UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam:	**INF 4300 / INF 9305 – Digital image analysis**
Date:	**Friday December 4, 2015**
Exam hours:	**09.00-13.00 (4 hours)**
Number of pages:	**6 pages of exercises**
Enclosures:	**1 page**
Allowed aid:	**Calculator**

- Read the entire exercise text before you start solving the exercises.
  Please check that the exam paper is complete.
  If you lack information in the exam text or think that some information is missing, you may make your own assumptions, as long as they are not contradictory to the "spirit" of the exercise.
  In such a case, you should make it clear what assumptions you have made.

- You should spend your time in such a manner that you get to answer all exercises shortly. If you get stuck on one question, move on to the next question.

- Your answers should be **short**, typically a few sentences and / or a sketch should be sufficient.

Good luck!!

Exercise 1: Representation of binary objects

One of the methods of representing the contour of binary objects is to traverse the boundary pixels clockwise and generate a chain of integer codes from 0 (=right) through 2 (=forward) to 7.

a) Given two chain codes 77553311 and 00664422, draw a simple sketch of the two objects.

b) Using Kulpas's 1977 expression for the length of the perimeter:

$$P_K = \frac{\pi}{8}\left(1+\sqrt{2}\right)\left(n_E + \sqrt{2}n_O\right)$$

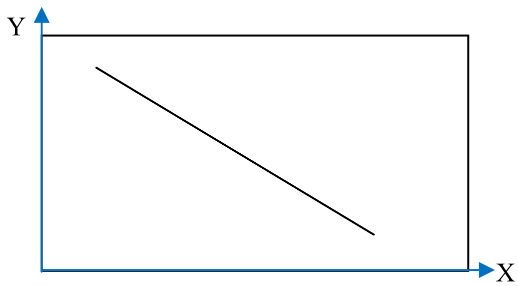
where n_E is the number of even chain elements and n_O the number of odd chain elements, what is the ratio of the lengths of the two perimeters?

c) Given two chains codes 77444411 and 00005533, how can we check whether these two codes represent the same shape?

Exercise 2: Hough transform of lines

The Hough transform is often used to detect and find the parameters of line segments in gradient images.

a) Copy the sketch below of a line segment in a 2D image. Explain the "normal representation" of a straight line in the Hough transform, and indicate the parameters involved.



b) Another representation of straight line segments in a 2D image is $Y = aX + b$. In the 8x8 pixel binary image below, several sets of co-linear pixels are present. Which of these sets can be correctly detected by an (a,b)-accumulator matrix having integer indexes along both the a and b axis if the accumulator threshold is $T \geq 4$? Please explain your reasoning without any computations!

0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0
0	0	1	0	1	0	0	0
0	0	0	1	0	0	0	0
0	1	1	0	0	0	1	0
0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0

c) Above we have seen the Hough transform as point-to-line mappings (PTLM). How do we get from this to point-to-point mappings (PTPM)?

For the PhD-students only :

d) The Hough transform has several weaknesses. Assuming an image of size *h x w*, Wallace introduced an alternative "Muff transform" 30 years ago:

As the basis for this parametrization, a bounding rectangle around the image is used. Each line in the image intersects this bounding box / perimeter at exactly two points. The distance of the first intersection (i.e. the nearest intersection from the origin along the perimeter) on the perimeter from the origin, and the distance between the first and second intersections are then used as the Muff parameters.

What is the size and resolution of this accumulator space,
and which weakness(es) does this transform solve?

Exercise 3: Classification

a) Consider a two-dimensional feature vector and a set of points in 2D feature space:
(-3,6) (-2,4) (-1, 2) (0,0) (1, -2) (2, -4) (3, -6)
Show that the covariance matrix between the two features is :

$$\Sigma = \begin{bmatrix} 4 & -8 \\ -8 & 16 \end{bmatrix}$$

Show all your calculations.

b) Given the two features defined above, would you base you classification on 1 or 2 features? Justify your answer.

c) The discriminant functions for a multivariate Gaussian classifier are given as:

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T \Sigma_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{n}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

Consider two classes with equal prior probability and

$$\boldsymbol{\mu}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \boldsymbol{\mu}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Sigma_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \quad \Sigma_2^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad |\Sigma_1| = 1 \quad |\Sigma_2| = 1$$

Can the discriminant function be simplified in this case?

d) Classify the point $x = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ by computing the value of the discriminant functions and assign it to the class corresponding to the highest probability.

e) Explain how classifier sensitivity and specitivity are computed, and discuss their importance for a medical classification problem.

For the PhD-students only:

f) Let us assume that we have a 2-class classification problem with a 1-dimensional feature vector f(x) which is exponentially distributed given the class-conditional parameter λ_i:

$$f(x) = \lambda_i \exp^{-\lambda_i x}$$

Find an expression for the decision boundary for this classification problem.

Exercise 4: Clustering

a) Describe how the K-means clustering algorithm works and which parameters it has.

b) K-means clustering with K=2 is done on the data points given in the scatter plot below. Three different strategies for initializing the clustering is to assign the initial cluster centers to
   a. minimum/maximum among the points in the data set
   b. the first K points
   c. K random data points.

In the table below the resulting cluster centers after each iteration in K- means clustering are given for two of these methods.

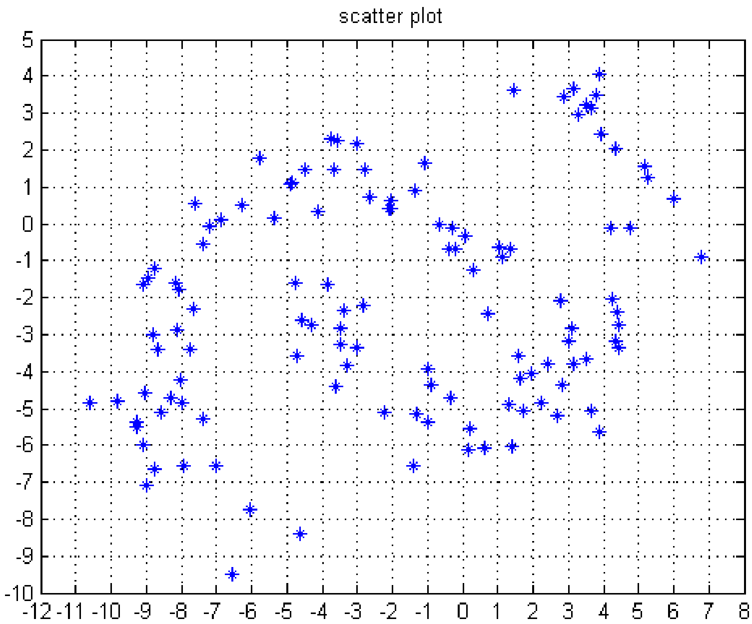
Discuss which of these two methods you would choose for this particular data set, and how different the clustering result would be.

Initialization	Cluster 1 mean	Cluster 2 mean
Iteration 0	(-10.6, -4.8)	(6.8, -0.9)
Iteration 1	(-6.4, -2.9)	(1.8, -1.4)
Iteration 2	(-6.4, -2.9)	(1.7, -1.5)
Iteration 3	(-6.4, -2.9)	(1.7, -1.5)
Iteration 4	(-6.4, -2.9)	(1.7, -1.5)
Iteration 5	(-6.4, -2.9)	(1.7, -1.5)
Iteration 6	(-6.4, -2.9)	(1.7, -1.5)

Table 1 Clustering, initialization using minmax points

Initialization	Cluster 1 mean	Cluster 2 mean
Iteration 0	(1.6, -3.6)	(-1.3, -5.1)
Iteration 1	(2.2, -0.9)	(-5.6, -3.1)
Iteration 2	(2.1, -1.5)	(-6.0, -2.8)
Iteration 3	(2.0, -1.6)	(-6.2, -2.7)
Iteration 4	(1.9, -1.7)	(-6.3, -2.6)
Iteration 5	(1.9, -1.7)	(-6.3, -2.6)
Iteration 6	(1.9, -1.7)	(-6.4, -2.6)

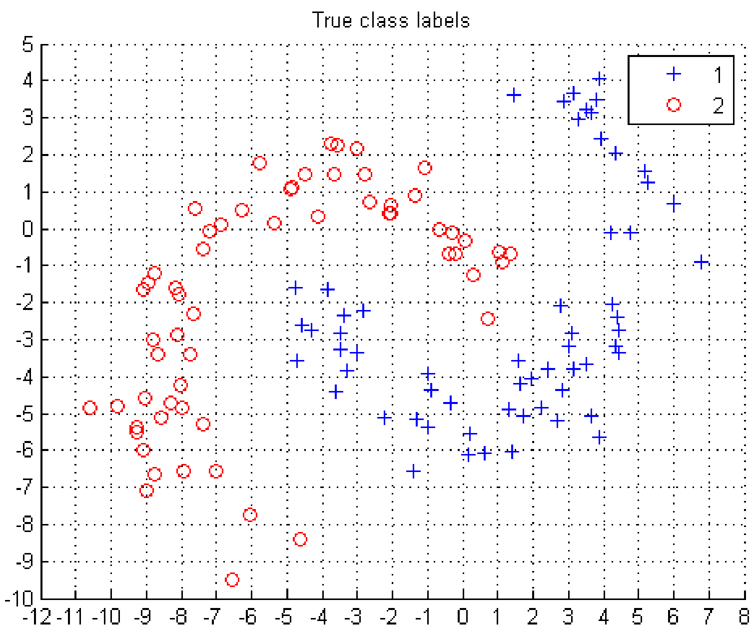
Table 2 - Cluster means after initialization by the first points in the data set



c) Select one of the initializations and indicate the decision boundaries the clustering would result in on the scatter plot in enclosure 1.

d) This data originates from a classification data set with known class labels. A scatter plot with class labels is given in the figure below.

Plot your estimated decision boundary from the clustering on the labelled scatter plot. Use the scatter plot to compute the confusion matrix for the clustering result.



e) Discuss if a Gaussian classifier with full class-conditional covariance matrix would perform well on this data set given the known class labels.

Exercise 5: Mathematical morphology on binary images

a) Explain what erosion and dilation do to a binary image.

b) Perform an erosion-based edge detection on the binary image given below, using a 3x3 plus-shaped structuring element that is symmetric around its origin.

0	0	0	**1**	**1**	0	**1**	**1**	0	0	0
0	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0
0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0
1	**1**	**1**	**1**	0	**1**	**1**	**1**	**1**	**1**	**1**
0	**1**	**1**	**1**	**1**	0	**1**	**1**	**1**	**1**	**1**
0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0
0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	0	0
0	0	0	0	0	**1**	0	**1**	0	0	0

c) Give the expression for the Bottom Hat operation, and apply this to the image given below. Illustrate the result of each step of the operation and describe the effect of the operators.

0	0	0	0	0	0	0	0	0	0	0
0	**1**	0	0	**1**	0	0	**1**	**1**	0	0
0	0	**1**	0	**1**	0	**1**	**1**	**1**	**1**	0
0	0	**1**	**1**	**1**	0	**1**	**1**	0	0	0
0	0	**1**	**1**	**1**	0	0	**1**	**1**	0	0
0	0	0	**1**	**1**	0	0	0	**1**	0	0
0	**1**	**1**	**1**	0	0	0	0	0	**1**	0
0	0	0	0	0	0	0	0	0	0	0

Good Luck!

Candidate number:

Enclosure, Exam INF4300/9305, December 4, 2015

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