INF 4300 2016
Classification 2
Exercises related to the lecture on 10.10.16

You will need your souce code for Exercise 1 when doing Mandatory Exercise part 2, so implement this now ☺

Exercise 1. Matlab exercise for classification based on a multivariate Gaussian classifier.

Step 1: Implement a Gaussian classifier using a d-dimensional feature vector
For the algorithm, see lecture foils. It is recommended that you use Matlab built-in functions for matrix inversion and computing the determinant, but write the remaining algorithm yourself. (You can use built-in functions mean() og cov())

Step 2: Train the classifier
Train the classifier on the image tm_train.png (found at https://www.uio.no/studier/emner/matnat/ifi/INF4300/h16/undervisningsmateriale/week6-classification1/week7/tm_train.png) using the 4 classes defined in the mask file tm_train.png. Estimates of the class-specific mean vector and covariance matrix are found on the lecture foils. You may use Matlab functions mean() and cov() here.

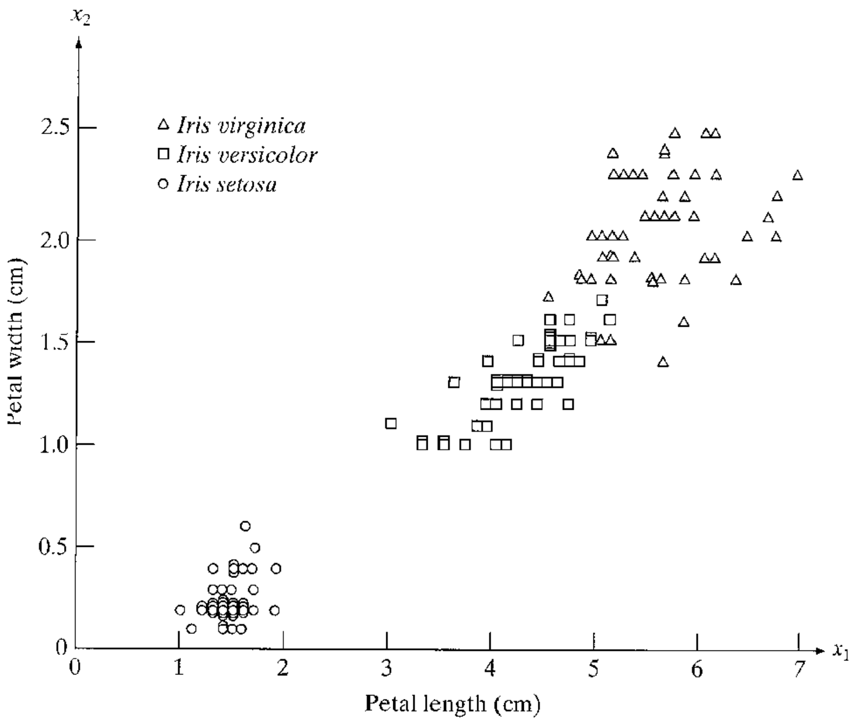
If you want to verify that your code gives the correct classification labels, check the resulting classification image you get when you classify the entire image tm.mat with the classification we produced (tm_classres.mat). In this image each pixel is assigned class labels 1-4.

Step 3: Find the classification accuracy for classification using all features.
Run the classification on the multivariate (6-band) input image.
Compute the percentage of correctly classified pixels when using all features, and compare it to using single features. The classification accuracy should be computed on the test image tm_test.png.

Also try the simplified covariance matrix $\Sigma=\sigma^2 I$. Which version gives the highest classification accuracy?

2. Finding the decision functions for a minimum distance classifier.

A classifier that uses diagonal covariance matrices is often called a minimum distance classifier, because a pattern is classified to class that is closest when distance is computed using Euclidean distance. See plot on next page.



a. In the above figure, find the class means just by looking at the plot.

b. If this data is classified using a minimum distance classifier, sketch the decision boundaries on the plot.

3. Discriminant functions

A classifier that uses Euclidean distance computes distance from pattern ***x*** to class *j* as:

$$D_j(x) = \|x - \mu_j\|$$

Show that classification with this rule is equivalent to using the discriminant function

$$d_j(x) = x^T \mu_j - \frac{1}{2}\mu_j^T \mu_j$$

4. Exercise 3

In a three-class two-dimensional problem the feature vectors in each class are normally distributed with covariance matrix

$$\Sigma = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix}$$

The mean vectors for the three classes are $[0.1, 0.1]^T$, $[2.1, 1.9]^T$, $[-1.5, 2.0]_T$.

Assuming that the classes are equally probable:

a. Classify the feature vector $[1.6, 1.5]^T$ according to a Bayesian classifier with the given covariance matrix.
b. Draw the curves of equal Mahalanobis distance from the class with mean $[2.1, 1.9]^T$.

5. Exercise 4

Given a two-class classification problem:

- $P(\omega 1) = P(\omega 2)$
- $p(x|(\omega 1) = N(\mu 1, \Sigma)$, $p(x|(\omega 2) = N(\mu 2, \Sigma)$
- $\mu 1 = [0\ 0]$; $\mu 2 = [3\ 3]$; $\Sigma = \begin{bmatrix} 1.1 & 0.3 \\ 0.3 & 1.9 \end{bmatrix}, \Sigma^{-1} = \begin{bmatrix} 0.95 & -0.15 \\ -0.15 & 0.55 \end{bmatrix}$

Classify feature vector x=[1.0 2.2] using Bayesian classification.