INF3580 – Semantic Technologies – Spring 2011

Lecture 9: Model Semantics & Reasoning

Martin Giese

22nd March 2011

Department of Informatics

University of Oslo

Today's Plan

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

Outline

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

Restricting RDF/RDFS

- We will simplify things by only looking at certain kinds of RDF graphs.
- No triples "about" properties, classes, etc., except RDFS
- Assume Resources are divided into four disjoint types:
  - *Properties* like `foaf:knows`, `dc:title`
  - *Classes* like `foaf:Person`
  - *Built-ins*, a fixed set including `rdf:type`, `rdfs:domain`, etc.
  - *Individuals* (all the rest, "usual" resources)
- All triples have one of the forms:

```
individual property individual .
individual rdf:type class .

class rdfs:subClassOf class .
property rdfs:subPropertyOf property .
property rdfs:domain class .
property rdfs:range class .
```

- Forget blank nodes and literals for a while!

Short Forms

- Resources and Triples are no longer all alike
- No need to use the same general triple notation
- Use alternative notation

Triples	Abbreviation
`indi prop indi .`	$r(i_1, i_2)$
`indi rdf:type class .`	$C(i_1)$
`class rdfs:subClassOf class .`	$C \sqsubseteq D$
`prop rdfs:subPropOf prop .`	$r \sqsubseteq s$
`prop rdfs:domain class .`	$\mathrm{dom}(r, C)$
`prop rdfs:range class .`	$\mathrm{rg}(r, C)$

- This is called "Description Logic" (DL) Syntax
- Used much in particular for OWL

Example

- Triples:

```
ws:romeo ws:loves ws:juliet .
ws:juliet rdf:type ws:Lady .

ws:Lady rdfs:subClassOf foaf:Person .
ws:loves rdfs:subPropertyOf foaf:knows .
ws:loves rdfs:domain ws:Lover .
ws:loves rdfs:range ws:Beloved .
```

- DL syntax, without namespaces:

$loves(romeo, juliet)$
$Lady(juliet)$

$Lady \sqsubseteq Person$
$loves \sqsubseteq knows$
$\mathrm{dom}(loves, Lover)$
$\mathrm{rg}(loves, Beloved)$

Interpretations for RDF

- To interpret the six kinds of triples, we need to know how to interpret
  - *Individual URIs* as real or imagined objects
  - *Class URIs* as sets of such objects
  - *Property URIs* as relations between these objects
- A *DL-interpretation* $\mathcal{I}$ consists of
  - A set $\Delta^{\mathcal{I}}$, called the *domain* (sorry!) of $\mathcal{I}$
  - For each individual URI i, an element $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
  - For each class URI C, a subset $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - For each property URI r, a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
- Given these, it will be possible to say whether a triple holds or not.

An example "intended" interpretation

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
- $romeo^{\mathcal{I}_1} =$  $juliet^{\mathcal{I}_1} =$ 
- $Lady^{\mathcal{I}_1} = \{$  $\}$ $Person^{\mathcal{I}_1} = \Delta^{\mathcal{I}_1}$

  $Lover^{\mathcal{I}_1} = Beloved^{\mathcal{I}_1} = \{$ ,  $\}$
- $loves^{\mathcal{I}_1} = \{ \langle$ ,  $\rangle, \langle$ ,  $\rangle \}$

  $knows^{\mathcal{I}_1} = \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_1}$

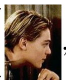
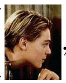
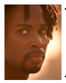
An example "non-intended" interpretation

- $\Delta^{\mathcal{I}_2} = \mathbb{N} = \{1, 2, 3, 4, \ldots\}$
- $romeo^{\mathcal{I}_2} = 17$
  $juliet^{\mathcal{I}_2} = 32$
- $Lady^{\mathcal{I}_2} = \{2^n \mid n \in \mathbb{N}\} = \{2, 4, 8, 16, 32, \ldots\}$
  $Person^{\mathcal{I}_2} = \{2n \mid n \in \mathbb{N}\} = \{2, 4, 6, 8, 10, \ldots\}$
  $Lover^{\mathcal{I}_2} = Beloved^{\mathcal{I}_2} = \mathbb{N}$
- $loves^{\mathcal{I}_2} = <= \{\langle x, y\rangle \mid x < y\}$
  $knows^{\mathcal{I}_2} = \leq = \{\langle x, y\rangle \mid x \leq y\}$
- Just because names (URIs) look familiar, they don't need to denote what we think!
- In fact, there is *no way* of ensuring they denote only what we think!

Validity in Interpretations

- Given an interpretation $\mathcal{I}$, define $\models$ as follows:
  - $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}}\rangle \in r^{\mathcal{I}}$
  - $\mathcal{I} \models C(i)$ iff $i^{\mathcal{I}} \in C^{\mathcal{I}}$
  - $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
  - $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
  - $\mathcal{I} \models \mathrm{dom}(r, C)$ iff $\mathrm{dom}\, r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
  - $\mathcal{I} \models \mathrm{rg}(r, C)$ iff $\mathrm{rg}\, r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- For a set of triples $\mathcal{A}$ (any of the six kinds)
- $\mathcal{A}$ is valid in $\mathcal{I}$, written
  $$\mathcal{I} \models \mathcal{A}$$
- iff $\mathcal{I} \models A$ for all $A \in \mathcal{A}$.

Validity Examples

- $\mathcal{I}_1 \models loves(juliet, romeo)$ because
  $\langle$ ,  $\rangle \in loves^{\mathcal{I}_1} = \{ \langle$ ,  $\rangle, \langle$ ,  $\rangle \}$
- $\mathcal{I}_2 \not\models Person(romeo)$ because
- $romeo^{\mathcal{I}_2} = 17 \notin Person^{\mathcal{I}_2} = \{2, 4, 6, 8, 10, \ldots\}$
- $\mathcal{I}_1 \models Lover \sqsubseteq Person$ because
  $Lover^{\mathcal{I}_1} = \{$ ,  $\} \subseteq Person^{\mathcal{I}_1} = \{$ , ,  $\}$
- $\mathcal{I}_2 \not\models Lover \sqsubseteq Person$ because
  $Lover^{\mathcal{I}_2} = \mathbb{N}$ and $Person^{\mathcal{I}_2} = \{2, 4, 6, 8, 10, \ldots\}$

Entailment

- Given a set of triples $\mathcal{A}$ (any of the six kinds)
- And a further triple T (also any kind)
- T is entailed by $\mathcal{A}$, written $\mathcal{A} \models T$
- iff
  - For any interpretation $\mathcal{I}$ with $\mathcal{I} \models \mathcal{A}$
  - $\mathcal{I} \models T$.
- Example:
  - $\mathcal{A} = \{\ldots, Lady(juliet), Lady \sqsubseteq Person, \ldots\}$ as before
  - $\mathcal{A} \models Person(juliet)$ because...
  - in *any* interpretation $\mathcal{I}$...
  - if $juliet^{\mathcal{I}} \in Lady^{\mathcal{I}}$ and $Lady^{\mathcal{I}} \subseteq Person^{\mathcal{I}}$ ...
  - then by set theory $juliet^{\mathcal{I}} \in Person^{\mathcal{I}}$
- *Not* about T being (intuitively) true or not
- Only about whether T is a *consequence* of $\mathcal{A}$

Countermodels

- If $\mathcal{A} \not\models T$,...
- then there is an $\mathcal{I}$ with
  - $\mathcal{I} \models \mathcal{A}$
  - $\mathcal{I} \not\models T$
- Vice-versa: if $\mathcal{I} \models \mathcal{A}$ and $\mathcal{I} \not\models T$, then $\mathcal{A} \not\models T$
- Such an $\mathcal{I}$ is called a *counter-model* (for the assumption that $\mathcal{A}$ entails T)
- To show that $\mathcal{A} \models T$ does *not* hold:
  - Describe an interpretation $\mathcal{I}$ (using your fantasy)
  - Prove that $\mathcal{I} \models \mathcal{A}$ (using the semantics)
  - Prove that $\mathcal{I} \not\models T$ (using the semantics)
- Countermodels for intuitively true statements are always unintuitive! (Why?)

Countermodel Example

- $\mathcal{A}$ as before:

$$\mathcal{A} = \{loves(romeo, juliet),\ Lady(juliet),\ Lady \sqsubseteq Person,\ loves \sqsubseteq knows,\ \mathrm{dom}(loves, Lover),\ \mathrm{rg}(loves, Beloved)\}$$

- Does $\mathcal{A} \models Lover \sqsubseteq Beloved$?
- Holds in $\mathcal{I}_1$ and $\mathcal{I}_2$.
- Try to find an interpretation with $\Delta^{\mathcal{I}} = \{a, b\}$, $a \neq b$.
- Interpret $romeo^{\mathcal{I}} = a$ and $juliet^{\mathcal{I}} = b$
- Then $\langle a, b\rangle \in loves^{\mathcal{I}}$, $a \in Lover^{\mathcal{I}}$, $b \in Beloved^{\mathcal{I}}$.
- With $Lover^{\mathcal{I}} = \{a\}$ and $Beloved^{\mathcal{I}} = \{b\}$, $\mathcal{I} \not\models Lover \sqsubseteq Beloved$!
- Choose

$$loves^{\mathcal{I}} = knows^{\mathcal{I}} = \{\langle a, b\rangle\} \qquad Lady^{\mathcal{I}} = Person^{\mathcal{I}} = \{b\}$$

to complete the count-model while satisfying $\mathcal{I} \models \mathcal{A}$

Outline

Simplifying Literals

- Literals can only occur as *objects* of triples
- Can be plain, with language tag, or with data type.
- The same predicate can be used with literals and resources:

```
ex:me ex:likes dbpedia:Berlin .
ex:me ex:likes "food" .
```

- We simplify things by:
  - ignoring language tags and data types, and
  - allowing either literal objects *or* literal objects for any predicate
- Five types of resources:
  - *Object Properties* like `foaf:knows`
  - *Datatype Properties* like `dc:title`, `foaf:name`
  - *Classes* like `foaf:Person`
  - *Built-ins*, a fixed set including `rdf:type`, `rdfs:domain`, etc.
  - *Individuals* (all the rest, "usual" resources)
- Why? – simpler, object/datatype split is in OWL

Allowed triples

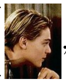
Allow only triples using object properties and datatype properties as intended

Triples	Abbreviation
`indi o-prop indi .`	$r(i_1, i_2)$
`indi d-prop "lit" .`	$a(i, l)$
`indi rdf:type class .`	$C(i_1)$
`class rdfs:subClassOf class .`	$C \sqsubseteq D$
`o-prop rdfs:subPropOf o-prop .`	$r \sqsubseteq s$
`d-prop rdfs:subPropOf d-prop .`	$a \sqsubseteq b$
`o-prop rdfs:domain class .`	$\text{dom}(r, C)$
`o-prop rdfs:range class .`	$\text{rg}(r, C)$

Interpretation with Literals

- Let Λ be the set of all literal values, i.e. all strings
- A *DL-interpretation* $\mathcal{I}$ consists of
  - A set $\Delta^{\mathcal{I}}$, called the *domain* of $\mathcal{I}$
  - Interpretations $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ as before
  - For each datatype property URI a, a relation $a^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Lambda$
- Semantics:
  - $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$ for object property r
  - $\mathcal{I} \models a(i, l)$ iff $\langle i^{\mathcal{I}}, l \rangle \in a^{\mathcal{I}}$ for datatype property a
  - $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$ for object properties r, s
  - $\mathcal{I} \models a \sqsubseteq b$ iff $a^{\mathcal{I}} \subseteq b^{\mathcal{I}}$ for datatype properties a, b
- Note: Literals l are in Λ, don't need to be interpreted.

Example: Interpretation with a Datatype Property

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
- $loves^{\mathcal{I}_1} = \{\langle$ ,  $\rangle, \langle$ ,  $\rangle\}$

  $knows^{\mathcal{I}_1} = \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_1}$
- $age^{\mathcal{I}_1} = \{\langle$ , "16" $\rangle, \langle$ , "almost 14" $\rangle, \langle$ , "13" $\rangle, \}$

Outline

Blank Nodes

- Remember: Blank nodes are just like resources...
- ...but without a "global" URI.
- Blank node has a local "blank node identifier" instead.

- A blank node *can* be used in several triples...
- ...but they have to be in the same "file" or "data set"
- Semantics of blank nodes require looking at a set of triples

- But we still need to interpret single triples.
- Solution: pass in blank node interpretation, deal with sets later!

Blank Node Valuations

- Given an interpretation $\mathcal{I}$ with domain $\Delta^{\mathcal{I}}$...
  - A *blank node valuation* β...
  - ...gives a domain element or literal value $\beta(b) \in \Delta^{\mathcal{I}} \cup \Lambda$...
  - ...for every blank node ID b
- Now define $\cdot^{\mathcal{I},\beta}$
  - $i^{\mathcal{I},\beta} = i^{\mathcal{I}}$ for individual URIs i
  - $l^{\mathcal{I},\beta} = l$ for literals l
  - $b^{\mathcal{I},\beta} = \beta(b)$ for blank node IDs b
- Interpretation:
  - $\mathcal{I}, \beta \models r(x, y)$ iff $\langle x^{\mathcal{I},\beta}, y^{\mathcal{I},\beta} \rangle \in r^{\mathcal{I}}$...
  - ...for any legal combination of URIs/literals/blank nodes x, y
  - ...and object/datatype property r

Sets of Triples with Blank Nodes

- Given a set $\mathcal{A}$ of triples with blank nodes...
- $\mathcal{I}, \beta \models \mathcal{A}$ iff $\mathcal{I}, \beta \models A$ for all $A \in \mathcal{A}$

- $\mathcal{A}$ is valid in $\mathcal{I}$

$$\mathcal{I} \models \mathcal{A}$$

  if there is a β such that $\mathcal{I}, \beta \models \mathcal{A}$

- I.e. if there exists some valuation for the blank nodes that makes all triples true.

Example: Blank Node Semantics

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
- $loves^{\mathcal{I}_1} = \{ \langle$ ,  $\rangle, \langle$ ,  $\rangle \}$ $knows^{\mathcal{I}_1} = \Delta^{\mathcal{I}_1} \times \Delta^{\mathcal{I}_1}$
- $age^{\mathcal{I}_1} = \{ \langle$ , "16" $\rangle, \langle$ , "almost 14" $\rangle, \langle$ , "13" $\rangle, \}$
- Let b_1, b_2, b_3 be blank nodes
- $\mathcal{A} = \{age(b_1, \texttt{"16"}), knows(b_1, b_2), loves(b_2, b_3), age(b_3, \texttt{"13"})\}$
- Valid in $\mathcal{I}_1$?
- Pick $\beta(b_1) = \beta(b_2) =$ , $\beta(b_3) =$ .
- Then $\mathcal{I}_1, \beta \models \mathcal{A}$
- So, yes, $\mathcal{I}_1 \models \mathcal{A}$.

Entailment with Blank Nodes

- Entailment is defined just like without blank nodes:
  - Given sets of triples $\mathcal{A}$ and $\mathcal{B}$,
  - $\mathcal{A}$ entails $\mathcal{B}$, written $\mathcal{A} \models \mathcal{B}$
  - iff for any interpretation $\mathcal{I}$ with $\mathcal{I} \models \mathcal{A}$, also $\mathcal{I} \models \mathcal{B}$.
- This expands to: for any interpretation $\mathcal{I}$
  - such that there exists a β_1 with $\mathcal{I}, \beta_1 \models \mathcal{A}$
  - there also exists a β_2 such that $\mathcal{I}, \beta_2 \models \mathcal{B}$
- Two different blank node valuations!
- Can evaluate the same blank node name differently in $\mathcal{A}$ and $\mathcal{B}$.
- Example:

$$\{loves(b_1, juliet), knows(juliet, romeo), age(juliet, \texttt{"13"})\}$$
$$\models \{loves(b_2, b_1), knows(b_1, romeo)\}$$

Monotonicity

- Assume $\mathcal{A} \models \mathcal{B}$
- Now add information to $\mathcal{A}$, i.e. $\mathcal{A}' \supseteq \mathcal{A}$
- Then $\mathcal{B}$ is still entailed: $\mathcal{A}' \models \mathcal{B}$
- We say that RDF/RDFS entailment is *monotonic*
- Needed to derive consequences under incomplete information (OWA)
- Non-monotonic reasoning:
  - $\{Bird \sqsubseteq CanFly, Bird(tweety)\} \models CanFly(tweety)$
  - $\{\ldots, Penguin \sqsubseteq Bird, Penguin(tweety), Penguin \sqsubseteq \neg CanFly\} \not\models CanFly(tweety)$
  - Interesting for human-style reasoning
  - Hard to combine with semantic web technologies

Outline

Two Kinds of Consequence?

- We now have two ways of describing logical consequence...

1. Using RDFS rules:

$$\frac{\texttt{:Lady rdfs:subClassOf :Person .} \qquad \texttt{:juliet a :Lady .}}{\texttt{:juliet a :Person .}}\ \text{rdfs9}$$

$$\frac{Lady \sqsubseteq Person \qquad Lady(juliet)}{Person(juliet)}\ \text{rdfs9}$$

2. Using the model semantics
   - If $\mathcal{I} \models Lady \sqsubseteq Person$ and $\mathcal{I} \models Lady(juliet)$...
   - ...then $Lady^{\mathcal{I}} \subseteq Person^{\mathcal{I}}$ and $juliet^{\mathcal{I}} \in Lady^{\mathcal{I}}$...
   - ...so by set theory, $juliet^{\mathcal{I}} \in Person^{\mathcal{I}}$...
   - ...and therefore $\mathcal{I} \models Person(juliet)$.
   - Together: $\{Lady \sqsubseteq Person, Lady(juliet)\} \models Person(juliet)$

- What is the connection between these two?

Entailment and Derivability

- Actually, two different notions!
- Entailment is defined using the model semantics.
- The rules say what can be *derived*
  - derivability
  - provability
- Entailment
  - is closely related to the *meaning* of things
  - higher confidence in model semantics than in a bunch of rules
  - *The* semantics given by the standard, rules are just "informative"
  - can't be directly checked mechanically (∞ many interpretations)
- Derivability
  - can be checked mechanically
  - forward or backward chaining
- Want these notions to correspond:
  - $\mathcal{A} \models \mathcal{B}$ iff $\mathcal{B}$ can be derived from $\mathcal{A}$

Soundness

- Two directions:
  1. If $\mathcal{A} \models \mathcal{B}$ then $\mathcal{B}$ can be derived from $\mathcal{A}$
  2. If $\mathcal{B}$ can be derived from $\mathcal{A}$ then $\mathcal{A} \models \mathcal{B}$
- Nr. 2 usually considered more important:
- If the calculus says that something is entailed then it is really entailed.
- The calculus gives no "wrong" answers.
- This is known as *soundness*
- The calculus is said to be *sound* (w.r.t. the model semantics)

Showing Soundness

- Soundness of every rule has to be (manually) checked!
- E.g. rdfs11,

$$\frac{A \sqsubseteq B \qquad B \sqsubseteq C}{A \sqsubseteq C}\ \text{rdfs11}$$

- Soundness means that
  - For any choice of three classes A, B, C
  - $\{A \sqsubseteq B, B \sqsubseteq C\} \models A \sqsubseteq C$
- Proof:
  - Let $\mathcal{I}$ be an arbitrary interpretation with $\mathcal{I} \models \{A \sqsubseteq B, B \sqsubseteq C\}$
  - Then by model semantics, $A^{\mathcal{I}} \subseteq B^{\mathcal{I}}$ and $B^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
  - By set theory, $A^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
  - By model semantics, $\mathcal{I} \models A \sqsubseteq C$
  - Q.E.D.
- This can be done similarly for all of the rules.
  - All given RDF/RDFS rules are sound w.r.t. the model semantics!

Completeness

- Two directions:
  1. If $\mathcal{A} \models \mathcal{B}$ then $\mathcal{B}$ can be derived from $\mathcal{A}$
  2. If $\mathcal{B}$ can be derived from $\mathcal{A}$ then $\mathcal{A} \models \mathcal{B}$
- Nr. 1 says that any entailment can be found using the rules.
- I.e. we have "enough" rules.
- Can't be checked separately for each rule, only for whole rule set
- Proofs are more complicated than soundness

Simple Entailment Rules

$$\frac{r(u,x)}{r(u,b_1)}\ \text{se1} \qquad \frac{r(u,x)}{r(b_1,x)}\ \text{se2}$$

Where b_1 is a blank node identifier, that either

- has not been used before in the graph, or
- has been used, but for the same URI/Literal x resp. u.

- Simple entailment is entailment
  - With blank nodes and literals
  - but without RDFS
  - and without RDF axioms like `rdf:type rdf:type rdf:Property .`
- se1 and se2 are complete for simple entailment, i.e.

  $\mathcal{A}$ simply entails $\mathcal{B}$
  iff $\mathcal{A}$ can be extended with se1 and se2 to $\mathcal{A}'$ with $\mathcal{B} \subseteq \mathcal{A}'$.

Simple Entailment Example

$\{loves(b_1, juliet), knows(juliet, romeo), age(juliet, \texttt{"13"})\}$

$loves(b_2, juliet) \quad (b_2 \to b_1)$

$loves(b_2, b_3) \quad (b_3 \to juliet)$

$knows(b_3, romeo) \quad (\text{reusing } b_3 \to juliet)$

$\models \{loves(b_2, b_3), knows(b_3, romeo)\}$

Rules for (simplified) RDF/RDFS

- See Foundations book, Sect. 3.3
- Many rules and axioms not needed for our “simplified” RDF/RDFS
  - `rdfs:range rdfs:domain rdfs:Class ...`
- Important rules for us:

$$\frac{\mathrm{dom}(r,A) \quad r(x,y)}{A(x)}\ \text{rdfs2} \qquad \frac{\mathrm{rg}(r,B) \quad r(x,y)}{B(y)}\ \text{rdfs3}$$

$$\frac{r \sqsubseteq s \quad s \sqsubseteq t}{r \sqsubseteq t}\ \text{rdfs5} \qquad \frac{}{r \sqsubseteq r}\ \text{rdfs6} \qquad \frac{r \sqsubseteq s \quad r(x,y)}{s(x,y)}\ \text{rdfs7}$$

$$\frac{A \sqsubseteq B \quad A(x)}{B(x)}\ \text{rdfs9} \qquad \frac{}{A \sqsubseteq A}\ \text{rdfs10} \qquad \frac{A \sqsubseteq B \quad B \sqsubseteq C}{A \sqsubseteq C}\ \text{rdfs11}$$

Complete?

- These rules are *not* complete for our RDF/RDFS semantics
- For instance

$$\{\mathrm{rg}(loves, Beloved), Beloved \sqsubseteq Person\} \models \mathrm{rg}(loves, Person)$$

- Because for every interpretation $\mathcal{I}$,
  - if $\mathcal{I} \models \{\mathrm{rg}(loves, Beloved), Beloved \sqsubseteq Person\}$
  - then by semantics, rg $loves^{\mathcal{I}} \subseteq Beloved^{\mathcal{I}}$ and $Beloved^{\mathcal{I}} \subseteq Person^{\mathcal{I}}$.
  - Therefore, by set theory, rg $loves^{\mathcal{I}} \subseteq Person^{\mathcal{I}}$
  - By semantics, $\mathcal{I} \models \mathrm{rg}(loves, Person)$
- But there is no way to derive this using the given rules
  - There is no rule which allows to derive a range statement.
- We could now add rules to make the system complete
- Won't bother to do that now. Will get completeness for OWL.

Outlook

- RDFS allows some simple modelling: “all ladies are persons”
- The following lectures will be about OWL
- Will allow to say things like
  - Every car has a motor
  - Every car has at least three parts of type wheel
  - A mother is a person who is female and has at least one child
  - The friends of my friends are also my friends
  - A metropolis is a town with at least a million inhabitants
  - . . . and many more
- Modeling will not be done by writing triples manually:
- Will use ontology editor Protégé.