INF3580 – Semantic Technologies – Spring 2011

Lecture 9: Model Semantics & Reasoning

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Today's Plan

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

Outline

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2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

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- Forget blank nodes and literals for a while!

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- Used much in particular for OWL

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- DL syntax, without namespaces:

$$loves(romeo, juliet)$$
$$Lady(juliet)$$

$$Lady \sqsubseteq Person$$
$$loves \sqsubseteq knows$$
$$\mathrm{dom}(loves, Lover)$$
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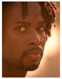
- Given these, it will be possible to say whether a triple holds or not.

An example “intended” interpretation

- $\Delta^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$

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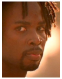
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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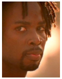
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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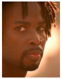
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- In fact, there is *no way* of ensuring they denote only what we think!

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- iff $\mathcal{I} \models A$ for all $A \in \mathcal{A}$.

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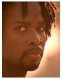
  $\mathit{Lover}^{\mathcal{I}_1} = \Big\{$  ,  $\Big\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \Big\{$  ,  ,  $\Big\}$

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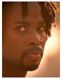
  $\mathit{Lover}^{\mathcal{I}_1} = \Big\{$  ,  $\Big\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \Big\{$  ,  ,  $\Big\}$
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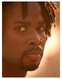
  $\mathit{Lover}^{\mathcal{I}_1} = \Big\{$ ,  $\Big\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$

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- Countermodels for intuitively true statements are always unintuitive! (Why?)

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- $\mathcal{A}$ as before:

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- With $Lover^{\mathcal{I}} = \{a\}$ and $Beloved^{\mathcal{I}} = \{b\}$, $\mathcal{I} \not\models Lover \sqsubseteq Beloved$!
- Choose

$$loves^{\mathcal{I}} = knows^{\mathcal{I}} = \{\langle a, b\rangle\} \qquad Lady^{\mathcal{I}} = Person^{\mathcal{I}} = \{b\}$$

to complete the count-model while satisfying $\mathcal{I} \models \mathcal{A}$

Outline

1 Repetition: RDF semantics

2 Literal Semantics

3 Blank Node Semantics

4 Entailment and Derivability

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- Why? – simpler, object/datatype split is in OWL

Allowed triples

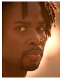
Allow only triples using object properties and datatype properties as intended

Triples	Abbreviation
`indi o-prop indi .`	$r(i_1, i_2)$
`indi d-prop "lit" .`	$a(i, l)$
`indi rdf:type class .`	$C(i_1)$
`class rdfs:subClassOf class .`	$C \sqsubseteq D$
`o-prop rdfs:subPropOf o-prop .`	$r \sqsubseteq s$
`d-prop rdfs:subPropOf d-prop .`	$a \sqsubseteq b$
`o-prop rdfs:domain class .`	$\mathrm{dom}(r, C)$
`o-prop rdfs:range class .`	$\mathrm{rg}(r, C)$

Interpretation with Literals

- Let Λ be the set of all literal values, i.e. all strings
- A *DL-interpretation* $\mathcal{I}$ consists of
  - A set $\Delta^{\mathcal{I}}$, called the *domain* of $\mathcal{I}$
  - Interpretations $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, and $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ as before
  - For each datatype property URI a, a relation $a^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Lambda$
- Semantics:
  - $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$ for object property r
  - $\mathcal{I} \models a(i, l)$ iff $\langle i^{\mathcal{I}}, l \rangle \in a^{\mathcal{I}}$ for datatype property a
  - $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$ for object properties r, s
  - $\mathcal{I} \models a \sqsubseteq b$ iff $a^{\mathcal{I}} \subseteq b^{\mathcal{I}}$ for datatype properties a, b
- Note: Literals l are in Λ, don't need to be interpreted.

Example: Interpretation with a Datatype Property

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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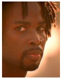
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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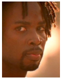
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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Outline

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

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- Solution: pass in blank node interpretation, deal with sets later!

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if there is a β such that $\mathcal{I}, \beta \models \mathcal{A}$
- I.e. if there exists some valuation for the blank nodes that makes all triples true.

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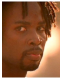
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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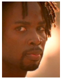
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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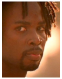
- $\Delta^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$
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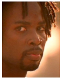
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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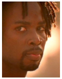
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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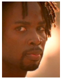
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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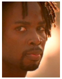
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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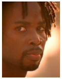
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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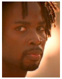
- $\Delta^{\mathcal{I}_1} = \{$  ,  ,  $\}$
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- So, yes, $\mathcal{I}_1 \models \mathcal{A}$.

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  - there also exists a β_2 such that $\mathcal{I}, \beta_2 \models \mathcal{B}$
- Two different blank node valuations!

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Outline

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2. Literal Semantics
3. Blank Node Semantics
4. Entailment and Derivability

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- This can be done similarly for all of the rules.

Showing Soundness

- Soundness of every rule has to be (manually) checked!
- E.g. rdfs11,

$$\frac{A \sqsubseteq B \qquad B \sqsubseteq C}{A \sqsubseteq C}\ \text{rdfs11}$$

- Soundness means that
  - For any choice of three classes A, B, C
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  - All given RDF/RDFS rules are sound w.r.t. the model semantics!

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- Won't bother to do that now. Will get completeness for OWL.

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