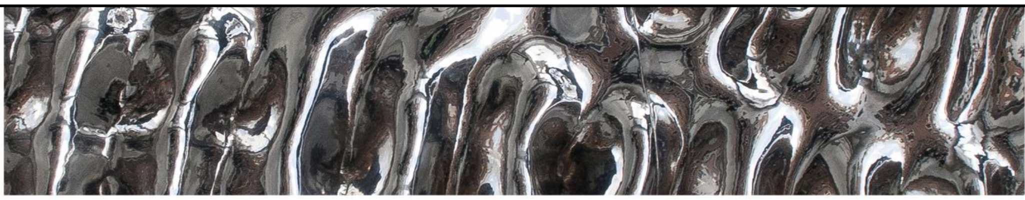

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**INF4420**

# Noise and Distortion

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## Outline

- Noise basics
- Component and system noise
- Distortion

## Introduction

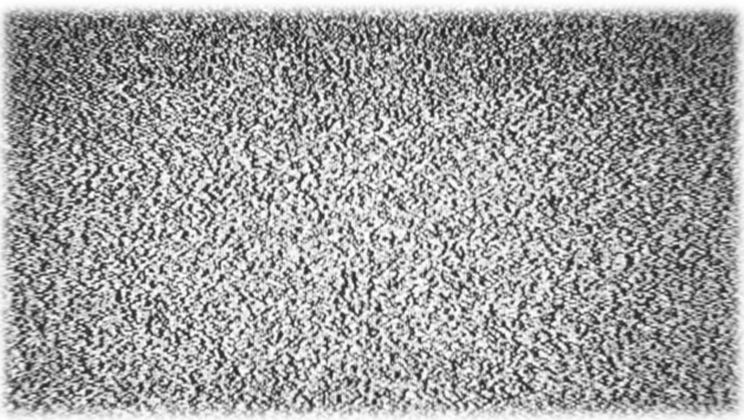
We have already considered one type of noise in the layout lecture: **Interference**—Other circuits or other parts of the circuit interfering with the signal. E.g. digital switching coupling through to an analog signal through the substrate or capacitive coupling between metal lines. We mitigate this interference with layout techniques.

True noise is random and zero mean!



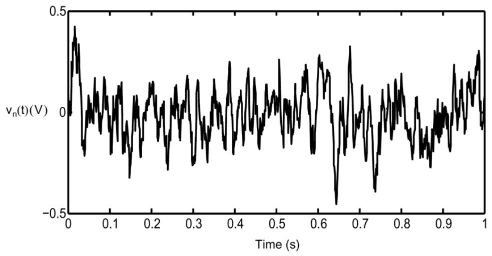


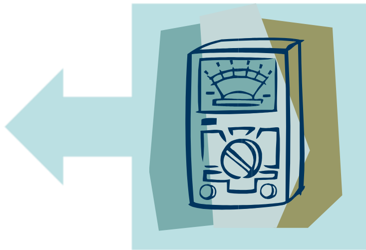
## Introduction



TV Static Stock Video by REC Room

# Introduction



We do not know the absolute value, but …

- We know the average value
- We know the statistical and spectral properties of the noise



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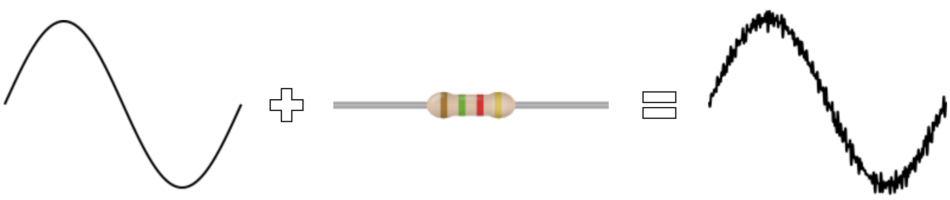
# Introduction



- Noise (power) is always compared to signal (power)
- How clearly can we distinguish the signal from the noise (SNR)

## Introduction

- Why is noise important?
- Signal headroom is reduced when the supply voltage is reduced.
- Supply voltage is reduced because of scaling (beneficial for digital, less power consumption)
- Noise is constant.
- Noise limits performance of our circuits (along with mismatch, etc.)





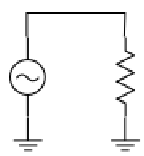
## Signal power

- Sine wave driving a resistor dissipates power
- The average voltage of the sine wave is zero
- The average power dissipated is *not* zero

$$\overline{P} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_A^2 \sin^2 x}{R}\, dx$$

Mean square value, RMS is $\frac{V_A}{\sqrt{2}}$

$$\overline{v^2} = \frac{1}{2\pi}\int_0^{2\pi} V_A^2 \sin^2 x\, dx = \boxed{\frac{V_A^2}{2}}$$



# White noise

White refers to the spectrum, constant PSD

$$V_n(f) = \text{constant}$$

Resistors exhibit white noise only (ideally). Random thermal motion of electrons (thermal noise)

$$V_R^2(f) = 4kTR \Leftrightarrow I_R^2 = \frac{4kT}{R}$$

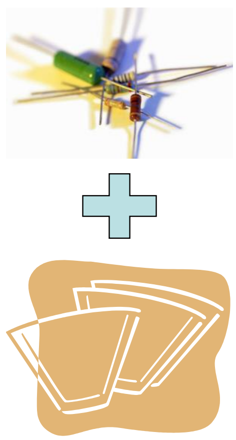
$1\ \text{k}\Omega$ resistor $\approx 4 \frac{\text{nV}}{\sqrt{\text{Hz}}}$ @ RT (useful to remember)



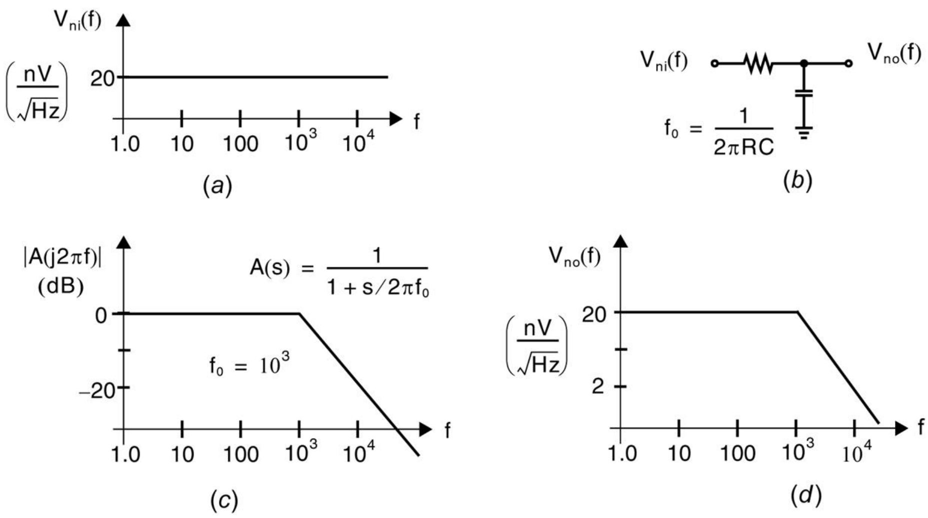


# Filtering noise

- In most circuits we have capacitors (noise free)
- There is always some parasitic capacitance present.
- White noise is filtered: the constant PSD is shaped.



## Filtering noise








## Total noise

We find the total noise by integrating from 0 to ∞.
This is a finite value because of the filtering.

$$\int_0^\infty \underbrace{\left|\frac{1}{1+j2\pi fRC}\right|^2}_{\text{1st order lowpass filter magnitude response}} \underbrace{4kTR}_{\text{Resistor noise}}\, df = \boxed{\frac{kT}{C}}$$

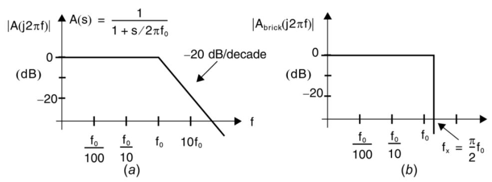
Important!

Total noise is independent of *R* and defined by *C*!

## Noise bandwidth

Find a frequency, such that when we integrate the ideal constant PSD up to this frequency, the total noise is identical. This is the equivalent noise BW.

For 1st order filtered noise: $\frac{\pi}{2} f_c = \frac{\pi}{2} \frac{1}{2\pi RC} = \frac{1}{4RC}$










## Adding noise sources

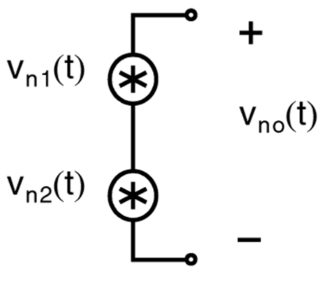
If the noise sources are uncorrelated (common assumption):

$$V_{no}^2 = V_{n1}^2 + V_{n2}^2$$

Generally:

$$V_{no}^2 = V_{n1}^2 + V_{n2}^2 + 2CV_{n1}V_{n2}$$

$C$ is the correlation coefficient.




(a)

## Signal to noise ratio

$$\mathrm{SNR} \equiv 10 \log_{10} \frac{\text{signal power}}{\text{noise power}} \text{ [dB]}$$

Example:

Best case signal swing, $V_A = \frac{V_{DD}}{2}$, noise is $\frac{kT}{C}$.

$$\mathrm{SNR} = \frac{C V_{DD}^2}{8kT}$$

If $V_{DD}$ is reduced, $C$ must increase: More power (-)



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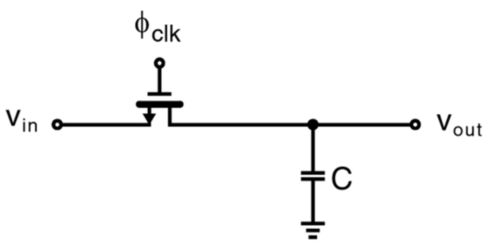
## Sampled noise

When sampling signals, the sampled spectrum only represents frequencies up to the Nyquist frequency (half the sampling frequency). Frequencies beyond are aliased down to this range. (More on this in a later lecture).

The total noise is still the same, $\frac{kT}{C}$.

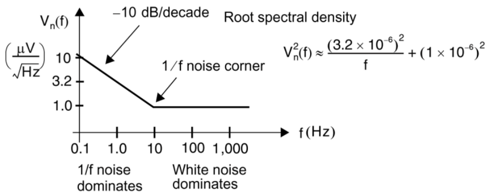


# 1/f noise

So far, we have considered white noise (with filtering).

Transistors exhibit significant flicker (1/f) noise. The PSD is proportional to the inverse of the frequency.

$$V_n^2(f) = \frac{\text{constant}}{f}$$



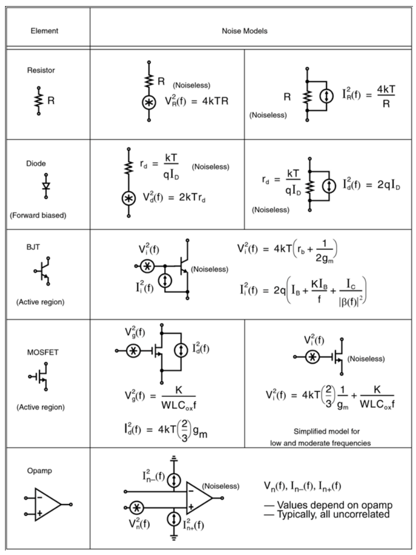







# Components

- Resistors exhibit white noise where voltage noise is proportional to resistance.
- Diodes exhibit white noise (shot noise) where noise current is proportional to diode current.



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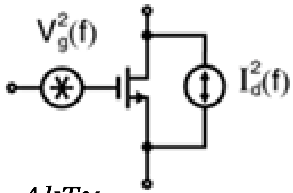
# MOSFET noise

**White noise** (channel resistance)




Triode: $I_d^2(f) = \frac{4kT}{r_{ds}}$

γ is process dependent, 2/3 for long-channel

Active: $I_d^2(f) = 4kT\gamma g_m$ or $V_g^2(f) = \frac{4kT\gamma}{g_m}$

**Flicker noise** (different models are used, we use the one from the textbook)

$$V_g^2(f) = \frac{K}{WLC_{ox}f}$$

K is process dependent

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# MOSFET noise

- Total MOSFET noise is white noise + flicker noise
- Higher $g_m$ attenuates the white noise contribution
- Larger devices (gate area) reduces the flicker noise
- We can reduce flicker noise using circuit techniques. E.g. in sampled analog systems we can sample the noise and subtract (correlated double sampling).

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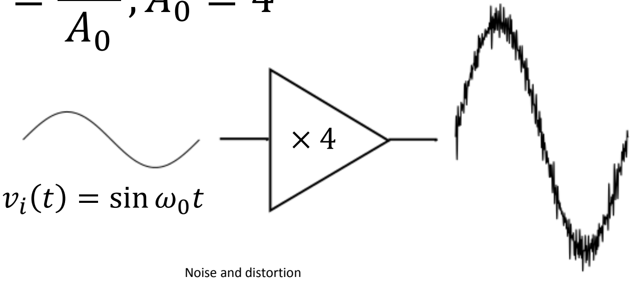
# Input referred noise

Amplifier not only amplifies the signal, but adds noise. Even though the amplifier noise is not physically present at the input, we can calculate an equivalent input noise, $v_{in}$.

$$v_{in} = \frac{v_{on}}{A_0}, A_0 = 4$$

$$v_o(t) = 4 \times v_i(t) + v_{on}(t)$$

Amplifier noise




$$v_i(t) = \sin \omega_0 t$$



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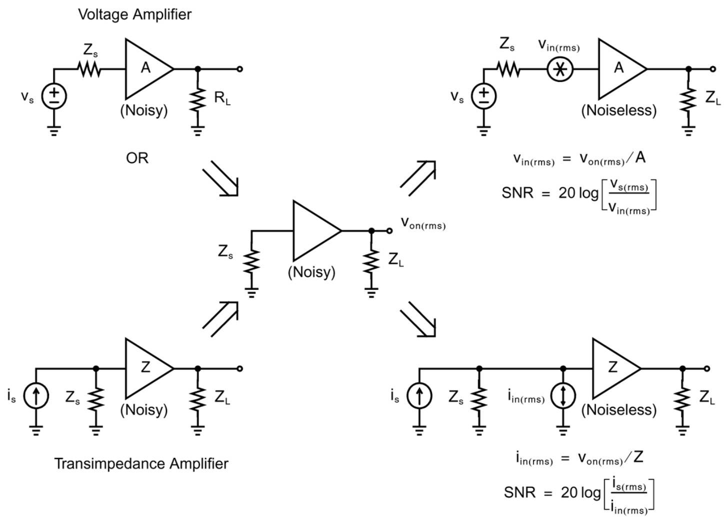
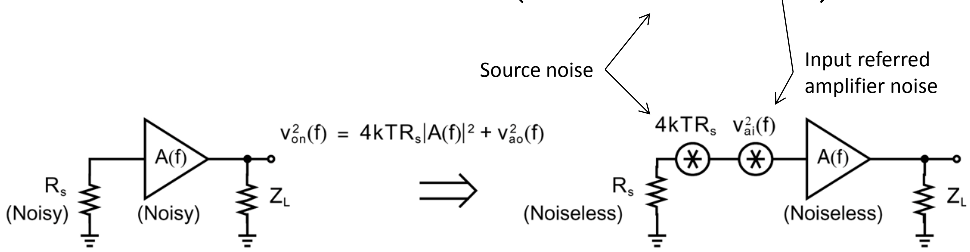
# Input referred noise



# Noise figure (NF)

A "figure of merit" for how much noise the amplifier adds, compared to the source.

$$\mathrm{NF}(f) = 10\log_{10}\left(\frac{4kTR_s + v_{ai}^2(f)}{4kTR_s}\right)$$



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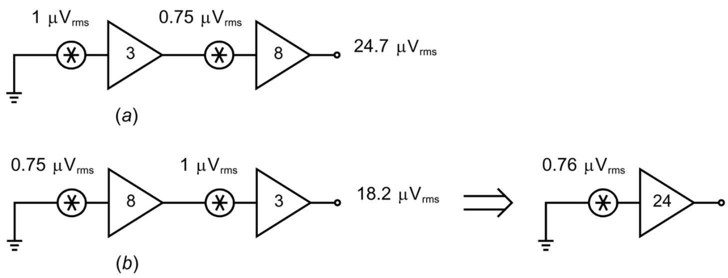
# Input referred noise

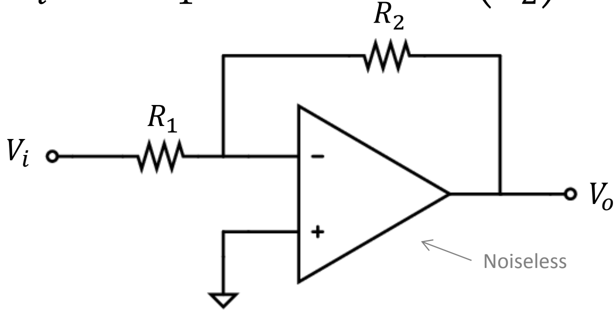
The amplifier may be a cascade of gain stages. High gain at the input stage attenuates noise contributed by later stages (important).



# Amplifier system example

Resistive feedback and noiseless opamp

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}, \quad V_{ni}^2 = V_{n1}^2 + \left(\frac{R_1}{R_2}\right)^2 V_{n2}^2$$



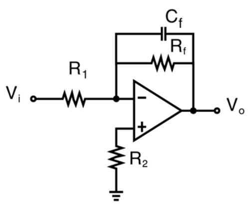




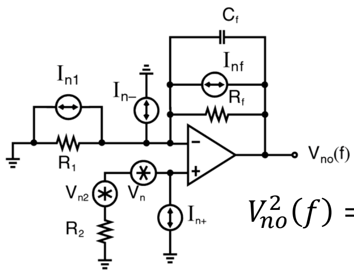

# Amplifier system example

$$V_{no1}^2(f) = \left(I_{n1}^2(f) + I_{nf}^2(f) + I_{n-}^2(f)\right) \left|\frac{R_f}{1 + j2\pi f R_f C_f}\right|^2$$

$$V_{no2}^2(f) = \left(I_{n+}^2(f) R_2^2 + V_{n2}^2(f) + V_n^2(f)\right) \left|1 + \frac{R_f R_1^{-1}}{1 + j2\pi f R_f C_f}\right|^2$$




(a)

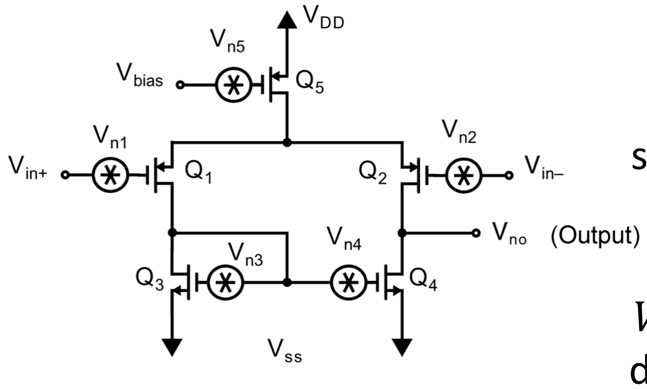



(b)

$$V_{no}^2(f) = V_{no1}^2(f) + V_{no2}^2(f)$$

## Differential pair example

Input stage → most significant noise




Several noise sources, we want the total input referred noise, $V_{ni}^2(f)$, assuming device symmetry.





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## Differential pair example

- Ignore $V_{n5}$, just modulates the bias current
- $V_{n1}$ and $V_{n2}$ are already at the input
- Find how $V_{n3}$ and $V_{n4}$ influence the output ($g_{m3}R_o$) and input refer, $\left(\frac{g_{m3}}{g_{m1}}\right)^2$

$$V_{ni}^2(f) = 2V_{n1}^2(f) + 2V_{n3}^2(f) \times \left(\frac{g_{m3}}{g_{m1}}\right)^2$$

$g_m = \sqrt{2\beta I_D} \rightarrow$

$$= 2V_{n1}^2(f) + 2V_{n3}^2(f) \times \frac{\beta_3}{\beta_1}$$

## Differential pair example

Contribution from white noise, $\frac{4kT\gamma}{g_m}$

$$8kT\gamma\frac{1}{g_{m1}}+8kT\gamma\frac{g_{m3}}{g_{m1}^2}$$

- Maximize $g_{m1}$ (small overdrive)
- Minimize $g_{m3}$ (large overdrive)

Contribution from flicker noise, $\frac{K}{WLC_{ox}f}$

$$\frac{2}{C_{ox}f}\left(\frac{K_1}{W_1L_1}+\frac{\mu_n}{\mu_p}\frac{K_3L_1}{W_1L_3^2}\right)$$

- Big devices helps
- Especially increased $W_1$ and $L_3$





## Distortion

- So far we have discussed noise, which is an important performance constraint
- The degree of non-linearity is another important performance limitation
- SNR improves with “stronger” input signals (because the noise remains constant)
- However, larger input amplitude adds more non-linearity.
- The sum of noise and distortion is important (SNDR). Increasing the amplitude improves SNDR up to some point where the non-linearity becomes significant. Beyond this point the SNDR deteriorates.

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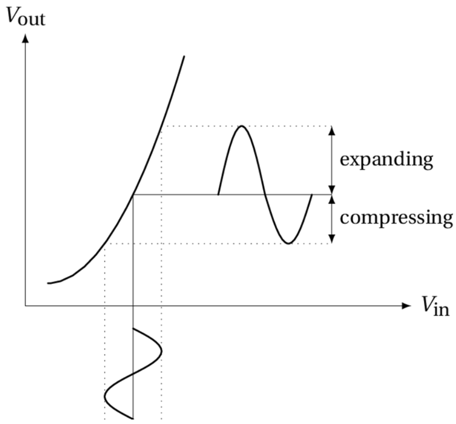
## Distortion

Amplification and non-linearity depends on the biasing point.

- Soft non-linearity (compression and expansion)
- Hard non-linearity (clipping)





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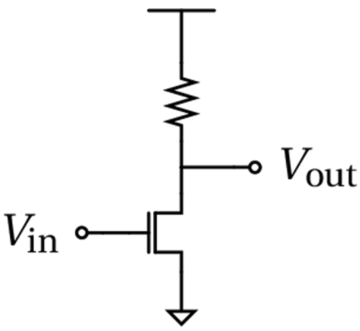
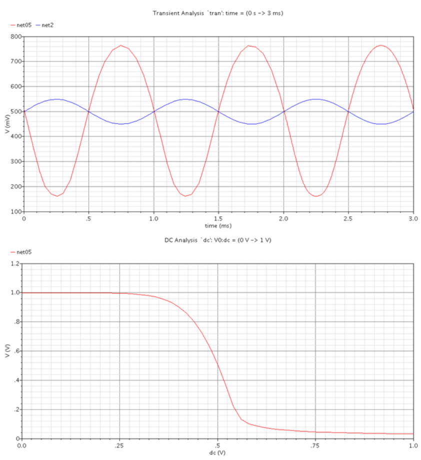
## Common source amplifier example



# Harmonic distortion

Using Taylor series allows us to study distortion independent of the specific shape of the non-linearity (e.g. common source).

Generic expression for total harmonic distortion (THD). *The non-linearity adds harmonics in the frequency domain.*



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# Harmonic distortion

$$y = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \cdots$$

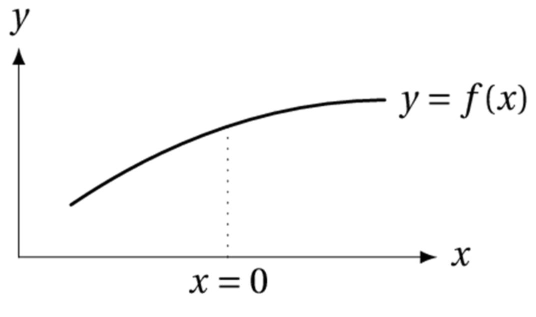
$$\alpha_0 = f(0)$$

$$\alpha_1 = \frac{df(0)}{dx}$$

$$\alpha_2 = \frac{1}{2}\frac{d^2 f(0)}{dx^2}$$

$$\alpha_3 = \frac{1}{6}\frac{d^3 f(0)}{dx^3}$$

$$\ldots$$



## Harmonic distortion

Using Taylor expansion we write the output of the amplifier as:

$$v_o(t) = \underbrace{a_0}_{\text{Output DC level}} + \underbrace{\alpha_1 v_i(t)}_{\text{Ideal gain}} + \underbrace{\alpha_2 v_i^2(t) + \alpha_3 v_i^3(t) + \cdots}_{\text{Distortion (harmonics)}}$$

Output DC level (Not important)

Ideal gain (This is the signal we want)

Distortion (harmonics)

In fully differential circuits, the even order terms, $\alpha_2, \alpha_4, \ldots$, cancels out.



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## Harmonic distortion

The first terms are the most significant

$$v_o(t) \approx \alpha_1 v_i(t) + \alpha_2 v_i^2(t) + \alpha_3 v_i^3(t)$$

In fully differential circuits we approximate the output as

$$v_o(t) \approx \alpha_1 v_i(t) + \alpha_3 v_i^3(t)$$

To analyze the linearity we assume a single tone input: $v_i(t) = A \cos \omega t$

# Harmonic distortion

From before, $v_i(t) = A\cos\omega t$

$v_o(t) \approx \alpha_1 v_i(t) + \alpha_2 v_i^2(t) + \alpha_3 v_i^3(t)$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \cos^3\theta = \frac{3\cos\theta + \cos 3\theta}{4}$$

$$v_o(t) \approx \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t)$$



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# Harmonic distortion

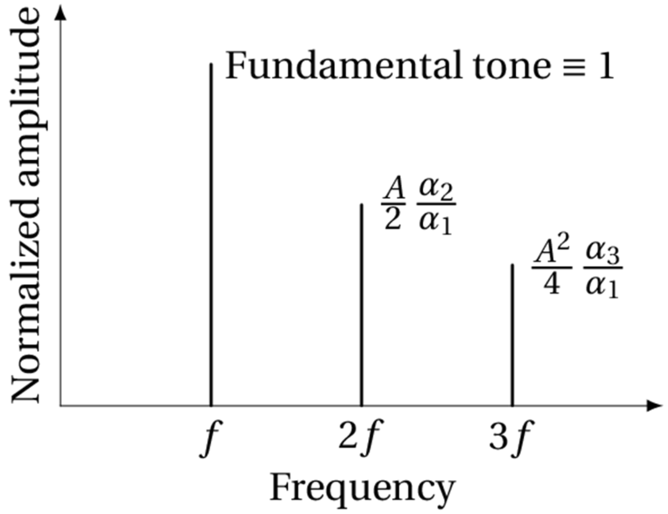
$$v_o(t) \equiv H_{D1}\cos\omega t + H_{D2}\cos 2\omega t + H_{D3}\cos 3\omega t$$

$$H_{D1} = \alpha_1 A + \frac{3}{4}\alpha_3 A^3 \approx \alpha_1 A$$

$$H_{D2} = \frac{\alpha_2}{2}A^2$$

$$H_{D3} = \frac{\alpha_3}{4}A^3$$

# Harmonic distortion







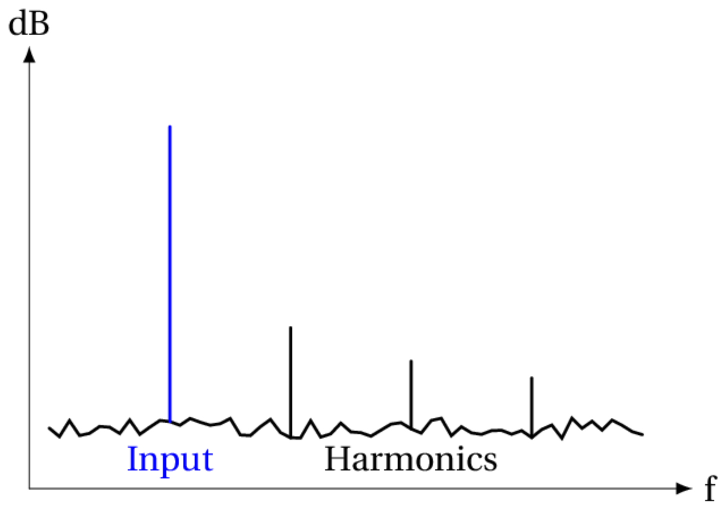

# Total harmonic distortion

As with noise, the ratio between the distortion and the signal is what we are interested in.

Total harmonic distortion (sum of all harmonics relative to the fundamental tone):

$$\mathrm{THD} = 10\log_{10}\frac{H_{D2}^2 + H_{D3}^2 + H_{D4}^2}{H_{D1}^2}$$

# Signal to noise and distortion (SNDR)



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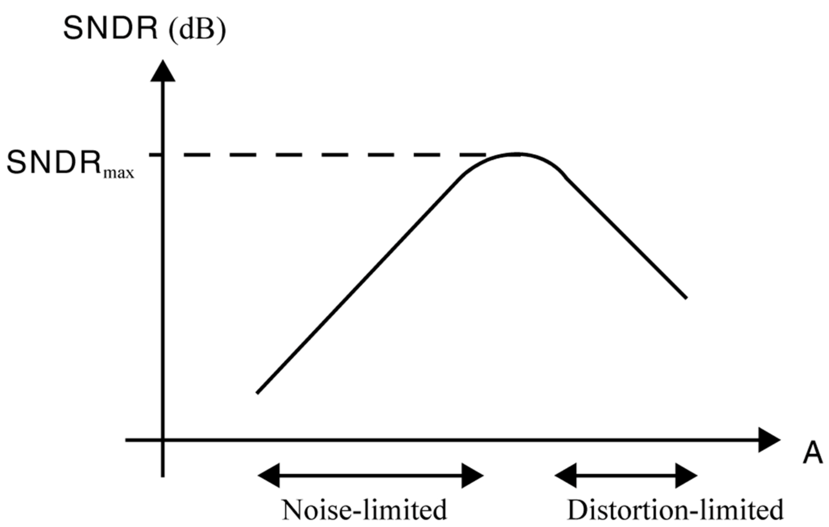
# Signal to noise and distortion (SNDR)



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# Third-order intercept point (IP3)

Using two input tones rather than one (same amplitude different frequencies)

$$v_{in}(t) = A\cos\omega_1 t + A\cos\omega_2 t$$

Gives rise to two new distortion components close to the input frequencies

$$\omega_1 - \Delta\omega \text{ and } \omega_2 + \Delta\omega \text{ where } \Delta\omega \equiv \omega_2 - \omega_1$$

This distortion increases as $A^3$. We use this to find the intercept point, and infer the third order distortion component.



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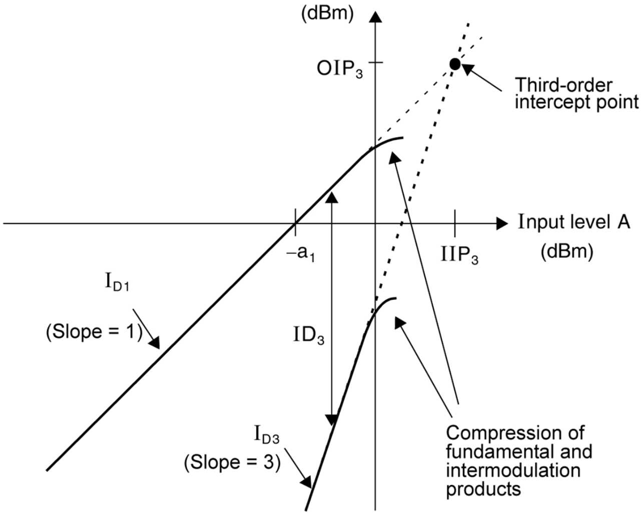
# Third-order intercept point (IP3)



# Spurious free dynamic range (SFDR)

Ratio between the input signal power and any "spurs" in the spectrum. Could be from harmonics, or feed through from clocks, etc.