (few days or less)
128	2^{128}	**Long-term** (several decades in the absence of quantum computers)
256	2^{256}	**Long-term** (also resistant against quantum computers – note that QC do not exist at the moment and might never exist)

Kerckhoff’s principles

- The system should be, if not theoretically unbreakable, unbreakable in practice.
- The design of a system should not require secrecy and compromise of the system should not inconvenience the correspondents (Kerckhoffs' principle).
- The key should be rememberable without notes and should be easily changeable
- The cryptograms should be transmittable by telegraph
- The apparatus or documents should be portable and operable by a single person
- The system should be easy, neither requiring knowledge of a long list of rules nor involving mental strain

Attack models:

Known ciphertext

Known plaintext

Chosen plaintext (adaptive)

Chosen ciphertext (adaptive)

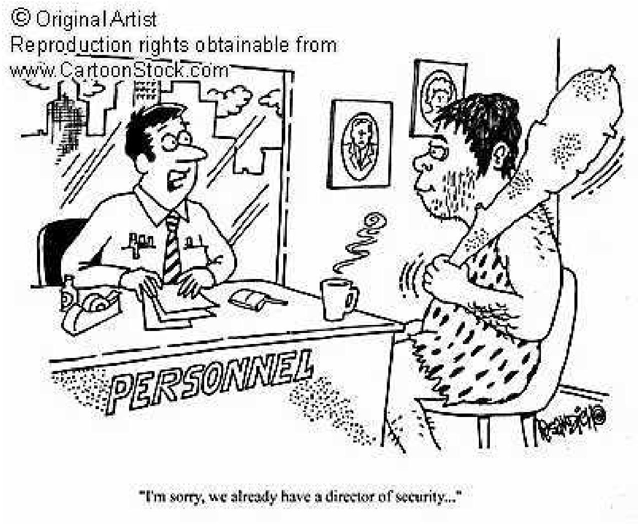
What are the goals of the attacker?

- Find the secret plaintext or part of the plaintext
- Find the encryption key
- Distinguish the encryption of two different plaintexts

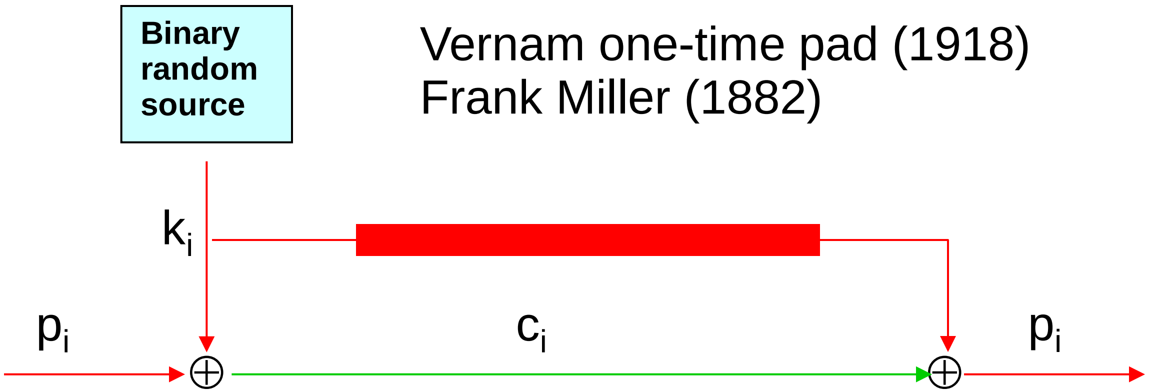
How clever is the attacker?

Does secure ciphers exist?

- What is a secure cipher?
  - Perfect security
  - Computational security
  - Provable security



A perfect secure crypto system



Vernam one-time pad (1918)
Frank Miller (1882)

$c_i = p_i \oplus k_i$
$p_i = c_i \oplus k_i = p_i \oplus k_i \oplus k_i = p_i$
Note: $a \oplus b = a + b \pmod 2$

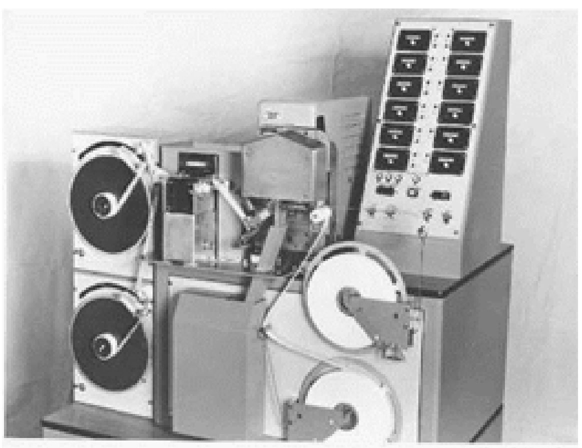
Offers perfect security assuming the key is perfectly random, of same length as The Message; and only used once. Proved by Claude E. Shannon in 1949.

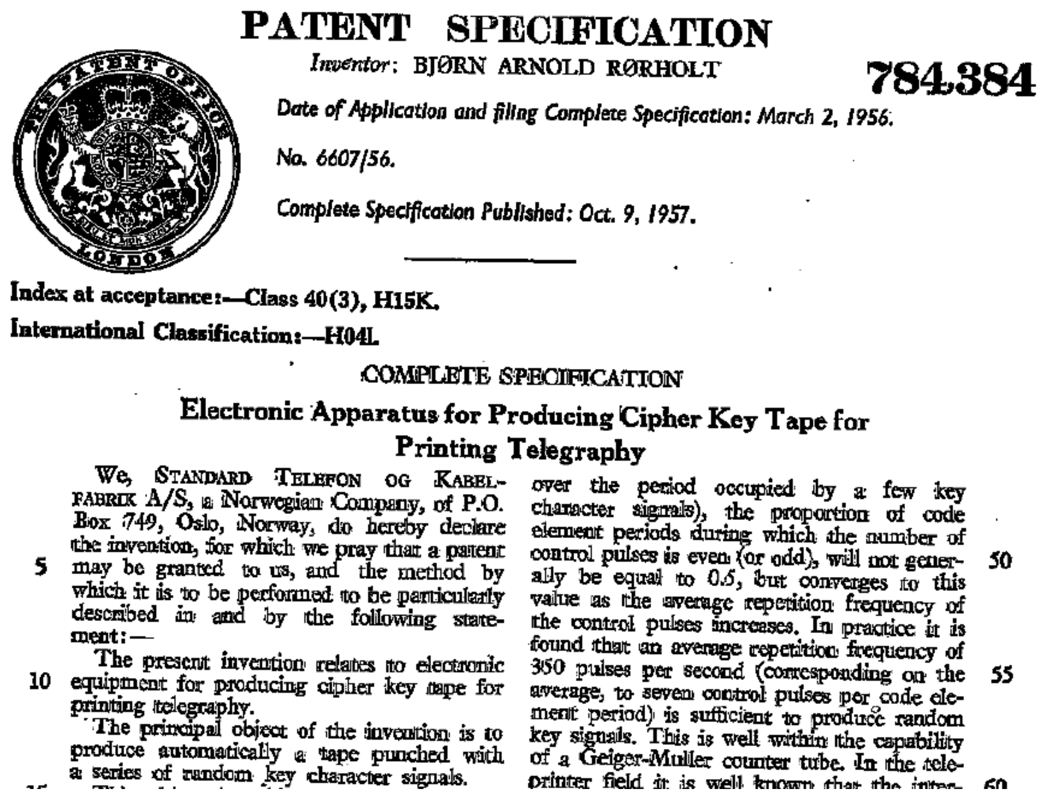
ETCRRM

- Electronic Teleprinter Cryptographic Regenerative Repeater Mixer (ETCRRM)
- Invented by the Norwegian Army Signal Corps in 1950
- Bjørn Rørholt, Kåre Mesingseth
- Produced by STK
- Used for "Hot-line" between Moskva and Washington
- About 2000 devices produced

White House Crypto Room 1960s

Producing key tape for the one-time pad





PATENT SPECIFICATION

Inventor: BJØRN ARNOLD RØRHOLT

784,384

Date of Application and filing Complete Specification: March 2, 1956.

No. 6607/56.

Complete Specification Published: Oct. 9, 1957.

Index at acceptance:—Class 40(3), H15K.

International Classification:—H04L.

COMPLETE SPECIFICATION

Electronic Apparatus for Producing Cipher Key Tape for Printing Telegraphy

We, STANDARD TELEFON OG KABELFABRIK A/S, a Norwegian Company, of P.O. Box 749, Oslo, Norway, do hereby declare the invention, for which we pray that a patent may be granted to us, and the method by which it is to be performed to be particularly described in and by the following statement:—

The present invention relates to electronic equipment for producing cipher key tape for printing telegraphy.

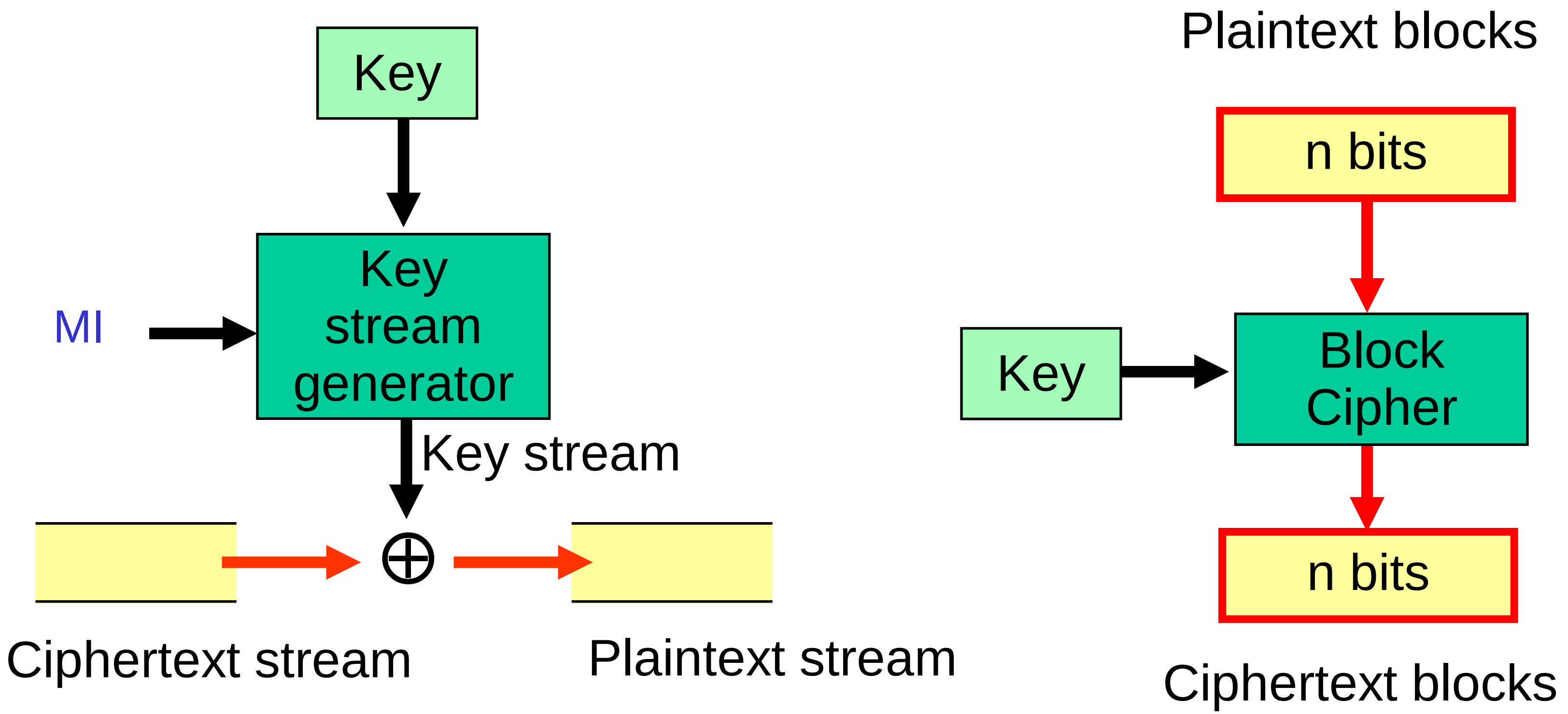
The principal object of the invention is to produce automatically a tape punched with a series of random key character signals.

over the period occupied by a few key character signals), the proportion of code element periods during which the number of control pulses is even (or odd), will not generally be equal to 0.5, but converges to this value as the average repetition frequency of the control pulses increases. In practice it is found that an average repetition frequency of 350 pulses per second (corresponding on the average, to seven control pulses per code element period) is sufficient to produce random key signals. This is well within the capability of a Geiger-Muller counter tube. In the teleprinter field it is well known that the inter-

Symmetric encryption

Is it possible to design secure and practical crypto?

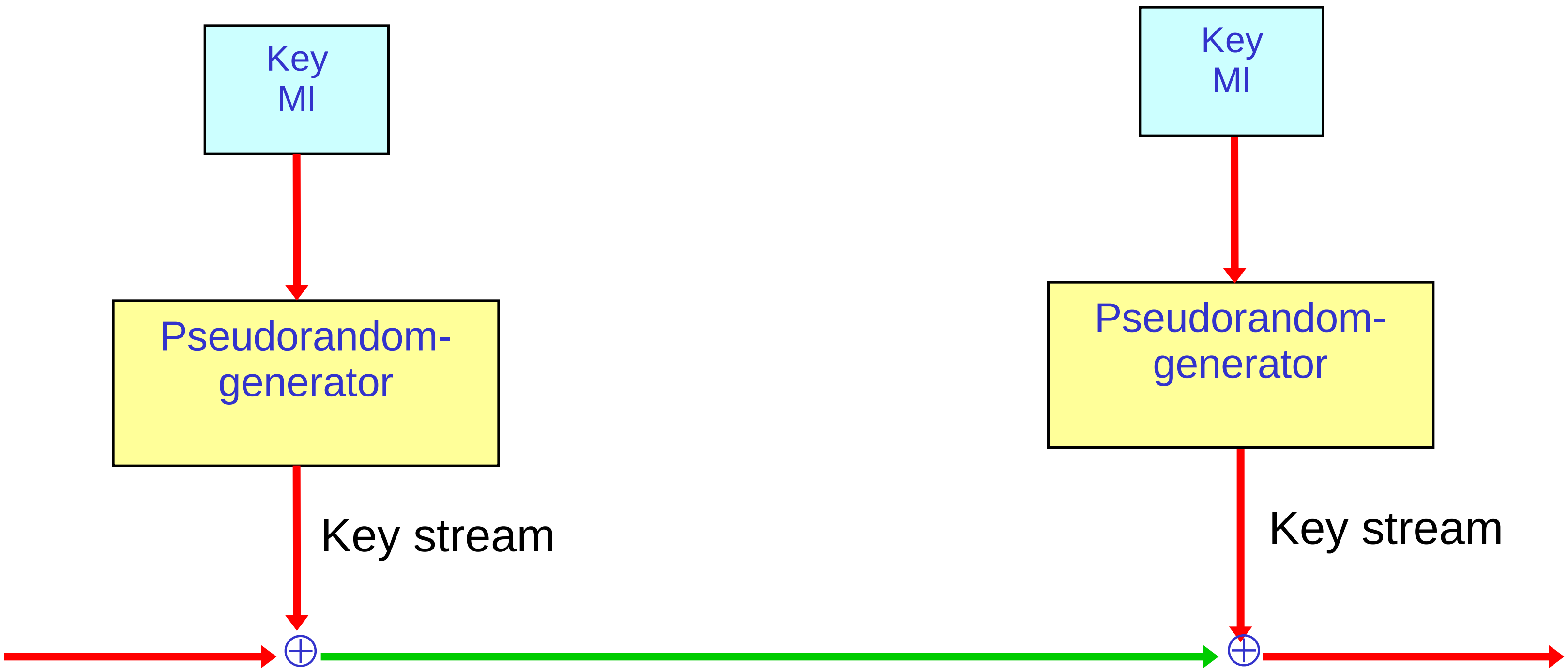
Stream Cipher vs. Block Cipher



Stream cipher

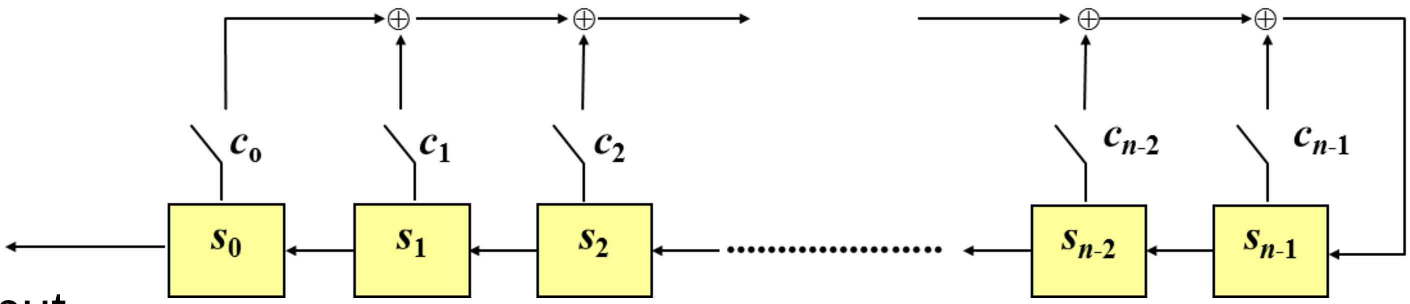
Block cipher

Symmetric stream cipher



LFSR

Linear feedback shift register



Output

Using n flip-flops we may generate a binary sequence of period $2^n - 1$

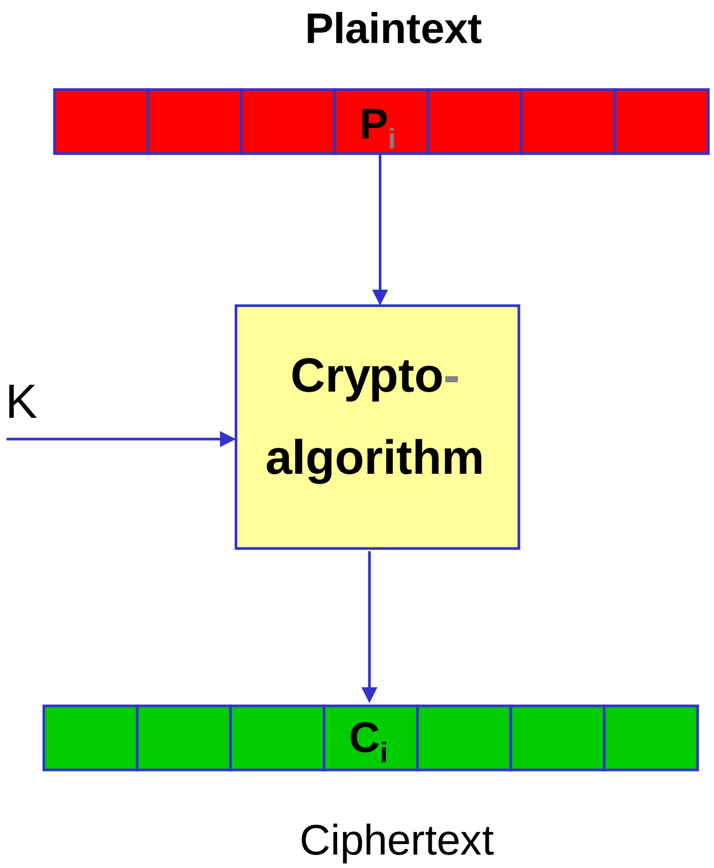
$$s_{n+i} = c_0 s_i + c_1 s_{i+1} + \cdots + c_{n-1} s_{i+n-1}$$

Note: The stream cipher is stateful

LFSR - properties

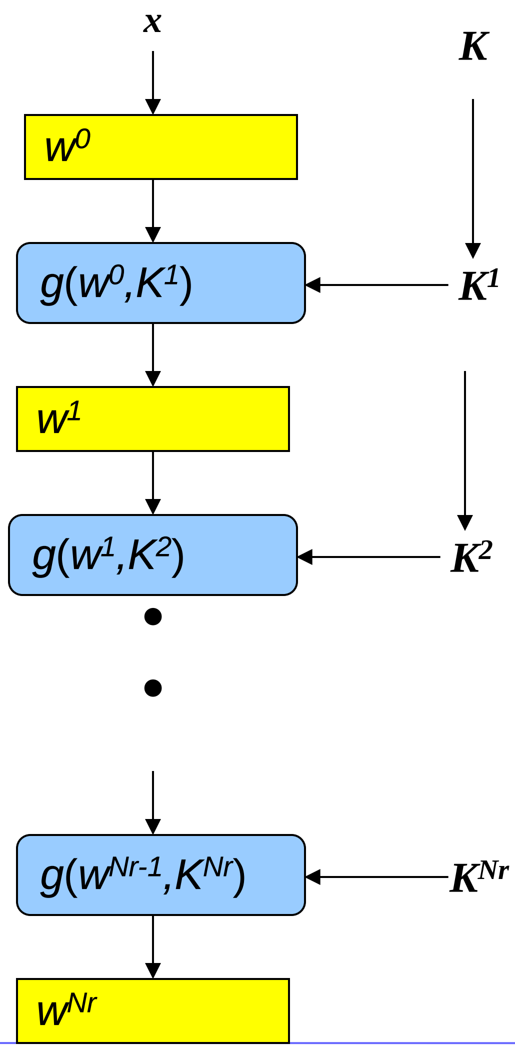
- Easy to implement in HW, offers fast clocking
- The output sequence is completely determined of the initial state and the feedback coefficients
- Using “correct” feedback a register of length *n* may generate a sequence with period 2^n-1
- The sequence will provide good statistical properties
- Knowing 2*n* consecutive bits of the key stream, will reveal the initial state and feedback
- The linearity means that a single LFSR is completely useless as a stream cipher, but LFSRs may be a useful building block for the design of a strong stream cipher

Symmetric block cipher



- The algorithm represents a family of permutations of the message space
- Normally designed by iterating a less secure round function
- May be applied in different operational modes
- Must be impossible to derive K based on knowledge of P and C

Itrerated block cipher design



Algorithm:

$w^0 \leftarrow x$

$w^1 \leftarrow g(w^0,K^1)$

$w^2 \leftarrow g(w^1,K^2)$

•

•

$w^{Nr-1} \leftarrow g(w^{Nr-2},K^{Nr-1})$

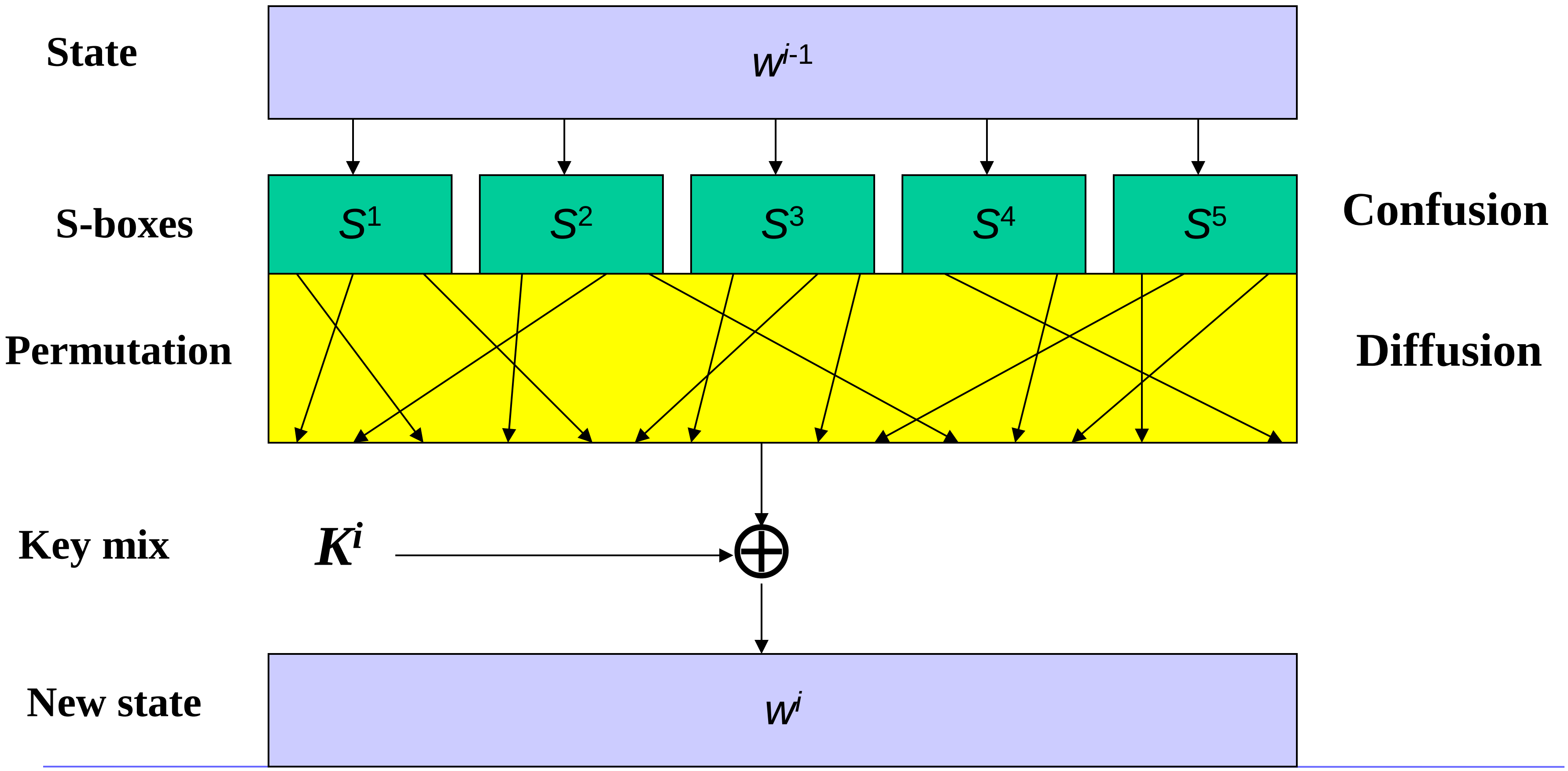
$w^{Nr} \leftarrow g(w^{Nr-1},K^{Nr})$

$y \leftarrow w^{Nr}$

NB! For a fixed *K*, *g* must be injective in order to decrypt *y*

Substitusjon-Permutasjon nettverk (SPN):

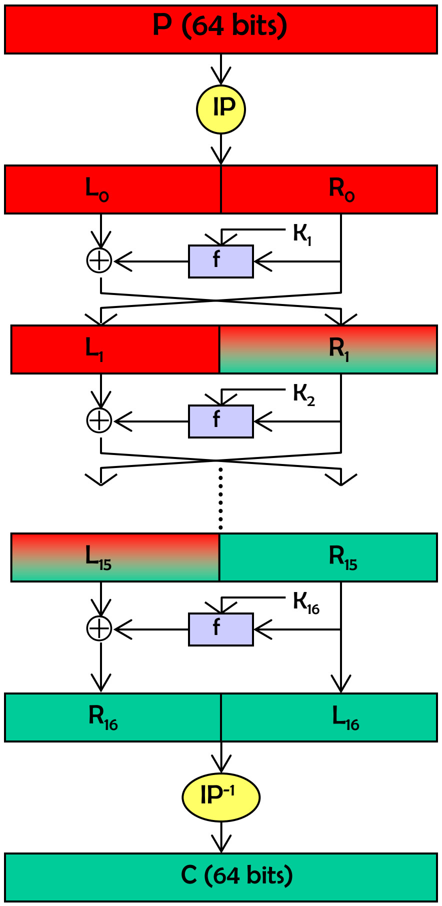
Round function *g* :



Data Encryption Standard

- Published in 1977 by the US National Bureau of Standards for use in unclassified government applications with a 15 year life time.
- 16 round Feistel cipher with 64-bit data blocks, 56-bit keys.
- 56-bit keys were controversial in 1977; today, exhaustive search on 56-bit keys is very feasible.
- Controversial because of classified design criteria, however no loop hole was ever found.

DES architecture



DES(P):

$(L_0, R_0) = IP(P)$

FOR i = 1 TO 16

$L_i = R_{i-1}$

$R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$

$C = IP^{-1}(R_{16}, L_{16})$

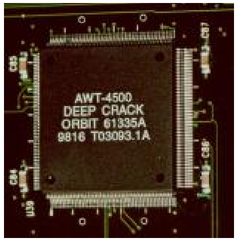
64 bit data block

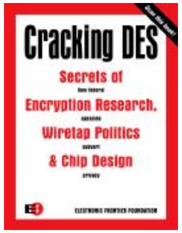
56 bit key

72.057.594.037.927.936

EFF DES-cracker

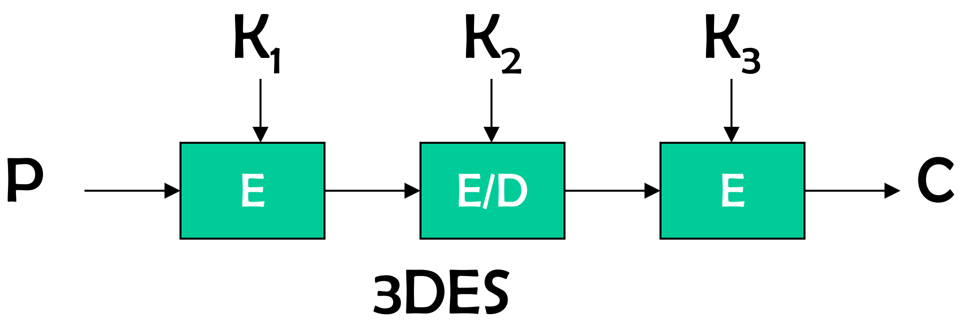
- Dedicated ASIC with 24 DES search engines
- 27 PCBs housing 1800 circuits
- Can test 92 billion keys per second
- Cost 250 000 $
- DES key found July 1998 after 56 hours search
- Combined effort DES Cracker and 100.000 PCs could test 245 billion keys per second and found key after 22 hours

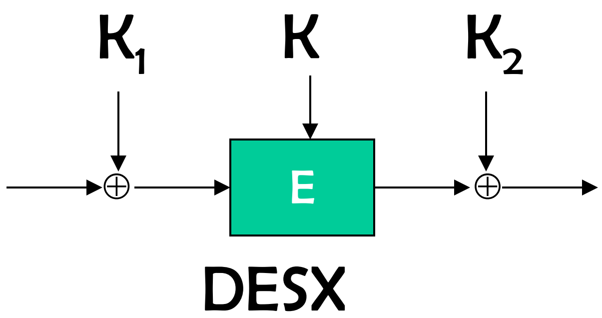




DES Status

- DES is the "work horse" which over 40 years have inspired cryptographic research and development
- "Outdated by now"!
- Single DES can not be considered as a secure block cipher
- Use 3DES (ANSI 9.52) or DESX

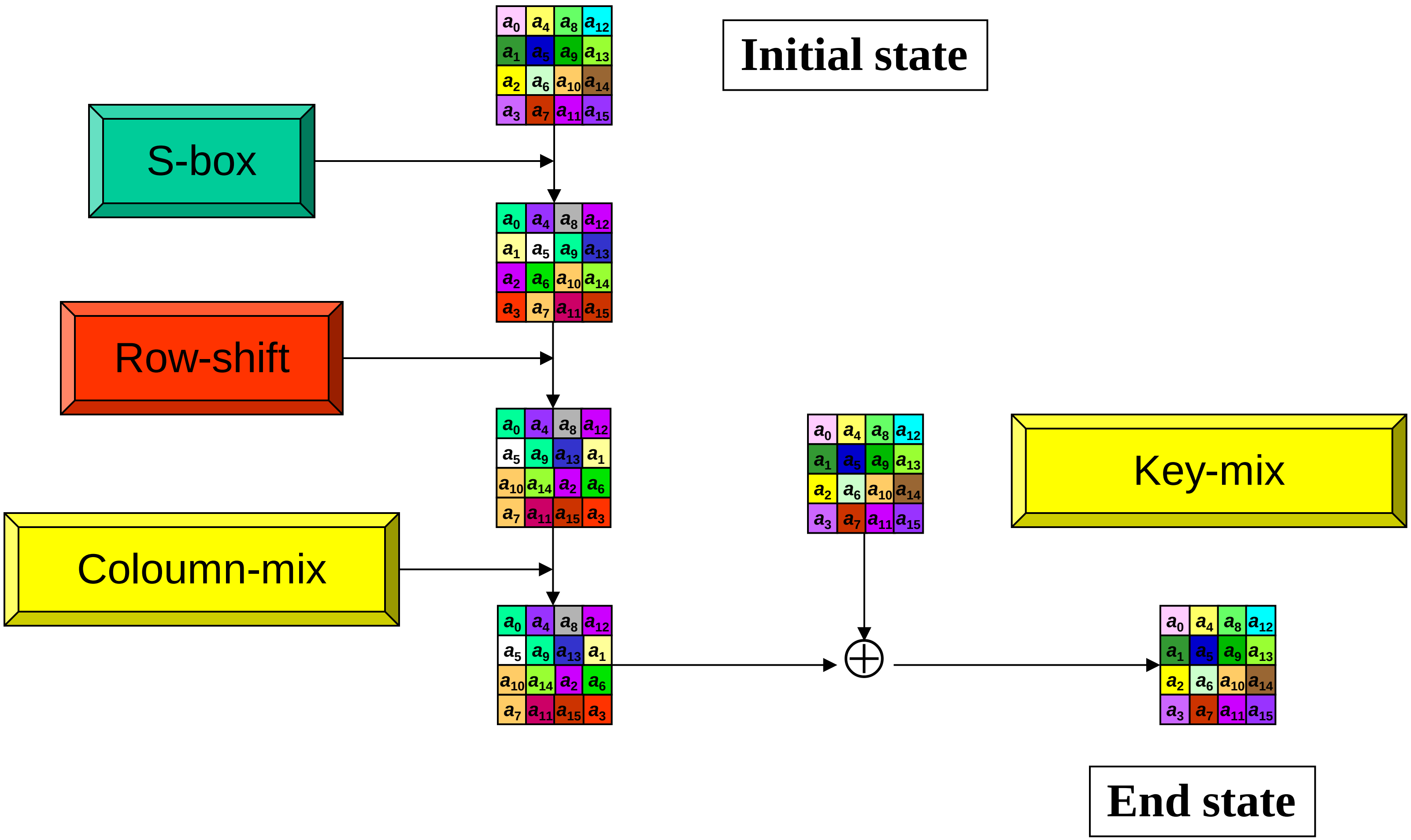




Advanced Encryption Standard

- Public competition to replace DES: because 56-bit keys and 64-bit data blocks were no longer adequate.
- Rijndael nominated as the new Advanced Encryption Standard (AES) in 2001 [FIPS-197].
- Rijndael (pronounce as “Rhine-doll”) designed by Vincent Rijmen and Joan Daemen.
- 128-bit block size (Note error in Harris p. 809)
- 128-bit, 196-bit, and 256-bit key sizes.
- Rijndael is <u>not</u> a Feistel cipher.

Rijndael round function



Rijndael encryption

1. Key mix (round key K_0)
2. N_r-1 rounds containing:
   a) Byte substitution
   b) Row shift
   c) Coloumn mix
   d) Key mix (round key K_i)
3. Last round containing:
   a) Byte substitution
   b) Row shift
   c) Key mix (round key K_{Nr})

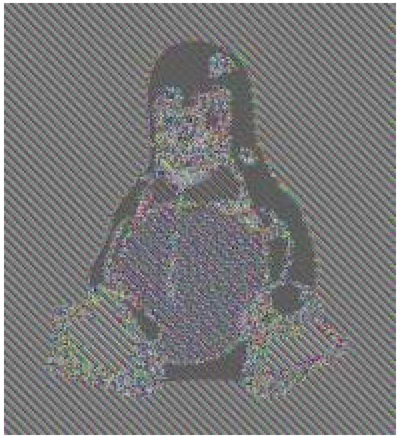
Key	Rounds
128	10
192	12
256	14

Block Ciphers: Modes of Operation

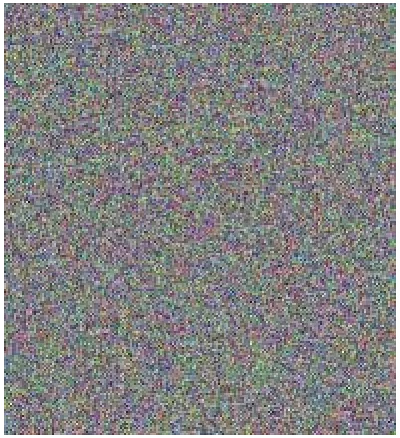
- Block ciphers can be used in different modes in order to provide different security services.
- Common modes include:
  - **E**lectronic **C**ode **B**ook (ECB)
  - **C**ipher **B**lock **C**haining (CBC)
  - **O**utput **F**eed**b**ack (OFB)
  - **C**ipher **F**eed**b**ack (CFB)
  - **C**oun**t**e**r** Mode (CTR)
  - **G**alois **C**ounter **M**ode (GCM) {Authenticated encryption}

Use a secure mode!

Plaintext



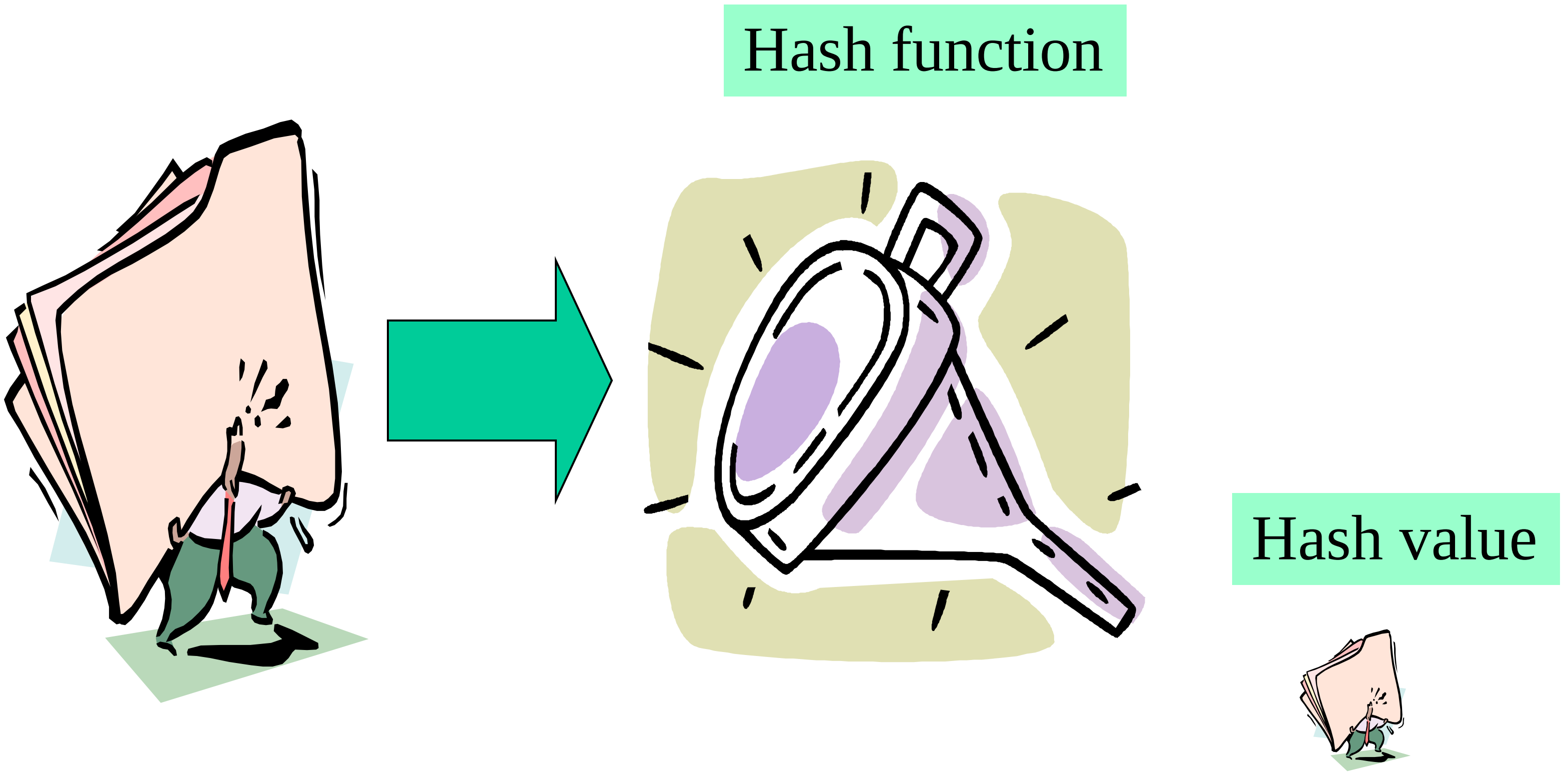
Ciphertext using ECB mode



Ciphertext using secure mode

Integrity Check Functions

Hash functions



Applications of hash functions

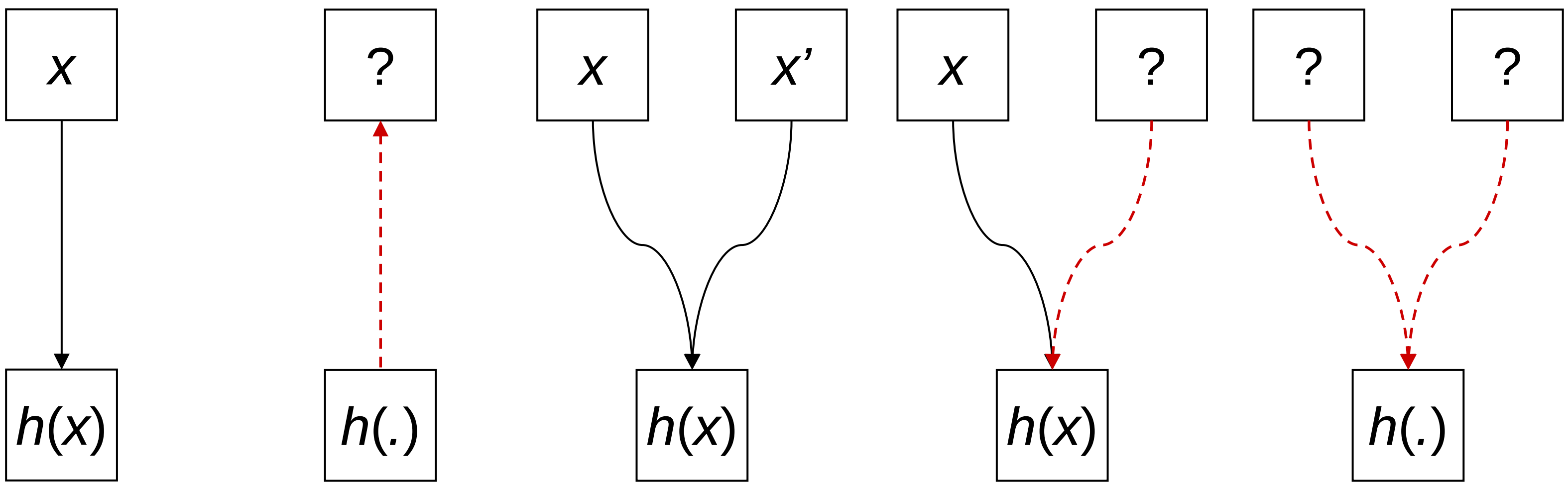
- Protection of password
- Comparing files
- Authentication of SW distributions
- Bitcoin
- Generation of Message Authentication Codes (MAC)
- Digital signatures
- Pseudo number generation/Mask generation functions
- Key derivation

Hash functions (message digest functions)

Requirements for a one-way hash function h:

1. Ease of computation: given x, it is easy to compute $h(x)$.
2. Compression: h maps inputs x of arbitrary bitlength to outputs $h(x)$ of a fixed bitlength n.
3. One-way: given a value y, it is computationally infeasible to find an input x so that $h(x)=y$.
4. Collision resistance: it is computationally infeasible to find x and x', where $x \neq x'$, with $h(x)=h(x')$ (note: two variants of this property).

Properties of hash functions



Ease of computation

Pre-image resistance

Collision

Weak collision resistance (2nd pre-image resistance)

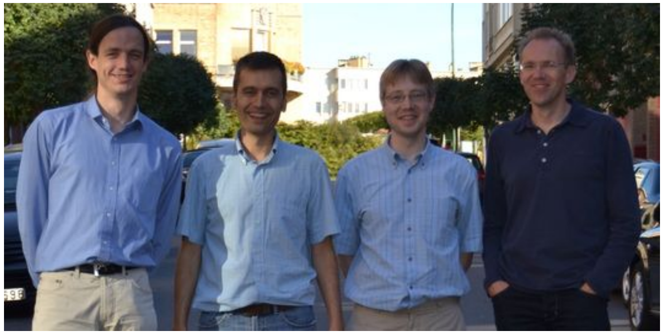
Strong collision resistance

Frequently used hash functions

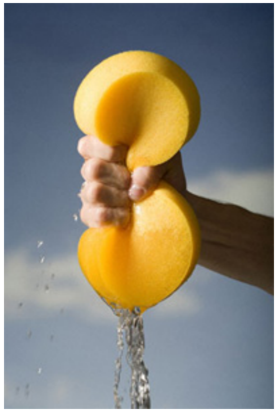
- MD5: 128 bit digest. Broken. Often used in Internet protocols but no longer recommended.
- SHA-1 (Secure Hash Algorithm):160 bit digest. Potential attacks exist. Designed to operate with the US Digital Signature Standard (DSA);
- SHA-256, 384, 512 bit digest. Still secure. Replacement for SHA-1 (SHA-2 family)
- RIPEMD-160: 160 bit digest. Still secure. Hash function frequently used by European cryptographic service providers.
- NIST competition for new secure hash algorithm, announcement of winner in 2012: SHA-3 = Keccak

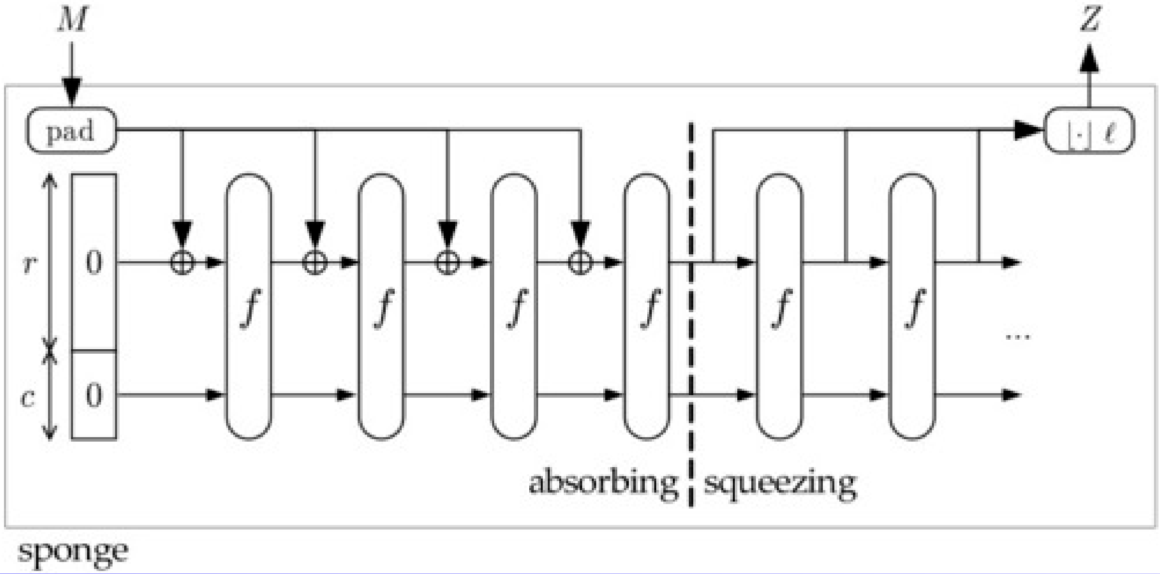
And the winner is?

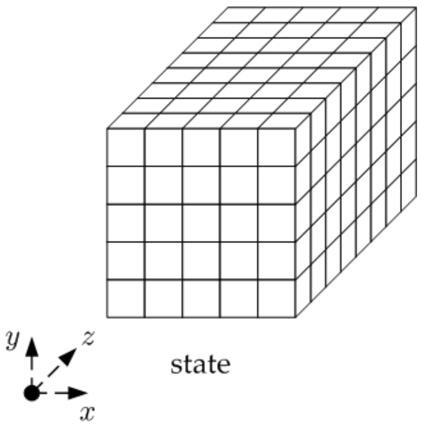
- NIST announced Keccak as the winner of the SHA-3 Cryptographic Hash Algorithm Competition on October 2, 2012, and ended the five-year competition.
- Keccak was designed by a team of cryptographers from Belgium and Italy, they are:
  - Guido Bertoni (Italy) of STMicroelectronics,
  - Joan Daemen (Belgium) of STMicroelectronics,
  - Michaël Peeters (Belgium) of NXP Semiconductors, and
  - Gilles Van Assche (Belgium) of STMicroelectronics.



Keccak and sponge functions



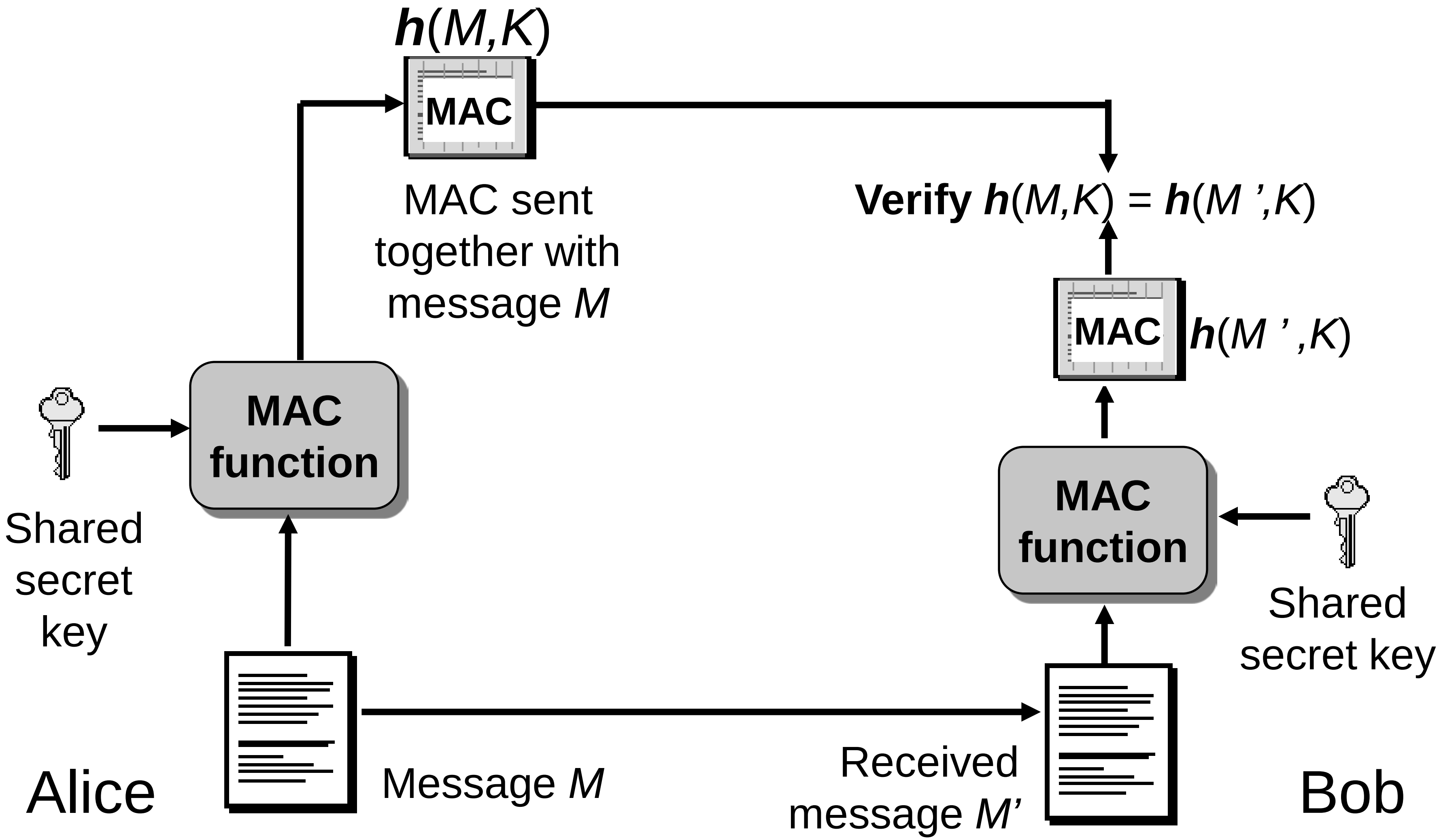




MAC and MAC algorithms

- MAC means two things:
  1. The computed message authentication code $h(M, k)$
  2. General name for algorithms used to compute a MAC
- In practice, the MAC algorithm is e.g.
  - HMAC (Hash-based MAC algorithm))
  - CBC-MAC (CBC based MAC algorithm)
  - CMAC (Cipher-based MAC algorithm)
- MAC algorithms, a.k.a. keyed hash functions, support data origin authentication services.

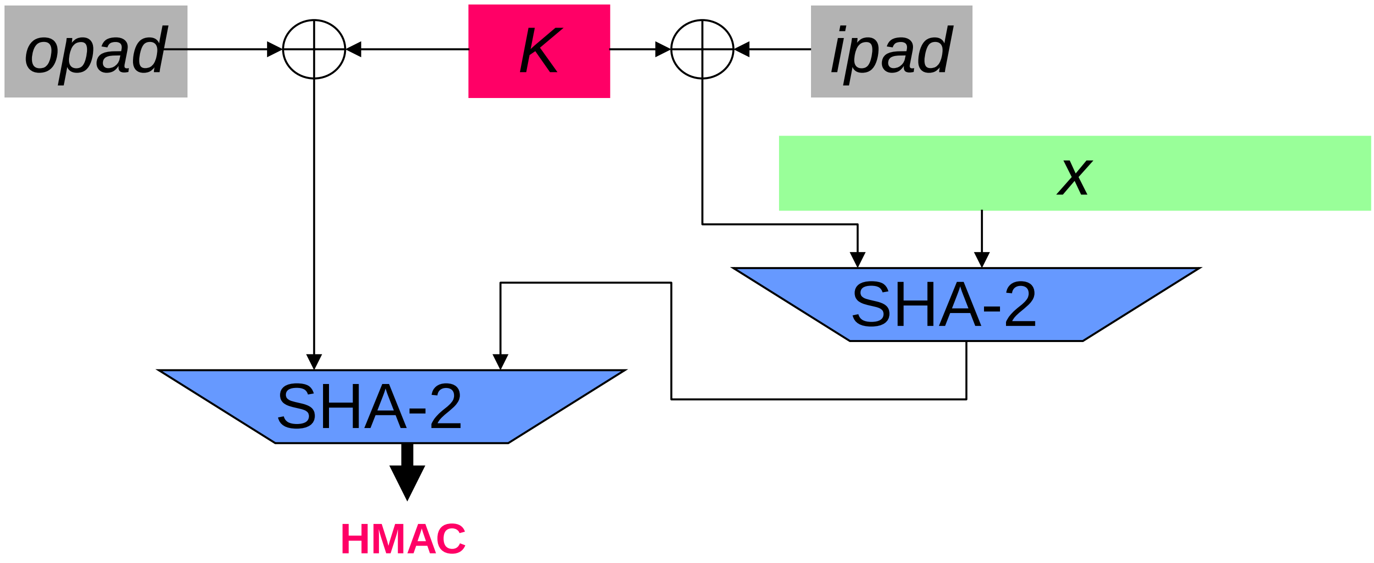
Practical message integrity with MAC



HMAC

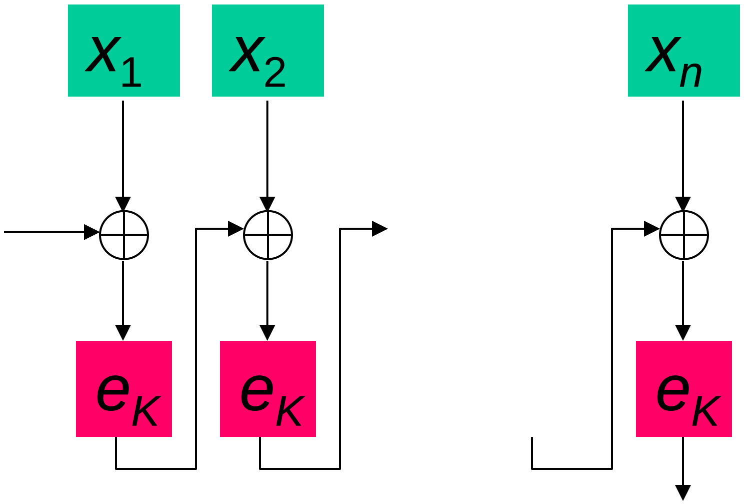
- Define: *ipad* = 3636….36 (512 bit)
- *opad* = 5C5C…5C (512 bit)

- $\text{HMAC}_K(x) = \text{SHA-1}((K \oplus opad) \,||\, \text{SHA-1}((K \oplus ipad) \,||\, x\,))$



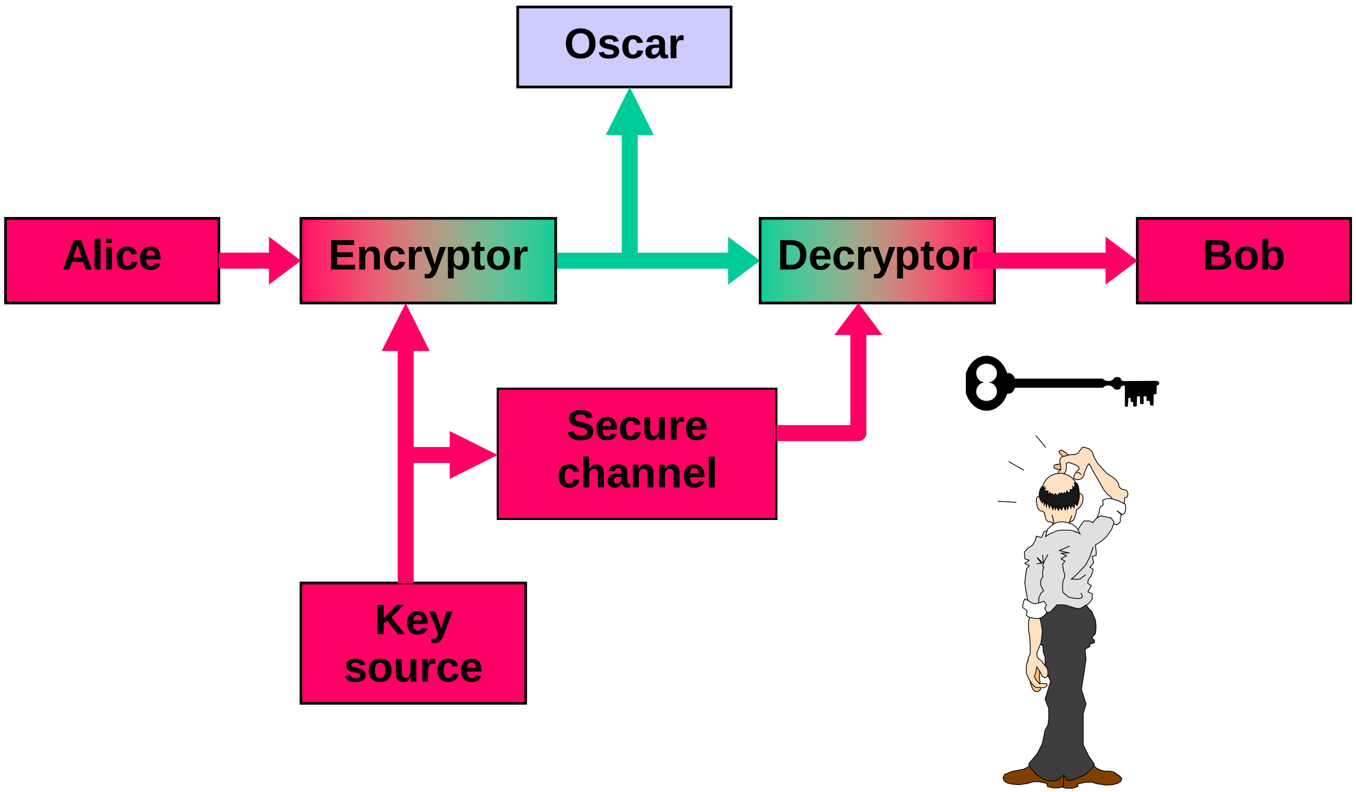
CBC-MAC

- **CBC-MAC($\boldsymbol{x}$, $\boldsymbol{K}$)**
- sett $x = x_1 \,||\, x_2 \,||\, \ldots. \,||\, x_n$
- IV $\leftarrow$ 00 … 0
- $y_0 \leftarrow$ IV
- **for** $i \leftarrow 1$ **to** n
- **do** $y_i \leftarrow e_K(y_{i-1} \oplus x_i)$
- **return** (y_n)

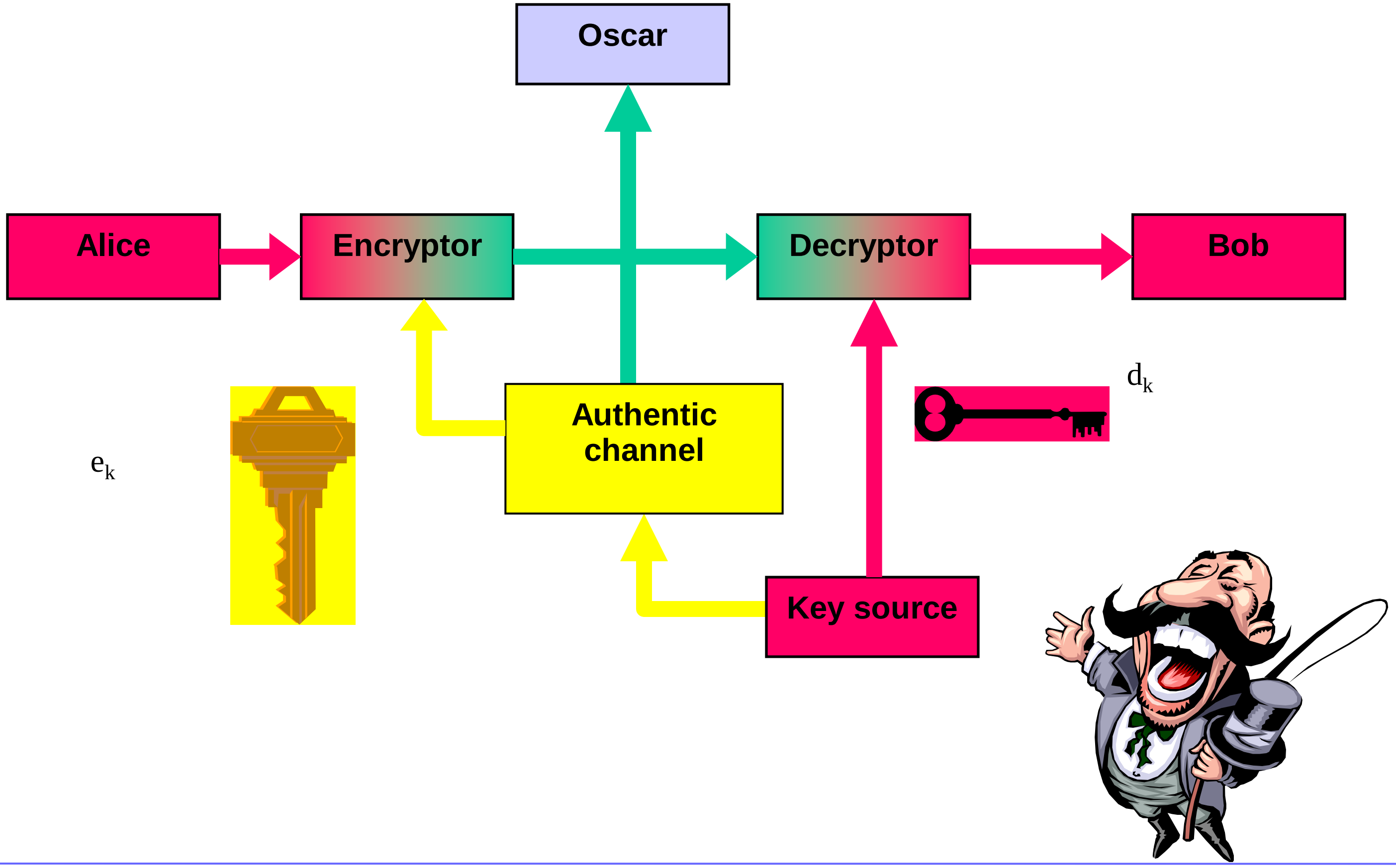


Public-Key Cryptography

Symmetric cryptosystem

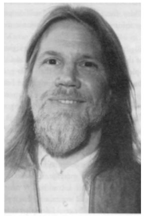


Asymmetric crypto system



Public key inventors?

Marty Hellman and Whit Diffie, Stanford 1976



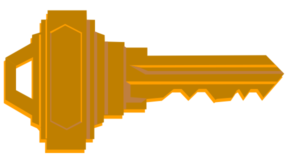
R. Rivest, A. Shamir and L. Adleman, MIT 1978

James Ellis, CESG 1970

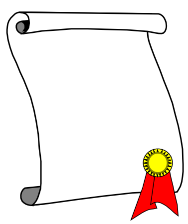
C. Cocks, M. Williamson, CESG 1973-1974

Asymmetric crypto

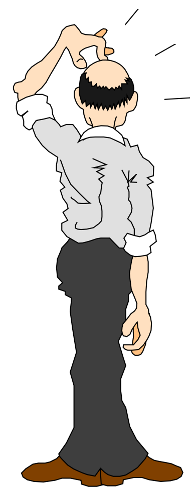
Public key cryptography was born in May 1975, the child of two problems and a misunderstanding!

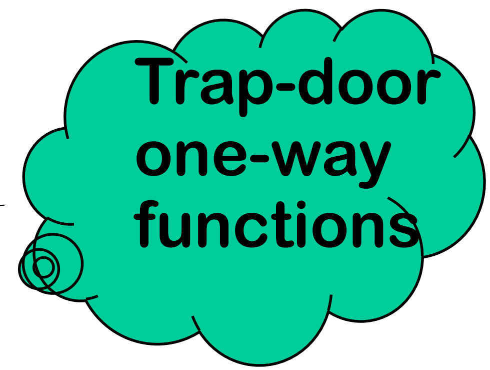


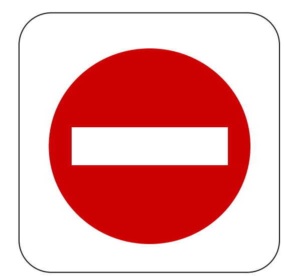
Key Distribution!



Digital signing!







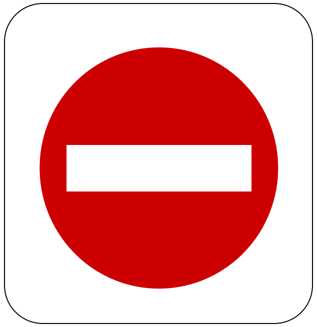
One-way functions

Modular power function

Given $n = pq$, where p and q are prime numbers. No efficient algoritms to find p and q.

Chose a positive integer b and define $f : Z_n \rightarrow Z_n$

$$f(x) = x^b \bmod n$$



Modular exponentiation

Given prime p, generator g and a modular power $a = g^x \pmod{p}$. No efficient algoritms to find x. $f : Z_p \rightarrow Z_p$

$$f(x) = g^x \bmod p$$

Diffie-Hellman key agreement (key exchange)
(provides no authentication)

Alice picks random integer a

$g^a \bmod p$ →

$g^b \bmod p$ ←

Computationally impossible to compute discrete logarithm

Bob picks random integer b

Alice computes the shared secret
$(g^b)^a = g^{ab} \bmod p$

Bob computes the same secret
$(g^a)^b = g^{ab} \bmod p$.

Example

- Z_{11} using $g = 2$:
  - $2^1 = 2 \pmod{11}$ $2^6 = 9 \pmod{11}$
  - $2^2 = 4 \pmod{11}$ $2^7 = 7 \pmod{11}$
  - $2^3 = 8 \pmod{11}$ $2^8 = 3 \pmod{11}$
  - $2^4 = 5 \pmod{11}$ $2^9 = 6 \pmod{11}$
  - $2^5 = 10 \pmod{11}$ $2^{10} = 1 \pmod{11}$

- $\log_2 5 = 4$
- $\log_2 7 = 7$
- $\log_2 1 = 10$ ($\equiv 0 \bmod 10$)

Example (2)

p =
30196626334536652266746444111852771272047217220445439805218819842806439806980163153421277779853237655786915947633907457862442472144616346714598423225826077976000905549946633556169688641786953396004062371399599729544977400404541673313622576825171747563463840240911791172271560696187007629722341591375265838579703621423172371480685909595288918038021190282938283683864372233025824059867626358694772029533769528178666567879514981999272674689885986300092124730492599541021908208672727813714852257201484474908352209019319074690727560652162418414435225636892749339867808955031056878928755875522700141844883356351776833964003

g =
17214844102945427204136512177889538496379881834679876598474115714966161705073026628129298835010174348250308006877834103702727269721499966768323290540216992770986728538508742382941595672248624817994917939749447675055374786840972654044030577846000645054950424877666860986820152109887355204363179653945098490724068905414681792636510652507946102434852166272721706635011474226289945817893390827991578201408649196984764863302981052471409215846871176739109049866118609117954454512573209668379576042056062096628325900231910090325301911333152181394803908610214937044613411740650800989334729586051242347771056691010439032429058

Finn *a* når

$g^a \pmod p$ =
44113216355065215159684488639683249149092460427650288245942898766876571824921690276662620979153820952830455103982849705054980427000258241321067445164291945709875449674237106754516103276658256727241360337237692098033897604855715556428192853384013674273248985055064876109463005314835390642583853176983615599073922523609689343385582696033895191791219150497333537020837218564219880414922079856566434665604898681669845852964624047443239120501341277499692338517113201830210812184500672101247270098803275601662656616757996322304239541426757926222214762596502305241986906124402779894141043268551743878130988606078310881106 17

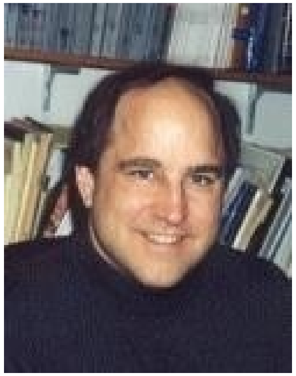
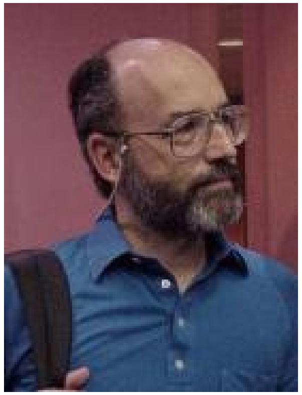
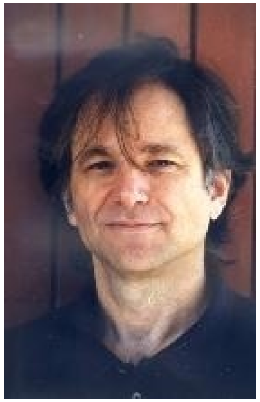
Solution

a =

71893136149709653804503478677866573695060790720621260648699193249561437588126371185
81694154929099396752251787268346548051895320171079663652680741564200286881487888963
19895353311170236034836658449187117723820644855184055305945501710227615558093657781
93109639893698220411548578601884177129022057550866690223052160523604836233675971504
25938247630127368253363295292024736143937779912318142315499711747531882501424082252
28164641111954587558230112140813226698098654739025636607106425212812421038155501562
37005192231836155067262308141154795194735834753570104459663325337960304941906119476
18181858300094662765895526963615406

It is easy to compute g^a (mod p) {0.016 s}, but it is computaionally infeasable to compute the exponent a from the g^a.

Ron Rivest, Adi Shamir and Len Adleman

- Read about public-key cryptography in 1976 article by Diffie & Hellman: *“New directions in cryptography”*
- Intrigued, they worked on finding a practical algorithm
- Spent several months in 1976 to re-invent the method for non-secret/public-key encryption discovered by Clifford Cocks 3 years earlier
- Named RSA algorithm

RSA parametre (textbook version)

- Bob generates two large prime numbers p and q and computes $n = p \cdot q$.
- He then computes a public encryption exponent e, such that
- $(e, (p-1)(q-1))\ ) = 1$ and computes the corresponding decryption exsponent d, by solving:

$$\boxed{d \cdot e \equiv 1 \pmod{(p-1)(q-1)}}$$

- Bob's public key is the pair $P_B = (e, n)$ and the corresponding private and secret key is $S_B = (d, n)$.

$$\boxed{\begin{array}{c}\text{Encryption: } C = M^e \pmod{n} \\ \text{Decryption: } M = C^d \pmod{n}\end{array}}$$

RSA toy example

- Set p = 157, q = 223. Then $n = p \cdot q$ =157 · 223 = 35011 and $(p-1)(q-1)$ = 156 ·222 = 34632
- Set encryption exponent: e = 14213 {gcd(34632,14213) = 1}
- Public key: (14213, 35011)
- Compute: $d = e^{-1} = 14213^{-1}$ (mod 34632) = 31613
- Private key: (31613, 35011)

- <u>Encryption:</u>
- Plaintext M = 19726, then C = 19726^{14213} (mod 35011) = 32986

- <u>Decryption:</u>
- Ciphererrtext C = 32986, then M = 32986^{31613}(mod 35011) = 19726

Factoring record– December 2009

- Find the product of
- p = 33478071698956898786044169848212690817704794983713768568
- 912431388982883793878002287614711652531743087737814467999489
- and
- q= 36746043666799590428244633799627952632279158164343087642676
- 032283815739666511279233373417143396810270092798736308917?

Answer:

n= 1230186684530117755130494958384962720772853569595334792197322
45215172640050726365751874520219978646938995647494277406384592
51925573263034537315482685079170261221429134616704292143116022
2124047927473779408066535141959745985690214341 3

Computation time ca. 0.0000003 s on a fast laptop!
RSA768 - Largest RSA-modulus that have been factored (12/12-2009)
Up to 2007 there was 50 000$ prize money for this factorisation!

Computational effort?

- Factoring using NFS-algorithm (Number Field Sieve)
- 6 mnd using 80 cores to find suitable polynomial
- Solding from August 2007 to April 2009 (1500 AMD64-år)
- 192 796 550 * 192 795 550 matrise (105 GB)
- 119 days on 8 different clusters
- Corresponds to 2000 years processing on one single core 2.2GHz AMD Opteron (ca. 2^{67} instructions)

Asymmetric Ciphers: Examples of Cryptosystems

- RSA: best known asymmetric algorithm.
  - RSA = Rivest, Shamir, and Adleman (published 1977)
  - Historical Note: U.K. cryptographer Clifford Cocks invented the same algorithm in 1973, but didn't publish.
- ElGamal Cryptosystem
  - Based on the difficulty of solving the discrete log problem.
- Elliptic Curve Cryptography
  - Based on the difficulty of solving the EC discrete log problem.
  - Provides same level of security with smaller key sizes.

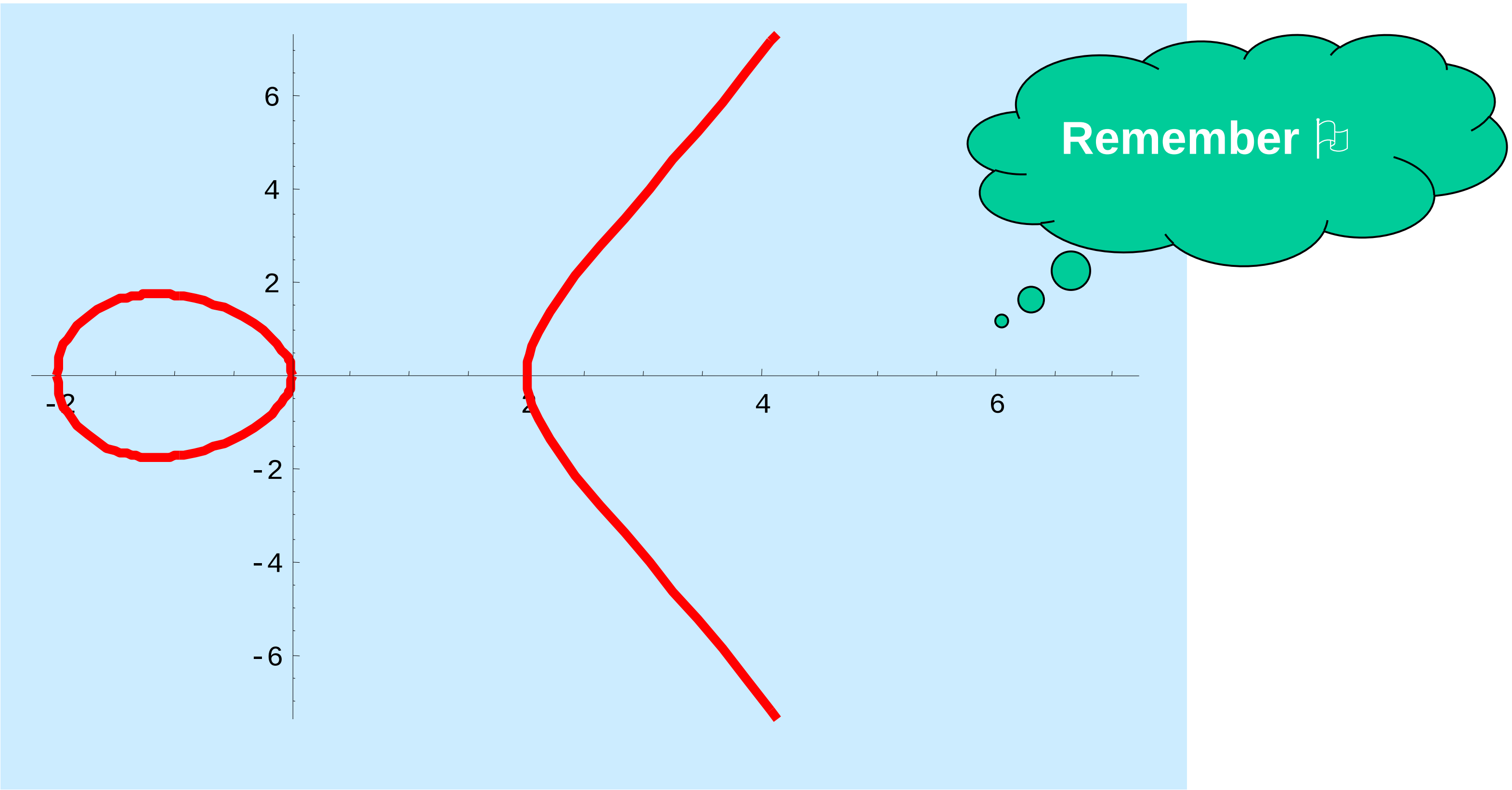
Elliptic curves

- Let $p > 3$ be a prime. An elliptic curve $y^2 = x^3 + ax + b$ over GF(p) = Z_p consist of all solutions $(x, y) \in Z_p \times Z_p$ to the equation

$$\boxed{y^2 \equiv x^3 + ax + b \pmod{p}}$$

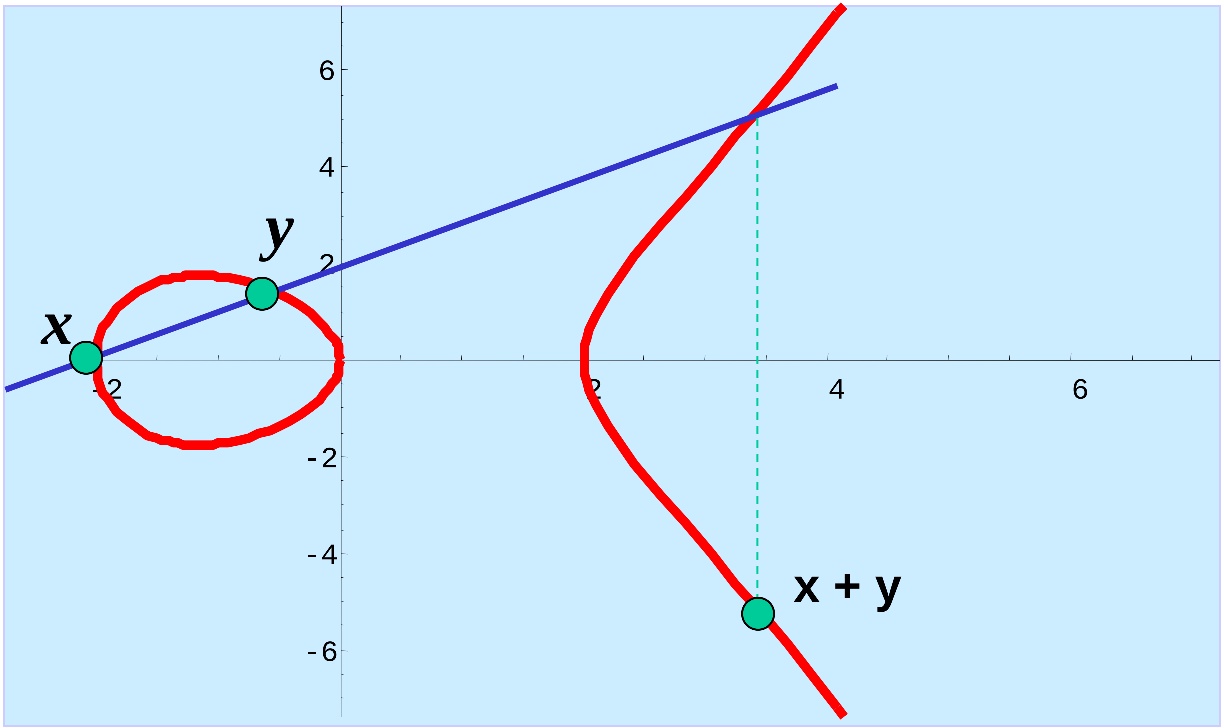
- where $a, b \in Z_p$ are constants such that $4a^3 + 27b^2 \neq 0 \pmod{p}$, together with a special point ⚐ which is denoted as *the point at infinity*.

Elliptic curve over R

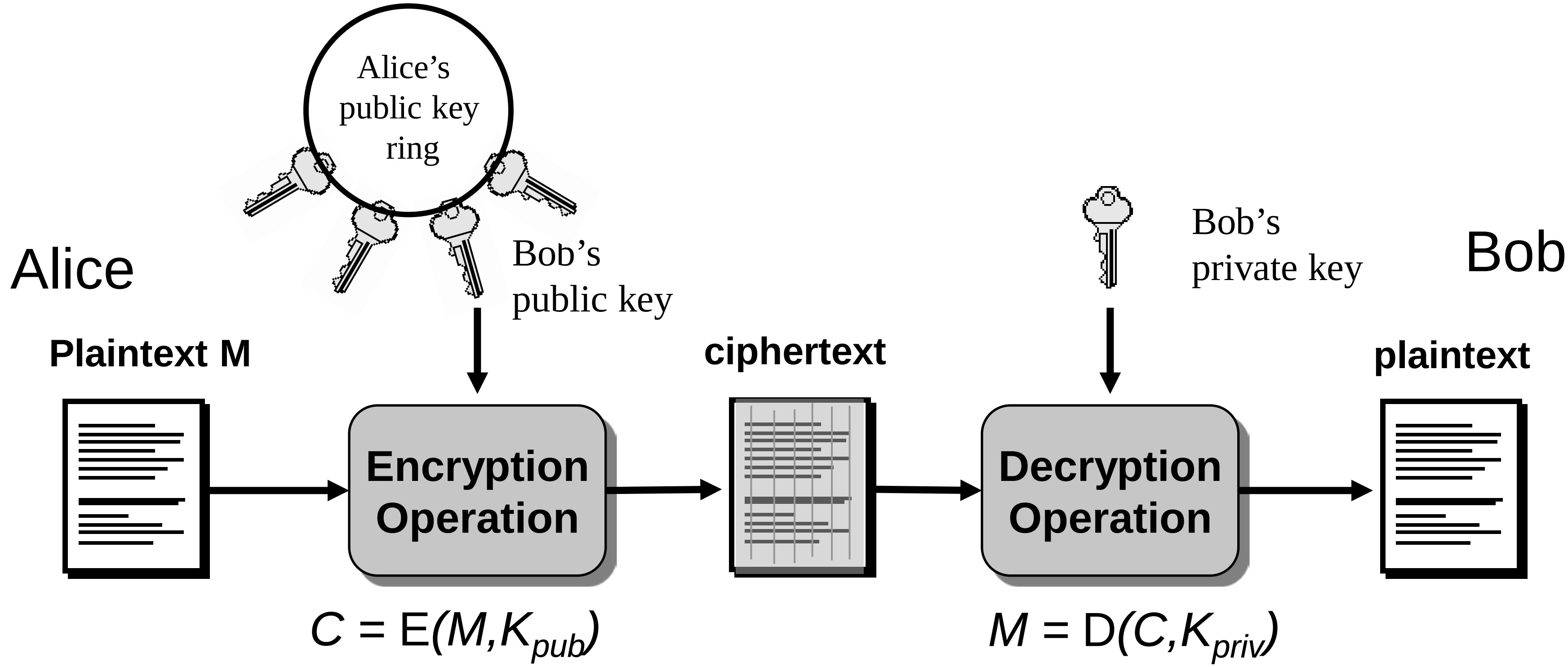


$$y^2 = x^3 - 4x$$

Point addition



Asymmetric Encryption: Basic encryption operation

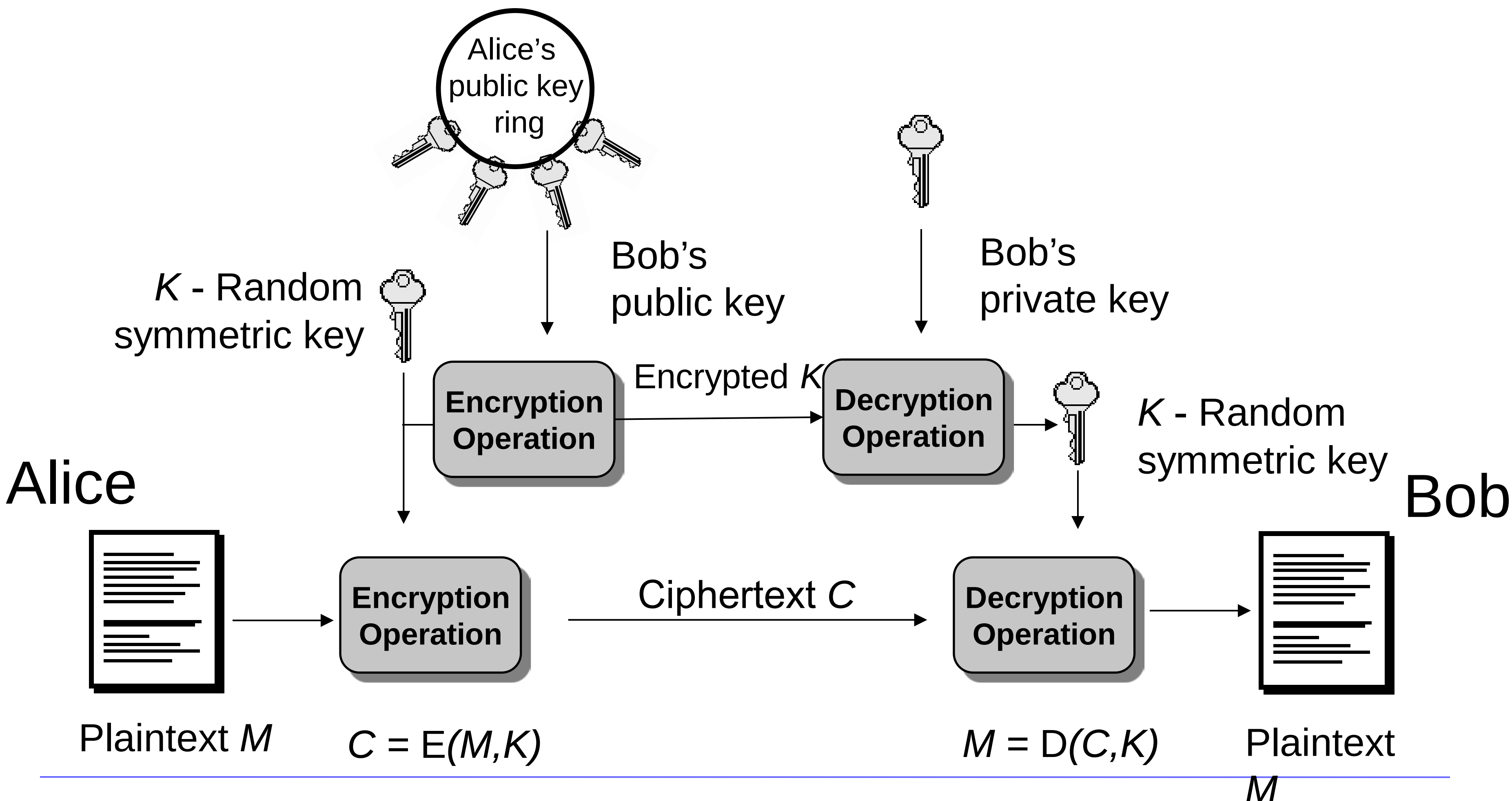


- In practice, large messages are not encrypted directly with asymmetric algorithms. Hybrid systems are used, where only symmetric session key is encrypted with asymmetric alg.

Hybrid Cryptosystems

- Symmetric ciphers are faster than asymmetric ciphers (because they are less computationally expensive), but ...
- Asymmetric ciphers simplify key distribution, therefore ...
- a combination of both symmetric and asymmetric ciphers can be used – a hybrid system:
  - The asymmetric cipher is used to distribute a randomly chosen symmetric key.
  - The symmetric cipher is used for encrypting bulk data.

Confidentiality Services: Hybrid Cryptosystems

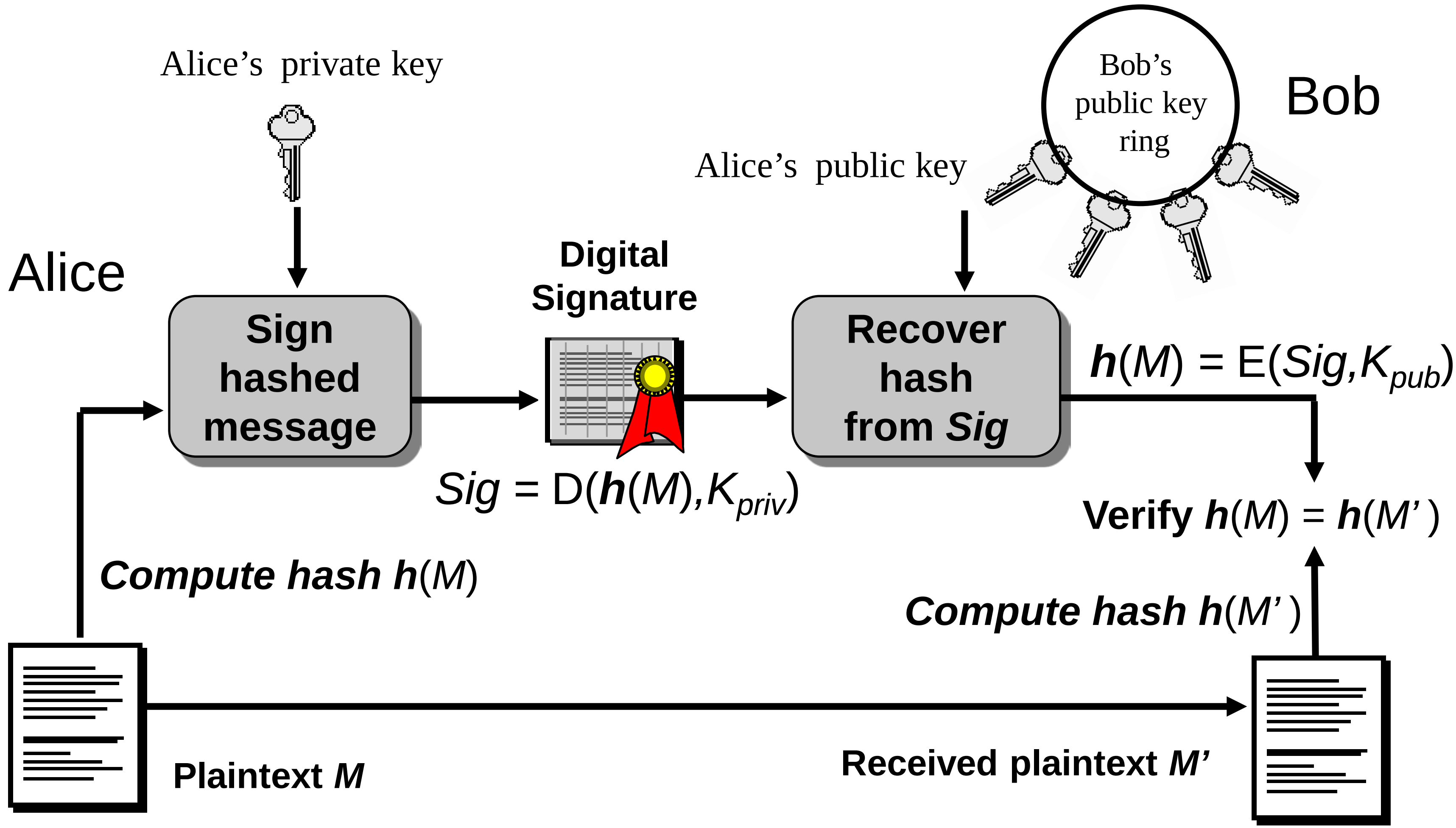


Digital Signatures

Digital Signature Mechanisms

- A MAC cannot be used as evidence that should be verified by a third party.
- Digital signatures used for non-repudiation, data origin authentication and data integrity services, and in some authentication exchange mechanisms.
- Digital signature mechanisms have three components:
  - key generation
  - signing procedure (private)
  - verification procedure (public)
- Algorithms
  - RSA
  - DSA and ECDSA

Practical digital signature based on hash value



Digital Signatures

- To get an authentication service that links a document to *A*'s name (identity) and not just a verification key, we require a procedure for *B* to get an authentic copy of *A*'s public key.
- Only then do we have a service that proves the authenticity of documents 'signed by *A*'.
- This can be provided by a PKI (Public Key Infrastructure)
- Yet even such a service does not provide non-repudiation at the level of persons.

Difference between MACs & Dig. Sig.

- MACs and digital signatures are both authentication mechanisms.
- MAC: the verifier needs the secret that was used to compute the MAC; thus a MAC is unsuitable as evidence with a third party.
  – The third party does not have the secret.
  – The third party cannot distinguish between the parties knowing the secret.
- Digital signatures can be validated by third parties, and can in theory thereby support both non-repudiation and authentication.

Key length comparison:

Symmetric and Asymmetric ciphers offering comparable security

AES Key Size	RSA Key Size	Elliptic curve Key Size
-	1024	163
128	3072	256
192	7680	384
256	15360	512

Another look at key lengths

Table 1. Intuitive security levels.

security level	volume of water to bring to a boil	bit-lengths: symmetric key	bit-lengths: cryptographic hash	bit-lengths: RSA modulus
teaspoon security	0.0025 liter	35	70	242
shower security	80 liter	50	100	453
pool security	2 500 000 liter	65	130	745
rain security	0.082 km^3	80	160	1130
lake security	89 km^3	90	180	1440
sea security	3 750 000 km^3	105	210	1990
global security	1 400 000 000 km^3	114	228	2380
solar security	-	140	280	3730

The eavesdropper strikes back!

MIT Technology Review

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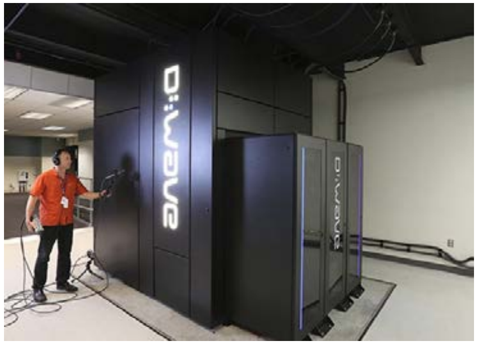
Computing

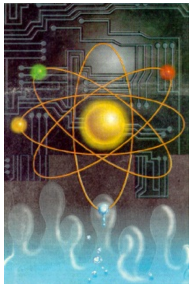
NSA Says It "Must Act Now" Against the Quantum Computing Threat

The National Security Agency is worried that quantum computers will neutralize our best encryption – but doesn't yet know what to do about that problem.

by Tom Simonite February 3, 2016



Quantum Computers



- Proposed by Richard Feynman 1982
- Boosted by P. Schor’s algorithm for integer factorization and discrete logarithm in quantum polynomial time
- Operates on qubit – superposition of 0 and 1
- IBM built a 7-bit quantum computer and could find the factors of the integer 15 using NMR techniques in 2001
- NMR does not scale
- Progress continues, but nobody knows if or when a large scale quantum computer ever can be constructed
- QC will kill current public key techniques, but does not mean an end to symmetric crypto
- Post Quantum Crypto (PQC) represents current research initiatives to develop crypto mechanisms that can resist quantum computer attacks!

Current world record of QF!

Table 5: *Quantum factorization records*

Number	# of factors	# of qubits needed	Algorithm	Year implemented	Implemented without prior knowledge of solution
15	2	8	Shor	2001 [2]	×
	2	8	Shor	2007 [3]	×
	2	8	Shor	2007 [3]	×
	2	8	Shor	2009 [5]	×
	2	8	Shor	2012 [6]	×
21	2	10	Shor	2012 [7]	×
143	2	4	minimization	2012 [1]	✓
56153	2	4	minimization	2012 [1]	✓
291311	2	6	minimization	not yet	✓
175	3	3	minimization	not yet	✓

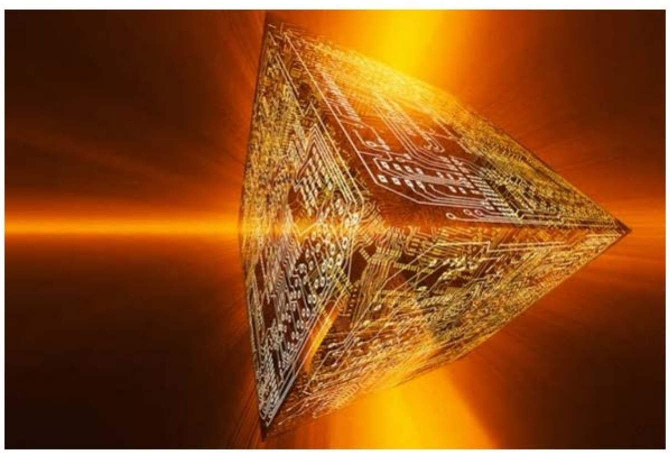
Scientific America Technology, Jan 2017

COMPUTING

Quantum Computers Ready to Leap Out of the Lab in 2017

Google, Microsoft and a host of labs and start-ups are racing to turn scientific curiosities into working machines

By Davide Castelvecchi, Nature magazine on January 4, 2017 **Véalo en español**



Credit: Mehau Kulyk Getty Images

Quantum computing has long seemed like one of those technologies that are 20 years away, and always will be. But 2017 could be the year that the field sheds its research-only image.

Computing giants Google and Microsoft recently hired a host of leading lights, and have set challenging goals for this year. Their ambition reflects a broader transition taking place at start-ups and academic research labs alike: to move from pure science towards engineering.

“People are really building things,” says Christopher Monroe, a physicist at the University of Maryland in College Park who co-founded the start-up IonQ in 2015. “I’ve never seen anything like that. It’s no longer just research.”

Brave new crypto world……………

Thank you!

End of lecture