

# INF3580 – Semantic Technologies – Spring 2010

## Lecture 10: OWL: Loose Ends

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13th April 2010



DEPARTMENT OF  
INFORMATICS



UNIVERSITY OF  
OSLO

# Today's Plan

- 1 Reminder: OWL
- 2 Cardinality restrictions
- 3 More about Datatypes
- 4 owl:sameAs and owl:differentFrom
- 5 Disjointness and Covering Axioms

# Outline

- 1 Reminder: OWL
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# ALC Semantics

## Interpretation

An interpretation  $\mathcal{I}$  fixes a set  $\Delta^{\mathcal{I}}$ , the *domain*,  $A^{\mathcal{I}} \subseteq \Delta$  for each atomic concept  $A$ , and  $R^{\mathcal{I}} \subseteq \Delta \times \Delta$  for each role  $R$

## Interpretation of concept descriptions

$$\begin{aligned}
 \top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
 \perp^{\mathcal{I}} &= \emptyset \\
 (\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
 (C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
 (C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
 (\forall R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid \forall b.(a, b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\} \\
 (\exists R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}
 \end{aligned}$$

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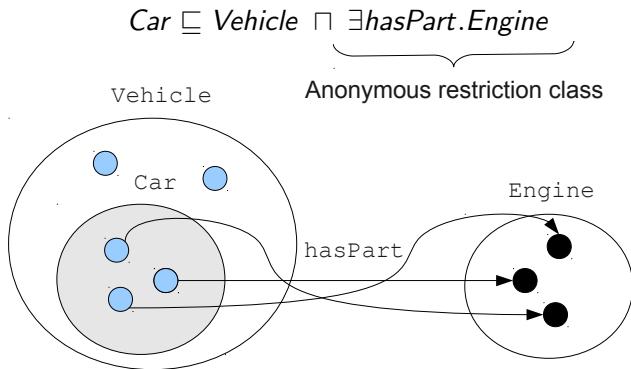
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- Example:
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  - $Car \sqsubseteq Vehicle \sqcap \exists hasPart.Engine$

# Existential restrictions illustrated



# A different perspective

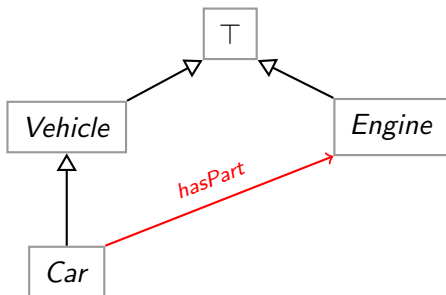


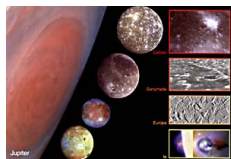
Figure: Connecting classes

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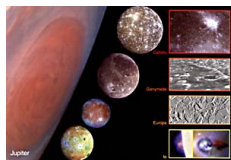
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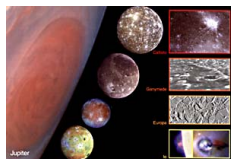
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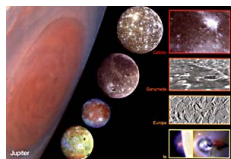
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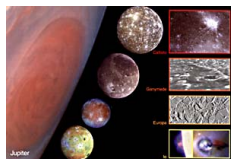
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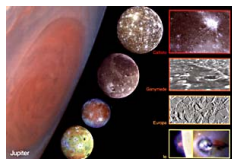
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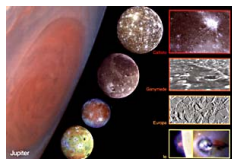
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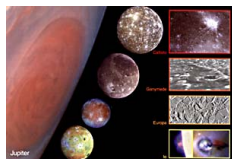
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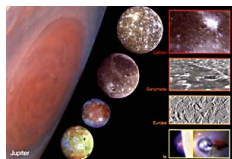
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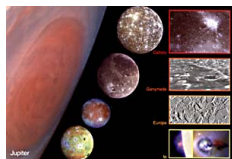
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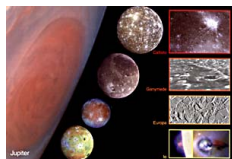
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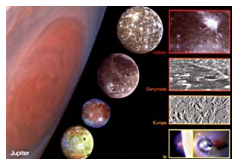
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$$SolarSystem \sqsubseteq \geq_1 hasPart. Star \sqcap \geq_1 hasPart. Planet$$



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- $R$  is functional  $\iff \leq_1 R.\top$



# Manchester Syntax

- $\leq_1$  *orbits.Star*  
orbits **max** 1 Star
- $\geq_8$  *hasPart.Planet*  
hasPart **min** 8 Planet

# The $\mathcal{ALCQ}$ Description Logic

## $\mathcal{ALCQ}$ concept descriptions

$C, D \rightarrow$	$A$		(atomic concept)
	$\top$		(universal concept)
	$\perp$		(bottom concept)
	$\neg C$		(atomic negation)
	$C \sqcap D$		(intersection)
	$C \sqcup D$		(union)
	$\forall R.C$		(value restriction)
	$\exists R.C$		(existential restriction)
	$\leq_n R.C$		(max. cardinality restriction)
	$\geq_n R.C$		(min. cardinality restriction)

# ALCQ Semantics

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# Cardinalities, non-unique names and open worlds

Cardinalities + the OWA and the NUNA is tricky, consider:

**TBox:**

*Ensemble*  $\sqsubseteq$  *ChamberEnsemble*  $\sqcup$  *Orchestra*

*ChamberEnsemble*  $\sqsubseteq \leq_1$  *firstViolin*. $\top$

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That is;

- Ensembles are either orchestras or chamber ensembles
- Chamber ensembles have only one instrument on each voice. . .
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## ABox:

Ensemble(oslo)

firstViolin(oslo, båtnes)

firstViolin(oslo, tønnesen)

## Musical taxons

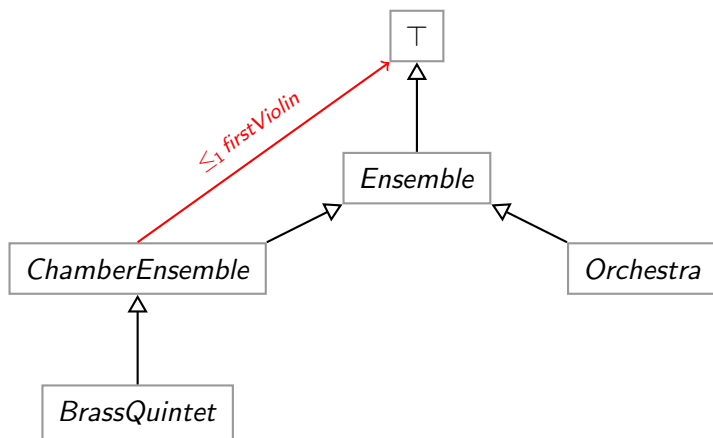


Figure: An ontology of ensembles

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- Possible to define more (dates, date ranges, etc.)

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  - `xsd:string`[`pattern "[01]*"`] – strings consisting of 0 and 1.

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  - Reasoner can find out that  $B \sqsubseteq A$ .

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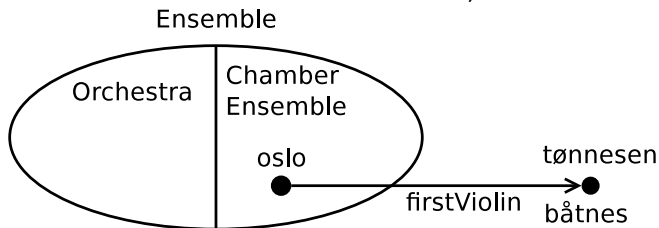
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`firstViolin(oslo, tønnesen)`

● Want to infer:  $Orchestra(oslo)$ 

## ● But:

$$tønnesen^I = båtnes^I$$



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- ... together imply *Orchestra*(oslo).
- OWL also provides an “allDifferent” construct for whole sets



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- How can a machine combine the information?

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- Note: only for individuals. For classes, use class equivalence axioms: en:Town owl:equivalentClass no:By .



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- Linked Open Data browsers treat them like other predicates

# Outline

- 1 Reminder: OWL
- 2 Cardinality restrictions
- 3 More about Datatypes
- 4 owl:sameAs and owl:differentFrom
- 5 Disjointness and Covering Axioms**



# Guys and Gals

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- First try:

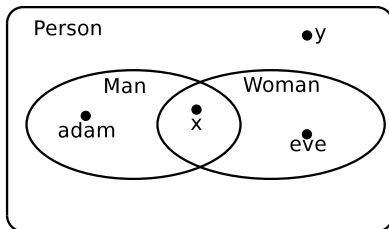
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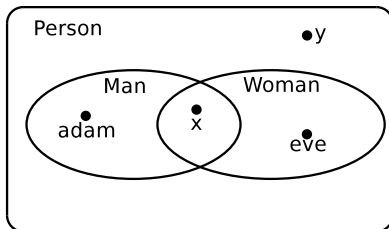


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- $x$  is both *Man* and *Woman*,  $y$  is neither but a *Person*.

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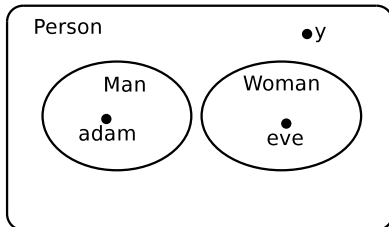
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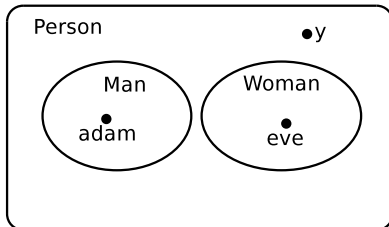
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- Specific support in OWL (`owl:disjointWith`) and Protégé

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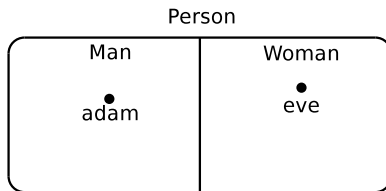
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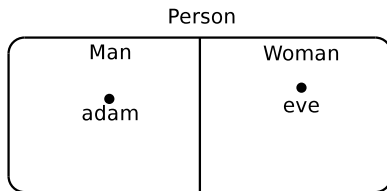


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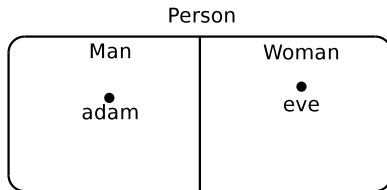


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- Compare to “abstract classes” in OO!

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- No disjointness axiom for *MeatEatingMammal* and *VeggieEatingMammal*!

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# Next Week

- Audun will take a recap:
- Some basic notions of sets and relations
- Repetition of logic, models, entailment, etc.