# INF3580/4580 – Semantic Technologies – Spring 2017

## Lecture 9: Model Semantics & Reasoning

Martin Giese

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Department of Informatics

University of Oslo

# Today's Plan

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Properties of Entailment by Model Semantics
5. Entailment and Derivability

# Outline

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2. Literal Semantics
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- Forget blank nodes and literals for a while!

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| `indi rdf:type class .` | $C(i_1)$ |
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| `prop rdfs:subPropertyOf prop .` | $r \sqsubseteq s$ |
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- Used much in particular for OWL

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ws:romeo ws:loves ws:juliet .
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- DL syntax, without namespaces:

$$loves(romeo, juliet)$$
$$Lady(juliet)$$

$$Lady \sqsubseteq Person$$
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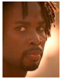
- Given these, it will be possible to say whether a triple holds or not.

# An example "intended" interpretation

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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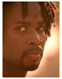
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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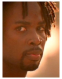
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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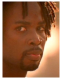
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- In fact, there is *no way* of ensuring they denote only what we think!

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- $\mathcal{A}$ is valid in $\mathcal{I}$, written

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- For a set of triples $\mathcal{A}$ (any of the six kinds)
- $\mathcal{A}$ is valid in $\mathcal{I}$, written

$$\mathcal{I} \models \mathcal{A}$$

- iff $\mathcal{I} \models A$ for all $A \in \mathcal{A}$.

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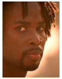
$\mathit{Lover}^{\mathcal{I}_1} = \Big\{$  ,  $\Big\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \Big\{$  ,  ,  $\Big\}$

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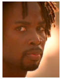
  $Lover^{\mathcal{I}_1} = \{$ ,  $\} \subseteq Person^{\mathcal{I}_1} = \{$ , ,  $\}$
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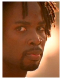
  $\mathit{Lover}^{\mathcal{I}_1} = \{$  ,  $\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \{$  ,  ,  $\}$
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  $\mathit{Lover}^{\mathcal{I}_2} = \mathbb{N}$ and $\mathit{Person}^{\mathcal{I}_2} = \{2, 4, 6, 8, 10, \ldots\}$

# Finding out stuff about Romeo and Juliet

Statements | Interpretations | The "Real World"

*loves*(*romeo*, *juliet*)
*Lady*(*juliet*)
*Lady* ⊑ *Person*
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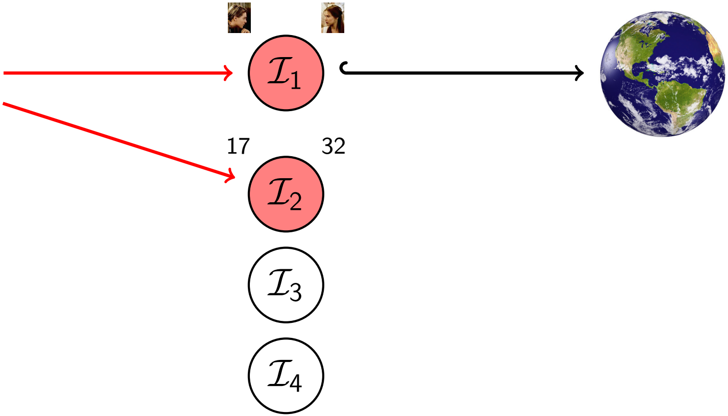

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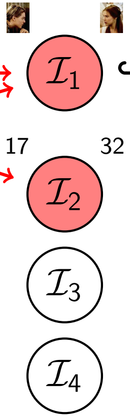


## The "Real World"

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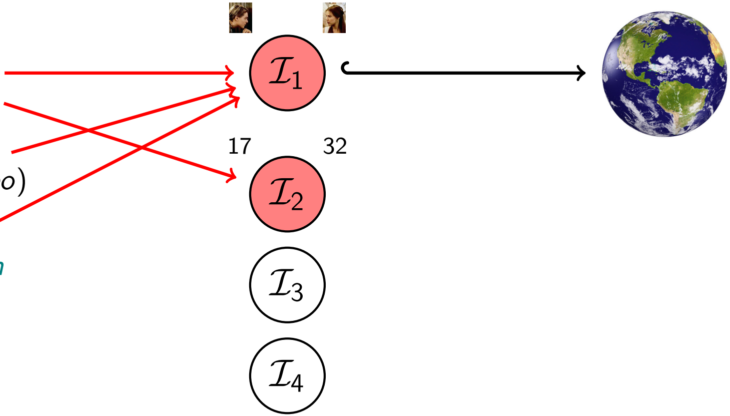

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- Countermodels for intuitively true statements are always unintuitive! (Why?)

# Countermodel Example

- $\mathcal{A}$ as before:

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- With $Lover^{\mathcal{I}} = \{a\}$ and $Beloved^{\mathcal{I}} = \{b\}$, $\mathcal{I} \not\models Lover \sqsubseteq Beloved$!

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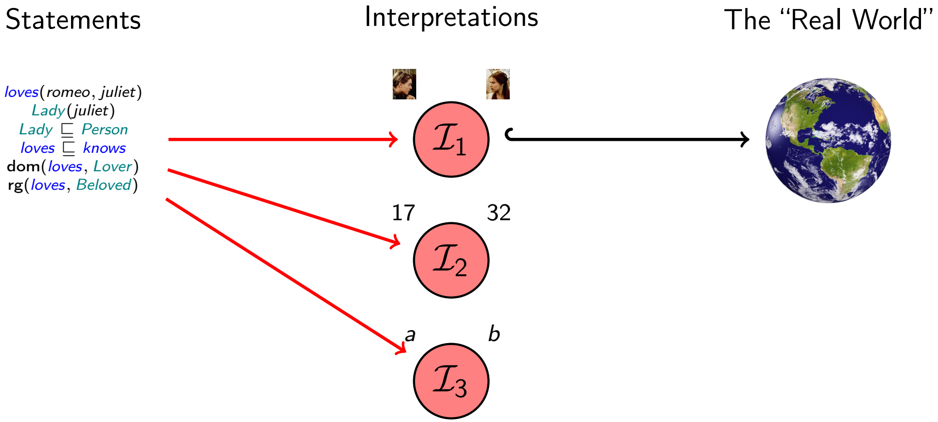
- With $Lover^{\mathcal{I}} = \{a\}$ and $Beloved^{\mathcal{I}} = \{b\}$, $\mathcal{I} \not\models Lover \sqsubseteq Beloved$!
- Choose

$$loves^{\mathcal{I}} = knows^{\mathcal{I}} = \{\langle a, b \rangle\} \qquad Lady^{\mathcal{I}} = Person^{\mathcal{I}} = \{b\}$$

to complete the count-model while satisfying $\mathcal{I} \models \mathcal{A}$

# Countermodels about Romeo and Juliet



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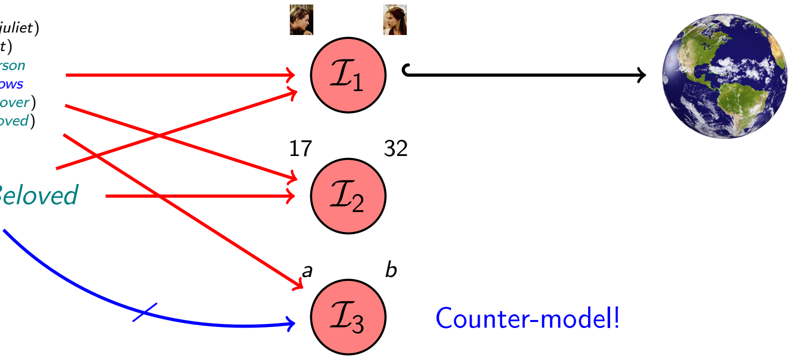

# Outline

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```
ex:me ex:likes dbpedia:Berlin .
```

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  - *Individuals* (all the rest, "usual" resources)
- Why? – simpler, object/datatype split is in OWL

# Allowed triples

Allow only triples using object properties and datatype properties as intended

| Triples | Abbreviation |
|---|---|
| `indi o-prop indi .` | $r(i_1, i_2)$ |
| `indi d-prop "lit" .` | $a(i, l)$ |
| `indi rdf:type class .` | $C(i_1)$ |
| `class rdfs:subClassOf class .` | $C \sqsubseteq D$ |
| `o-prop rdfs:subPropertyOf o-prop .` | $r \sqsubseteq s$ |
| `d-prop rdfs:subPropertyOf d-prop .` | $a \sqsubseteq b$ |
| `o-prop rdfs:domain class .` | $\mathrm{dom}(r, C)$ |
| `o-prop rdfs:range class .` | $\mathrm{rg}(r, C)$ |

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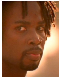
- Note: Literals $l$ are in $\Lambda$, don't need to be interpreted.

# Example: Interpretation with a Datatype Property

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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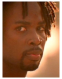
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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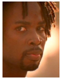
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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# Outline

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- Solution: pass in blank node interpretation, deal with sets later!

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- I.e. if there exists some valuation for the blank nodes that makes all triples true.

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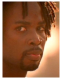
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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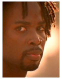
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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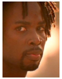
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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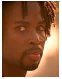
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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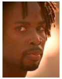
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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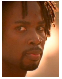
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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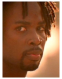
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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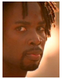
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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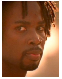
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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$$\models \{\mathit{loves}(b_2, b_1), \mathit{knows}(b_1, \mathit{romeo})\}$$

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  - In some models of $KB$, harald has a father, in others not.

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- (requires blank node IDs in $\mathcal{A}$ and $\mathcal{B}$ to be disjoint)

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- Won't bother to do that now. Will get completeness for OWL.

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