Chess Algorithms Theory and Practice

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Complexity of a Chess Game

- **20** possible start moves, **20** possible replies, etc.
- **400** possible positions after **2** ply (half moves)
- **197 281** positions after **4** ply
- 7^{13} positions after 10 ply (5 White moves and 5 Black moves)
- **Exponential explosion**!
- Approximately **40 legal moves** in a typical position
- There exists about $\mathbf{10^{120}}$ possible chess games

Solving Chess, is it a myth?

Chess Complexity Space

- The estimated number of possible chess games is 10^{120}
  - Claude E. Shannon
  - 1 followed by 120 zeroes!!!
- The estimated number of reachable chess positions is 10^{47}
  - Shirish Chinchalkar, 1996
- Modern GPU's performs 10^{13} flops
- If we assume one million GPUs with 10 flops per position we can calculate 10^{18} positions per second
- It will take us 1 600 000 000 000 000 000 000 years to solve chess

Assuming Moore's law works in the future

- Todays top supercomputers delivers 10^{16} flops
- Assuming 100 operations per position yields 10^{14} positions per second
- Doing retrograde analysis on supercomputers for 4 months we can calculate 10^{21} positions.
- When will Moore's law allow us to reach 10^{47} positions?
- Answer: in 128 years, or around year 2142!

http://chessgpgpu.blogspot.no/2013/06/solving-chess-facts-and-fiction.html

History of Computer Chess

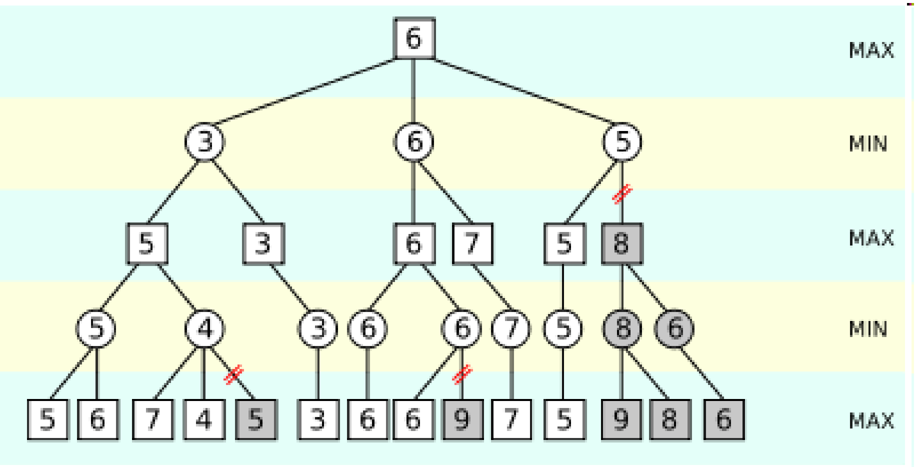
- Chess was a good fit for computers:
  - **Clearly defined rules**
  - Game of **complete information**
  - **Easy to evaluate (judge) positions**
  - **Search tree is not too small or too big**
- 1950: Programming a Computer for Playing Chess (Claude Shannon)
- 1951: First chess playing program (on paper) (Alan Turing)
- 1958: First computer program that can play a complete chess game
- 1981: Cray Blitz wins a tournament in Mississippi and achieves master rating
- 1989: Deep Thought loses 0-2 against World Champion Garry Kasparov
- 1996: Deep Blue wins a game against Kasparov, but loses match 2-4
- 1997: Upgraded Dee Blue wins 3.5-2.5 against Kasparov
- 2005: Hydra destroys GM Michael Adams 5.5-0.5
- 2006: World Champion Vladimir Kramnik looses 2-4 against Deep Fritz (PC chess engine)
- 2014: Magnus Carlsen launches “Play Magnus “ app on iOS where anyone can play against a chess engine that emulates the World Champion’s play at 21 different ages (5 to 25 years).

Chess Compared to Go

- Go is played on a 19x19 square board where a new stone is placed on any free square each move (and never moved around)
- Go has a much higher branching factor (starting with 361 and slowly descending) and much more complicated leaf node evaluation
- For many years the best Go programs had amateur rating only
- In 2016 Alpha Go surprisingly beat Lee Sedol (9-dan profession) 4-1 using a combination of machine learning (deep neural network) and Monte Carlo tree search algorithm.
- Alpha Go beat Ke Jie (ranked no. 1 in the world) 3-0 in 2017 and retired afterwards.

Search Trees and Position Evaluation

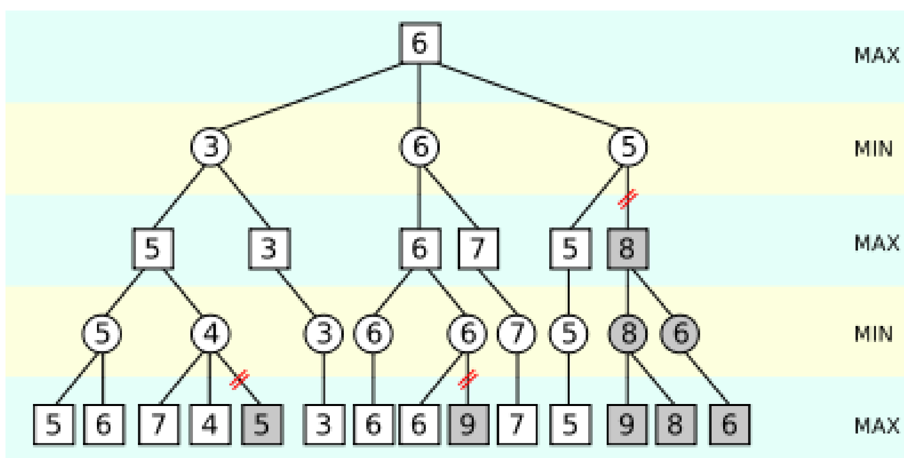
- Search trees (nodes are positions, edges are legal chess moves)
- Leaf nodes are end positions which needs to be evaluated (judged)
- A simple judger: Check mate? If not, count material
- Nodes are marked with a numeric evaluation value



Minimax: The Basic Search Algorithm

- Minimax: Assume that both White and Black plays the best moves. We maximizes White's score
- Perform a **depth-first search** and **evaluate** the **leaf nodes**
- Choose child node with **highest value** if it is **White** to move
- Choose child node with **lowest value** if it is **Black** to move
- **Branching factor** is **40** in a typical chess position

White
Black
White
Black
White



ply = 0
ply = 1
ply = 2
ply = 3
ply = 4

NegaMax – "Simplified" Minimax

Minimax

```
int maxi( int depth ) {
  if ( depth == 0 )
    return evaluate();
  int max = -∞;
  for ( all moves) {
    score = mini( depth - 1 );
    if( score > max )
      max = score;
  }
  return max;
}
```

```
int mini( int depth ) {
  if ( depth == 0 )
    return -evaluate();
  int min = + ∞;
  for ( all moves) {
    score = maxi( depth - 1 );
    if( score < min )
      min = score;
  }
  return min;
}
```

NegaMax

max(a, b) == -min(-a, -b)

```
int negaMax( int depth ) {
  if ( depth == 0 ) return evaluate();
  int max = - ∞;
  for ( all moves)  {
    score = -negaMax( depth - 1 );
    if( score > max )
      max = score;
  }
  return max;
}
```

Node explosion

A typical middle-game position has 40 legal moves.

Depth	Node count	Time at 10M nodes/sec
1	40	0.000004 s
2	1 600	0.00016 s
3	64 000	0.0064 s
4	2 560 000	0.256 s
5	102 400 000	10.24 s
6	4 096 000 000	6 min 49,6 s
7	163 840 000 000	4 h 33 min 4 s
8	6 553 600 000 000	7 d 14 h 2 min 40 s

- 10 M nodes per second (nps) is realistic for modern chess engines
- Modern engines routinely reach depths 25-35 ply at tournament play
- But they only have a few minutes per move, so they should only be able to go 5-6 ply deep
- How do they then get to depth 25 so easily?

Pruning Techniques

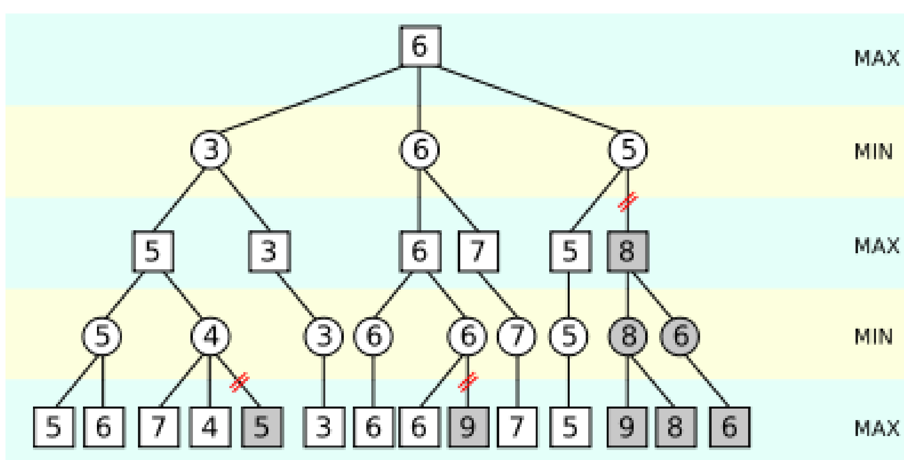
- The complexity of searching *d* ply ahead is O(b*b*…*b) = $O(b^d)$
- With a branching factor (*b*) of 40 it is crucial to be able to prune the search tree

Alpha-Beta Pruning

“Position is so good for White (or Black) that the opponent with best play will not enter the variation that gives the position.”

- Use previous known max and min values to limit the search tree
- Alpha value: White is guaranteed this score or better (start value: -∞)
- Beta value: Black is guaranteed this score or less (start value: +∞)
- If Alpha is higher than Beta, then the position will never occur assuming best play
- If search tree below is evaluated left to right, then we can skip the greyed-out sub trees
- Regardless of what values we get for the grey nodes, they will not influence the root node score

White | ply = 0
Black | ply = 1
White | ply = 2
Black | ply = 3
White | ply = 4



Analyze the Best Move First

- Even with alpha-beta pruning, if we always start with the worst move, we still get O(b*b*..*b) = $O(b^d)$
- If we always start with the best move (also recursive) it can be shown that complexity is O(b*1*b*1*b*1…) = $O(b^{d/2}) = O(\sqrt{b^d})$
- We can **double** the **search depth** without using more resources
- Conclusion: It is very important to try to **start** with the **strongest moves first**

Killer-Move Heuristics

- Killer-move heuristics is based on the assumption that a **strong move** which gave a **large pruning** of a sub tree, might also be a strong move in **other nodes** in the search tree
- Therefore we start with the killer moves in order to maximize search tree pruning

Zero-Move Heuristics

- Alpha-Beta cutoff: “The position is so good for White (or Black) that the opponent with best play will avoid the variation resulting in that position”
- Zero-Move heuristics is based on the fact that in most positions it is an **advantage** to be the **first player to move**
- Let the player (e.g. White) who has just made a move, play another move (**two moves in a row**), and perform a shallower (2-3 ply less) and therefore cheaper search from that position
- If the shallower search gives a cutoff value (e.g. bad score for White), it means that most likely the search tree can be **pruned** at this position without performing a deeper search, since **two moves in a row did not help**
- Very effective pruning technique!
- Cavecats: Check and endgames (where a player can be in “trekktvang” – every move worsens the position)

Iterative Deeper Depth-First Search (IDDFS)

- Since it is so important to evaluate the best move first, it might be worthwhile to execute a **shallower search** first and then use the resulting **alpha/beta cutoff values** as **start values** for a **deeper search**
- Since the **majority** of search **nodes** are on the **lowest level** in a balanced search tree, it is relatively cheap to do an extra shallower search

Search Tree Extensions

- PC programs today can compute **25-35 ply ahead** (Deep Blue computed 12 ply against Kasparov in 1997, Hydra (64 nodes with FPGAs) computed at least 18 ply)
- It is important to **extend** the search in leaf nodes that are "**unstable**"
- Good **search extensions** includes all moves that gives **check** or **captures** a piece
- The longest search extensions are typically **double** the average length of the search tree!

Transposition Table

- **Same position** will commonly occur from **different move orders**
- All chess engines therefore has a **transposition table** (position cache)
- Implemented using a **hash table** with chess position as key
- Doesn’t have to evaluate large sub trees over and over again
- Chess engines typically uses half of available memory to hash table – proves how important it is

Other challenges

- Move generator (hardware / software)
  - Hydra (64 nodes Xeon cluster, FPGA chips) computed 200 millions positions per second, approximately the same as Deep Blue (on older ASIC chip sets)
  - Hydra computed 18+ ply ahead while Deep Blue only managed 12 (Hydra prunes search tree better)
  - Komodo 10 chess engine calculates 3-4 mill moves/second on my Surface Book (Intel i7 @ 2.6 GHz with 3 cores) and computes 20+ ply in less than 5 seconds and 25+ ply in less than 30 seconds
- Efficient data structure for a chess board (0x88, bitboards)
- Opening library suited for a chess computer
- Position evaluation:
    - Traditionally chess computers has done **deep searches** with a **simple evaluation function**
    - But one of the best PC chess engines today, Rybka, sacrifices search depth for a **complex position evaluation** and better search heuristics

Endgame Tablebases

- Chess engines plays endgames with 3-7 pieces left on the board perfectly by **looking up best move in huge tables**
- These endgame databases are called **Tablebases**
- Retrograde analyses: Tablebases are generated by starting with **final positions** (check mate, steal mate or insufficient mating material (e.g. king vs. king)) and then **compute backwards** until all nodes in search tree are marked as win, draw or loose
- Using complex **compression** algorithms (Nalimov, Syzygy)
- The newer Syzygy compression format uses less than 200 GB for all endgames with up to 6 piezes (compared to over 1 TB for Nalimov tablebases)

Lomonosov Tablebases

- All 7 piece endgames (except 6 pieces vs a lone king) calculated for the first time in 2013 on the Lomonosov supercomputer in Moscow State University.
- Took 6 months to generate
- Needed 140 TB of storage
- Longest forced mate:
  White to mate in 545 moves!

- See http://chessok.com/?page_id=27966, http://tb7.chessok.com/

Demo

- Demo: ChessBase with chess engine Komodo 10 and Stockfish 7
- Best open source UCI chess engine (and may be best overall):
  - Stockfish (stockfishchess.org)

Thank you

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