Problem set corresponding to Dino's 3rd lecture (Complexity)

1 (Prob. 14 in the Compendium)

Describe an algorithm for HAMILTONICITY. What is the complexity of your algorithm? (Important concept: Exponential-time algorithms.)

2. (Problem 15 in the Compendium)

Prove that HAMILTONICITY is in NP by showing a non-deterministic algorithm for HAMILTONICITY. What is the complexity of that algorithm? (Important concept: NP.)

3. (Problem 16 in the Compendium)

Prove that the problem of finding the shortest path in a graph is in P. (Important concept: Polynomial-time algorithms.)

4. (Problem 17 in the Compendium)

Prove that if the problem of finding the longest simple path in a graph can be solved in polynomial time, then so can HAMILTONICITY. (Important concept: Studying complexity of problems by using reductions.)

5. (Problem 18 in the Compendium

We do not know whether NON-HAMILTONICITY is in the class NP. What is the difficulty? (Important insight: What do problems that are beyond NP look like?)

6. (Problem 19 in the Compendium)

Prove that the TSP (Traveling Salesperson's Problem) optimization problem is at least as difficult as HAMILTONICITY in the following sense: If the TSP can be solved in polynomial time so can HAMILTNICITY. (Important concepts: Reduction. Representing optimization problems by decision problems.)