INF3580 – Semantic Technologies – Spring 2011

Lecture 12: OWL: Loose Ends

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Today's Plan

1. Reminder: OWL
2. Disjointness and Covering Axioms
3. Keys
4. More about Datatypes
5. What can't be expressed in OWL 2

Outline

$\mathcal{ALCQ}$ Semantics

Interpretation

An interpretation $\mathcal{I}$ fixes a set $\Delta^{\mathcal{I}}$, the *domain*, $A^{\mathcal{I}} \subseteq \Delta$ for each atomic concept A, and $R^{\mathcal{I}} \subseteq \Delta \times \Delta$ for each role R

Interpretation of concept descriptions

$$
\begin{aligned}
\top^{\mathcal{I}} &= \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} &= \emptyset \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\forall R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid b \in C^{\mathcal{I}} \text{ for all } b \text{ with } \langle a, b\rangle \in R^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid b \in C^{\mathcal{I}} \text{ for some } b \text{ with } \langle a, b\rangle \in R^{\mathcal{I}}\} \\
(\leq_n R.C)^{\mathcal{I}} &= \{a \in \Delta^{\mathcal{I}} \mid \#\{b \mid \langle a, b\rangle \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \leq n\} \\
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  - and `owl:differentFrom`: $a \neq b$

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  $Person \sqcap \exists knows.Self$

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- All people who know themselves:
  $\mathit{Person} \sqcap \exists \mathit{knows}.\mathit{Self}$
- Manchester Syntax:
  `Person and knows Self`

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- Add a little more to OWL, and this is lost!

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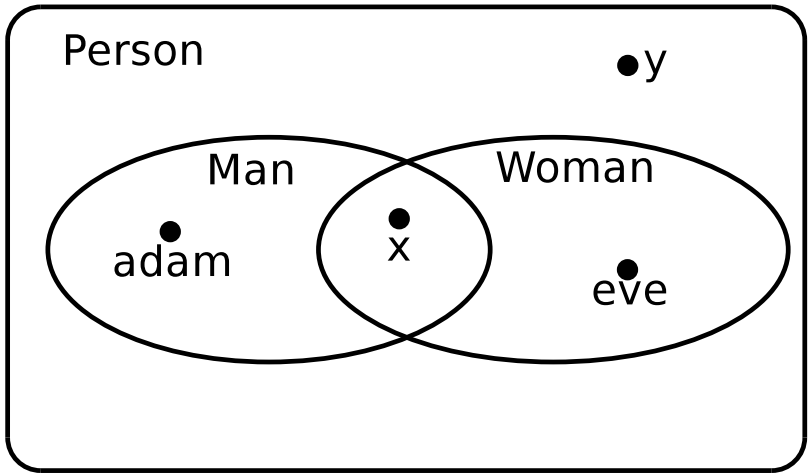

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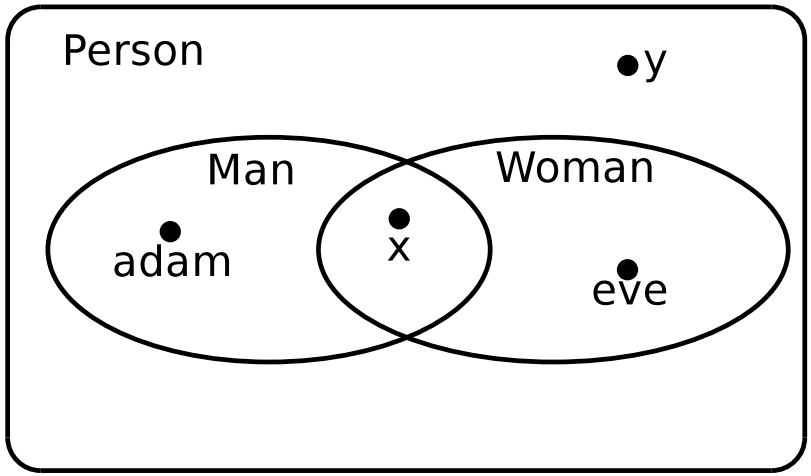

- x is both *Man* and *Woman*, y is neither but a *Person*.

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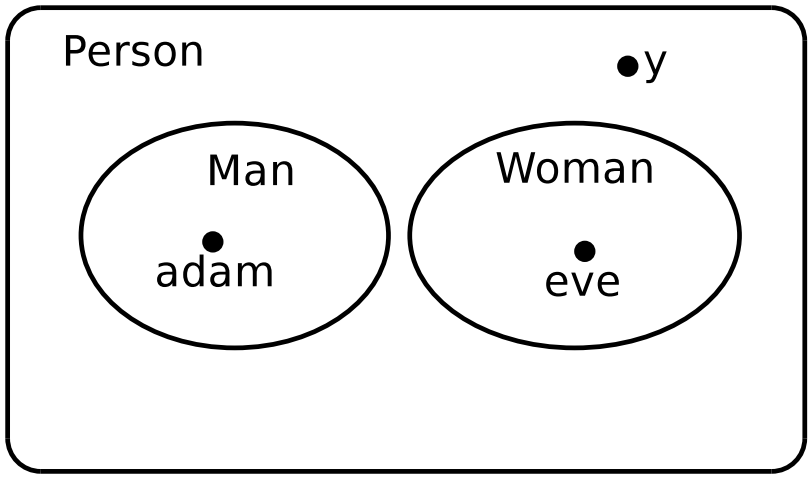

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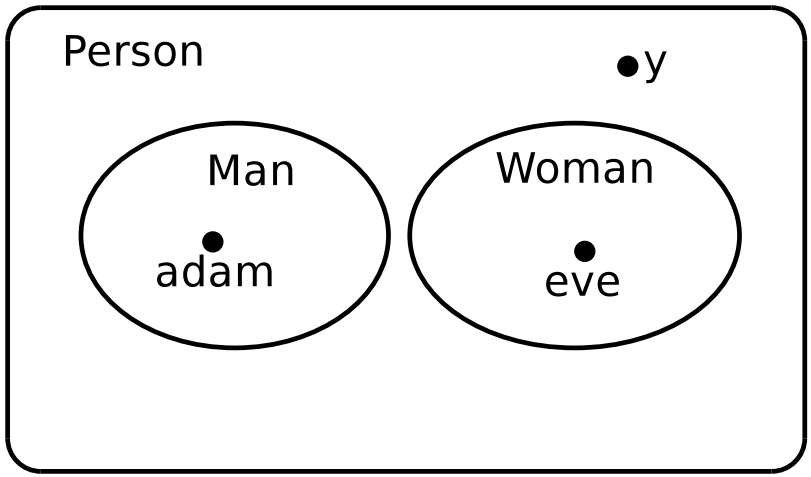

- Specific support in OWL (`owl:disjointWith`) and Protégé

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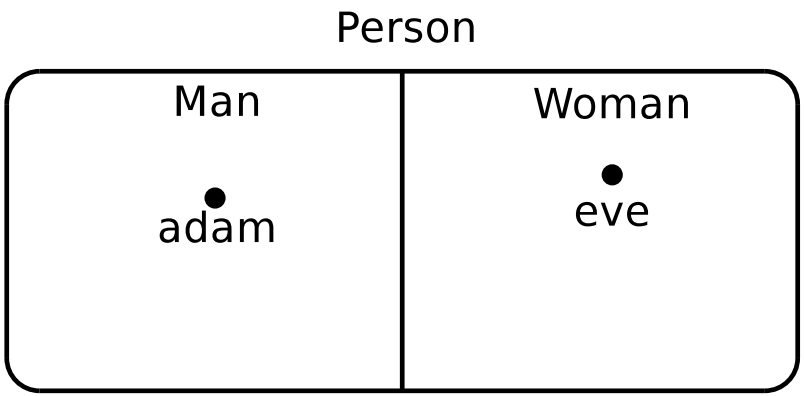

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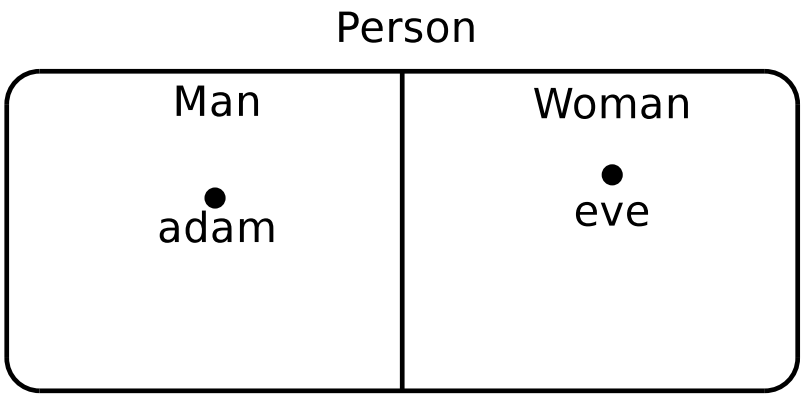

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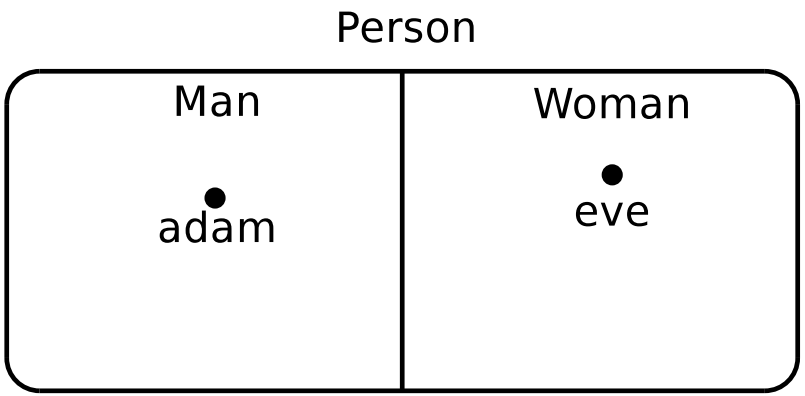

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- Compare to "abstract classes" in OO!

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- No disjointness axiom for *MeatEatingMammal* and *VeggieEatingMammal*!

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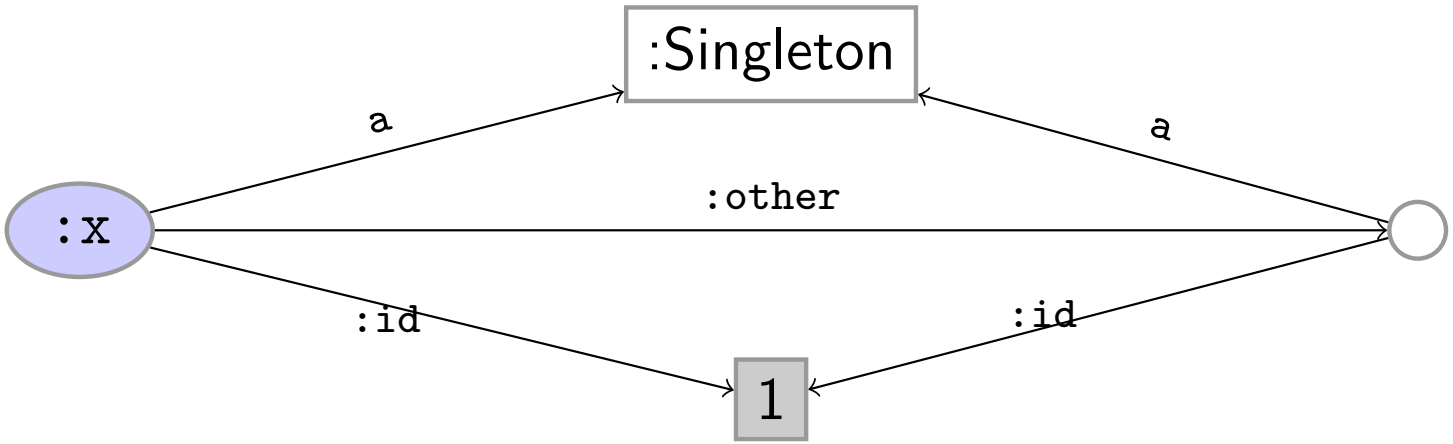

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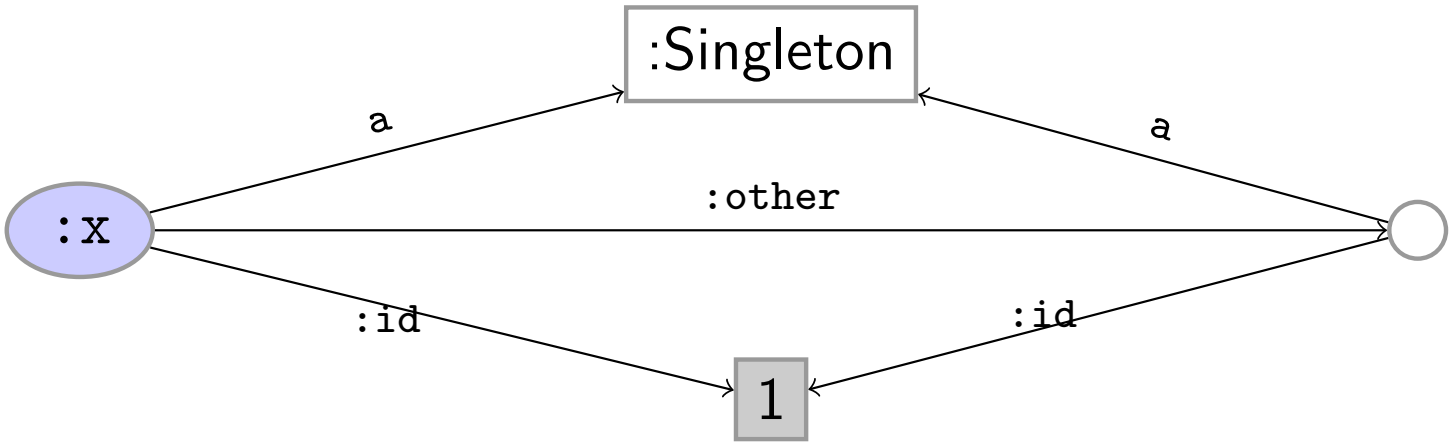

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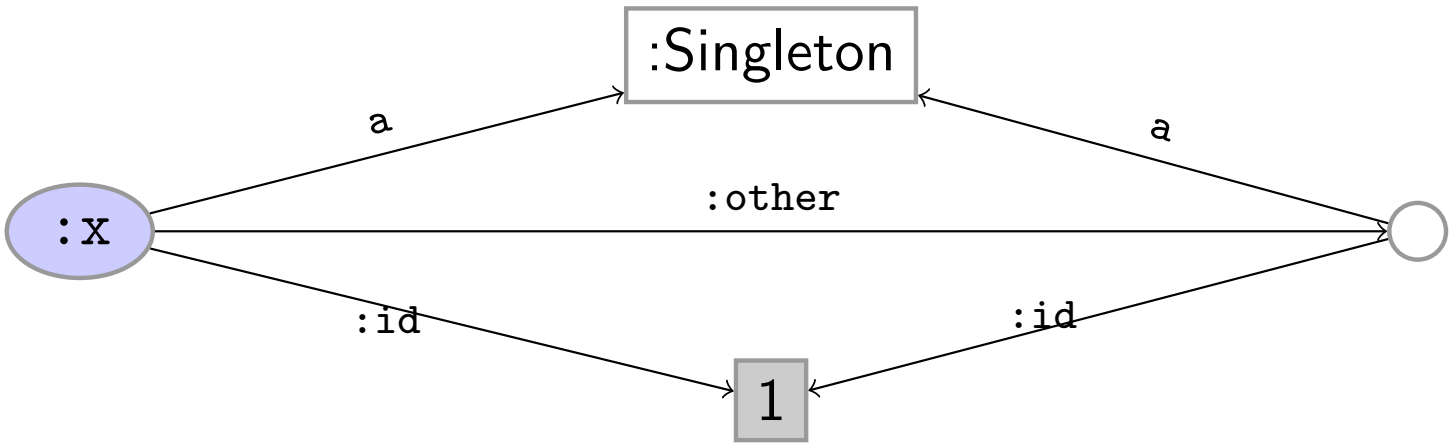

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  - domain *Whisky*
  - range *int*
  - relates the whisky to each year it is in the cask.

e.g. `:young :casked "2000"^^int, "2001"^^int, "2002"^^int`

- $Scotch \sqsubseteq Whisky \sqcap \geq_3 casked.int$
- Works, but...
- Can't express e.g. that the years are consecutive
  - Knowing a whisky is casked in 2000 and 2009 doesn't imply it is casked for 10 years.
- Reasoning about $\geq_n$ often works by generating n sample instances
  - $Town \equiv \geq_{10000} inhabitant.Person$
  - $Metropolis \equiv \geq_{1000000} inhabitant.Person$
  - Will kill almost any reasoner

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- Possible to define more (dates, date ranges, etc.)

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  - `xsd:string[pattern "[01]*"]` – strings consisting of 0 and 1.

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- Note: often makes best sense with functional properties

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  - Reasoner can find out that $B \sqsubseteq A$.

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  - We look at some examples, not proofs

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- Given terms

  hasSibling *Male*

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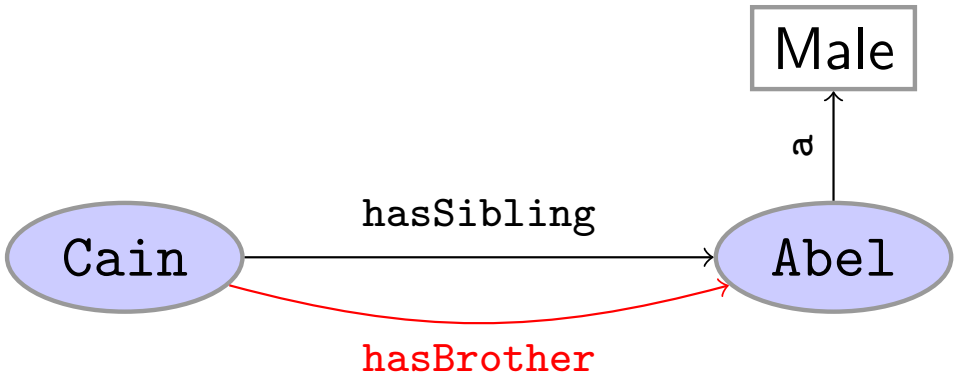
- . . . a brother is *defined* to be a sibling who is male



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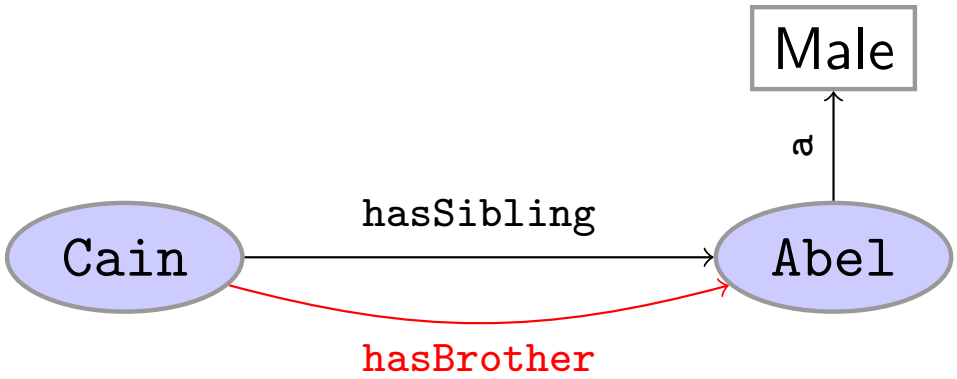

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$$hasBrother \sqsubseteq hasSibling$$
$$\forall hasBrother.Male \quad \text{or: } \mathrm{rg}(hasBrother, Male)$$
$$\exists hasSibling.Male \sqsubseteq \exists hasBrother.\top$$

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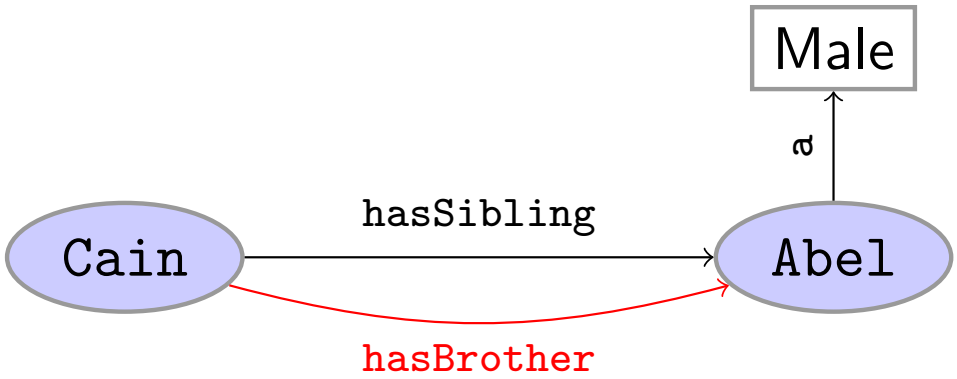

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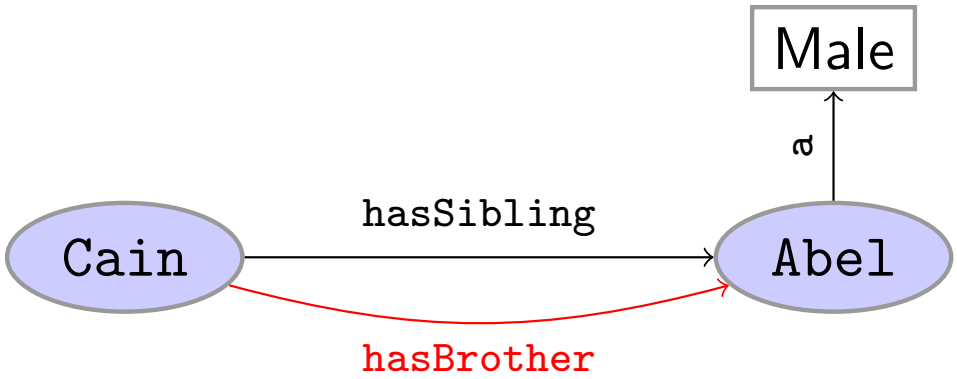

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  - (probably mostly an "accident" in the OWL 2 specification)

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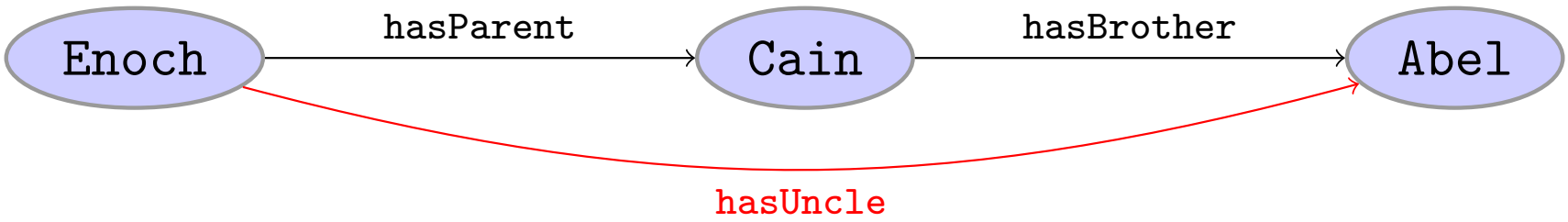

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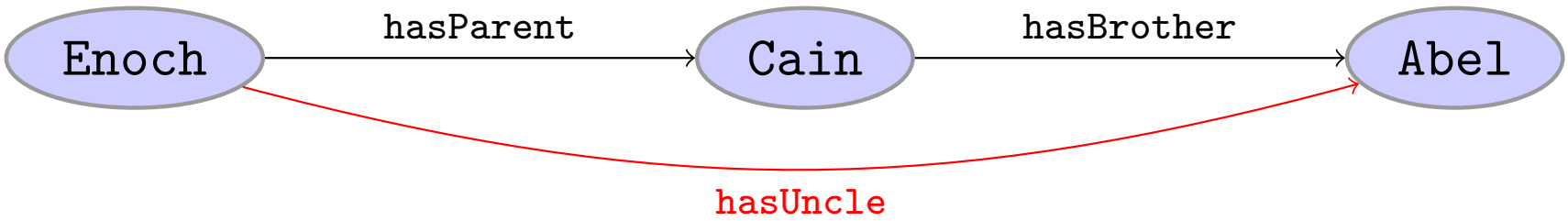

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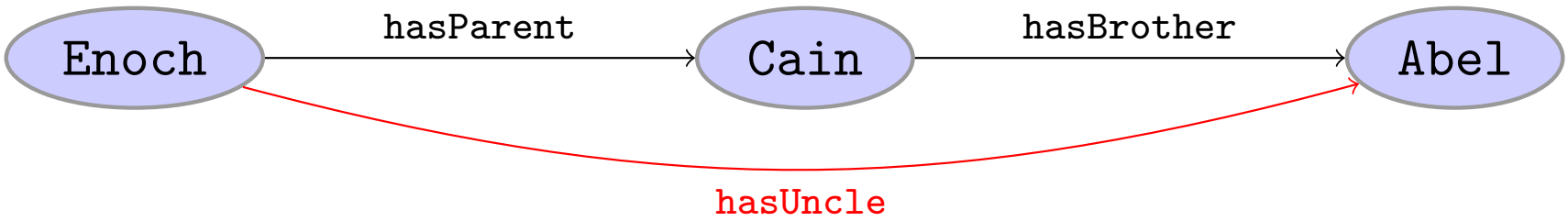

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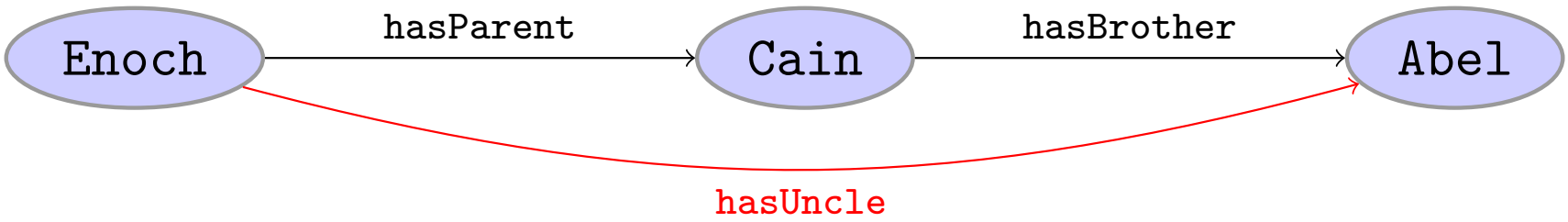

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- properties cannot be declared sub-properties of property chains.
  - (can become problematic for reasoning in some constellations)

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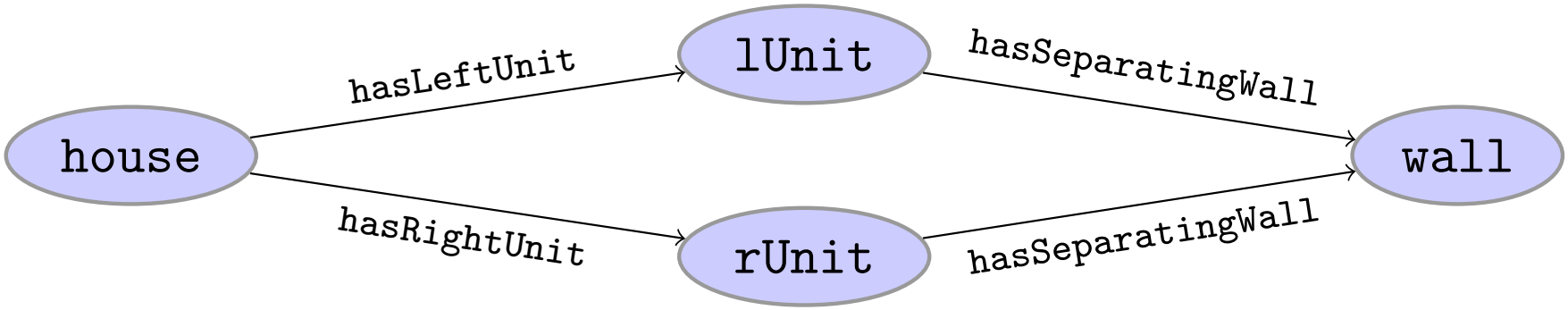

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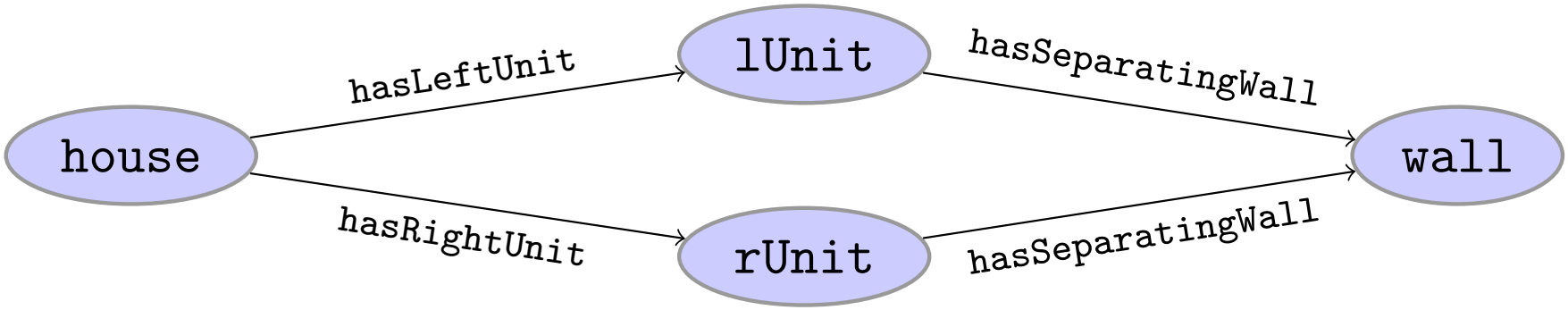

- Try...

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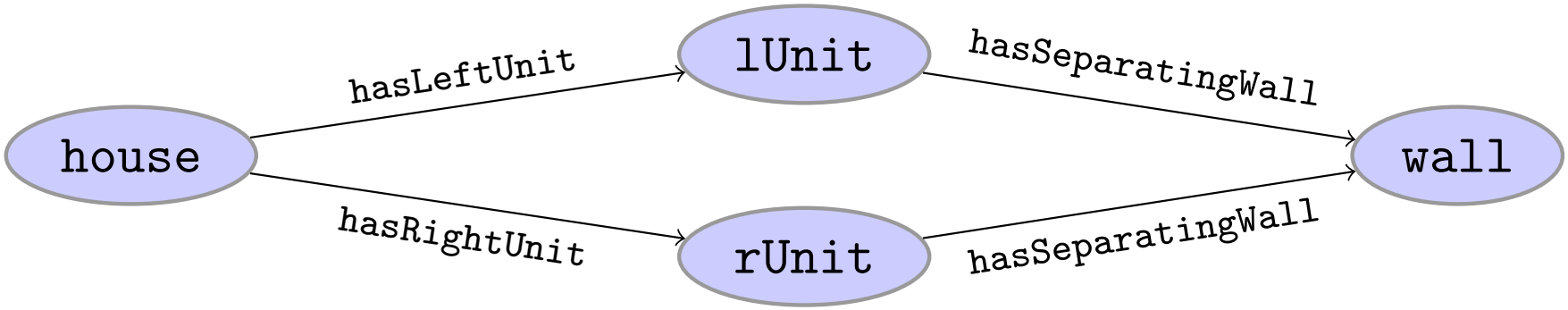

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$$SemiDetached \sqsubseteq \exists hasLeftUnit.Unit \sqcap \exists hasRightUnit.Unit$$
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- And now what?

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$$\mathit{TwinParent} \equiv \mathit{Person} \sqcap \geq_2 \mathit{hasChild}.\exists \mathit{hasBirthday}[\ldots]$$

Connecting Datatype Properties

- Given terms

$$\mathit{Person} \qquad \mathit{hasChild} \qquad \mathit{hasBirthday}$$

- A twin parent is defined to be a person who has two children with the same birthday.
- Try...

$$\begin{aligned}\mathit{TwinParent} \equiv \mathit{Person} \quad &\sqcap \ \exists \mathit{hasChild}.\exists \mathit{hasBirthday}[\ldots] \\ &\sqcap \ \exists \mathit{hasChild}.\exists \mathit{hasBirthday}[\ldots]\end{aligned}$$

- No way to connect the two birthdays to say that they're the same.
  - (and no way to say that the children are *not* the same)
- Try...

$$\mathit{TwinParent} \equiv \mathit{Person} \sqcap \geq_2 \mathit{hasChild}.\exists \mathit{hasBirthday}[\ldots]$$

- Still no way of connecting the birthdays!

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  - Outside the intended scope of Description Logics

After the Easter Holidays

- More (practical) details about SPARQL
- RDF on the Web: Linked Open Data and RDFa
- Exporting relational databases as RDF with D2R
- Guest lecture: commercial projects with RDF