INF5110 – Compiler Construction

Scanning

Spring 2016

Outline

Outline

1. Scanning

what's a scanner?

- Input: source code.[a]
- Output: sequential stream of tokens

[a] The argument of a scanner is often a *file name* or an *input stream* or similar.

- *regular expressions* to describe various token classes
- (deterministic/nondeterminstic) finite-state automata (FSA, DFA, NFA)
- implementation of FSA
- regular expressions $\to$ NFA
- NFA $\leftrightarrow$ DFA

What's a scanner?

- other names: lexical scanner, lexer, tokenizer

A scanner's functionality

Part of a compiler that takes the source code as input and translates this stream of characters into a stream of tokens.

- char's typically language independent.[1]
- *tokens* already language-specific.[2]
- works always "left-to-right", producing one *single token* after the other, as it scans the input[3]
- it "segments" char stream into "chunks" while at the same time "classifying" those pieces ⇒ tokens

[1] Characters are language-independent, but perhaps the encoding may vary, like ASCII, UTF-8, also Windows-vs.-Unix-vs.-Mac newlines etc.

[2] There are large commonalities across many languages, though.

[3] No theoretical necessity, but that's how also humans consume or "scan" a source-code text. At least those humans trained in e.g. Western languages.

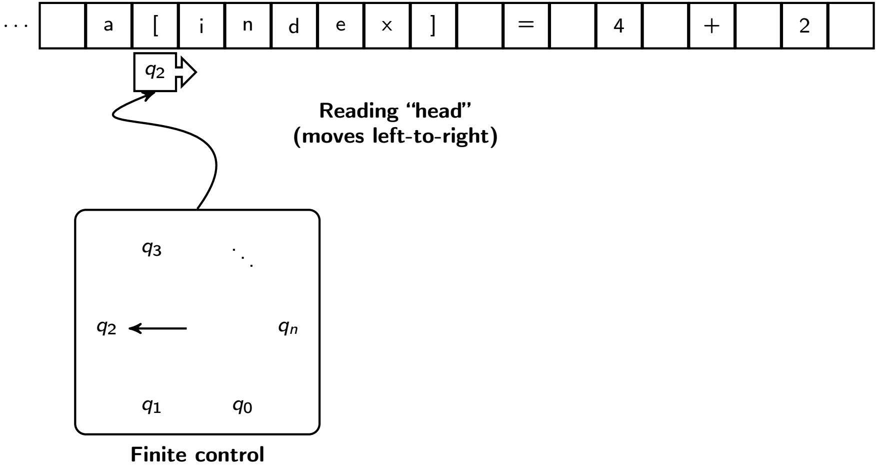
Typical responsibilities of a scanner

- segment & classify char stream into tokens
- typically described by "rules" (and regular expressions)
- typical language aspects covered by the scanner
  - describing *reserved words* or *key words*
  - describing format of *identifiers* (= "strings" representing variables, classes . . .)
  - comments (for instance, between // and `NEWLINE`)
  - *white space*
    - to segment into tokens, a scanner typically "jumps over" white spaces and afterwards starts to determine a new token
    - not only "blank" character, also `TAB`, `NEWLINE`, etc.
- lexical rules: often (explicit or implicit) *priorities*
  - *identifier* or *keyword*? $\Rightarrow$ keyword
  - take the *longest* possible scan that yields a valid token.

Rule of thumb

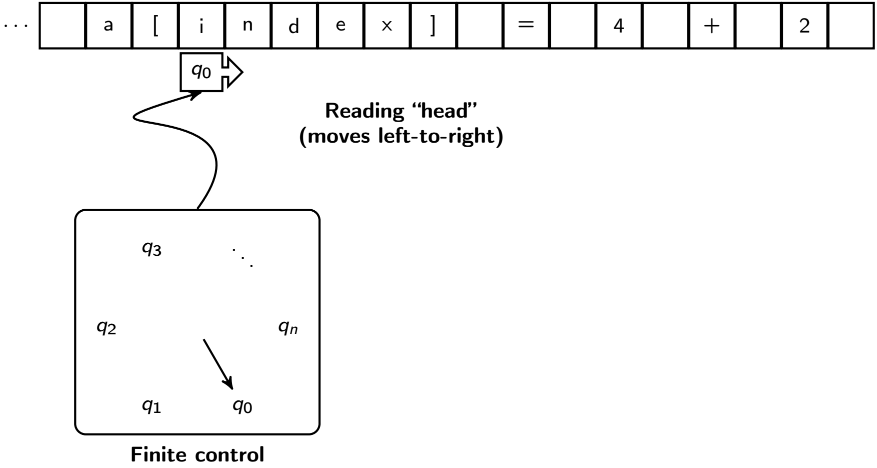
Everything about the source code which is so simple that it can be captured by reg. expressions belongs into the scanner.

How does scanning roughly work?



`a[index] = 4 + 2`

How does scanning roughly work?



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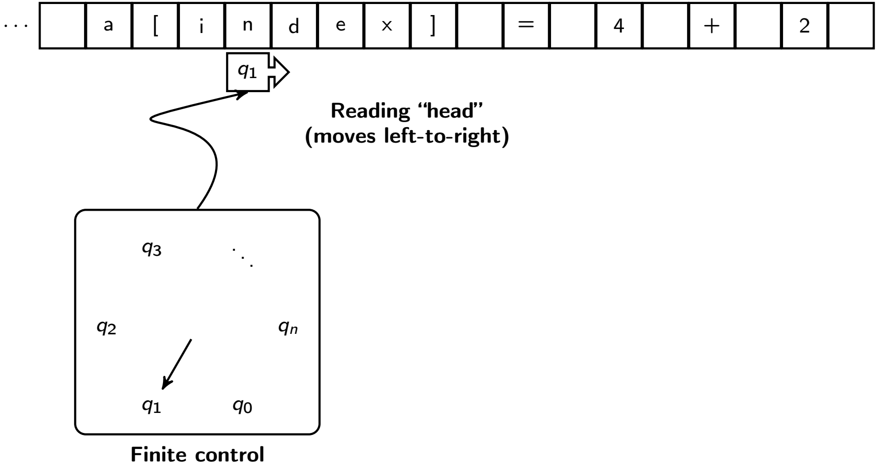
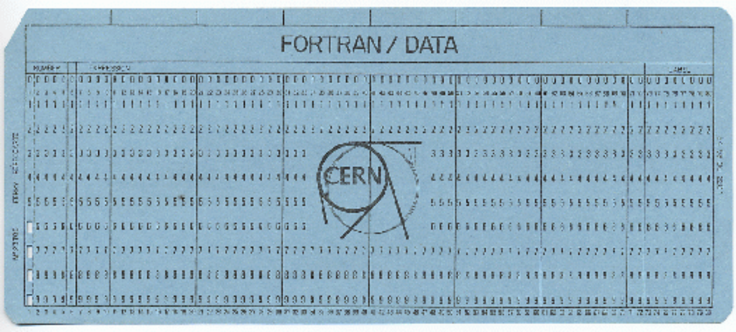


`a[index] = 4 + 2`

- usual invariant in such pictures (by convention): arrow or head points to the *first* character to be *read next* (and thus *after* the last character having been scanned/read last)
- in the scanner *program* or procedure:
  - analogous invariant, the arrow corresponds to a *specific variable*
  - contains/points to the next character to be read
  - name of the variable depends on the scanner/scanner tool
- the *head* in the pic: for illustration, the scanner does not really have a "reading head"
  - remembrance of Turing machines, or
  - the old times when perhaps the program data was stored on a tape.[4]

[4] Very deep down, if one still has a magnetic disk (as opposed to SSD) the secondary storage still has "magnetic heads", only that one typically does not parse *directly* char by char from disk. . .

- in the days of the pioneers
  - main memory was *smaaaaaaaaaall*
  - compiler technology was not well-developed (or not at all)
  - programming was for *very* few "experts".[5]
  - Fortran was considered a very high-level language (wow, a language so complex that you had to compile it . . .)



[5] There was no computer science as profession or university curriculum.

Lexical aspects = those dealt with a scanner

- whitespace *without* "meaning":

  `I F( X 2.  EQ. 0) TH E N` vs. `IF ( X2.  EQ.0 ) THEN`

- no reserved words!

  `IF (IF.EQ.0) THEN THEN=1.0`

- general obscurity tolerated:

  `DO99I=1,10` vs. `DO99I=1.10`

```
DO 99 I=1,10
 —
 —
99 CONTINUE
```

Fortran scanning: remarks

- Fortran (of course) has evolved from the pioneer days ...
- no keywords: nowadays mostly seen as *bad* idea[6]
- treatment of white-space as in Fortran: not done anymore: `THEN` and `TH EN` *are* different things in all languages
- however:[7] both considered "the same":

```
if␣b␣then␣..
```

```
if␣␣␣b␣␣␣␣then␣..
```

- since concepts/tools (and much memory) were missing, Fortran scanner and parser (and compiler) were
  - quite simplistic
  - syntax: designed to "help" the lexer (and other phases)

[6] It's mostly a question of language *pragmatics*. The lexers/parsers would have no problems using `while` as variable, but humans tend to have.

[7] Sometimes, the part of a lexer / parser which removes whitespace (and comments) is considered as separate and then called *screener*. Not very common though.

A scanner classifies

- "good" classification: depends also on later phases, may not be clear till later

Rule of thumb

Things being treated equal in the syntactic analysis (= parser, i.e., subsequent phase) should be put into the same category.

- terminology not 100% uniform, but most would agree:

Lexemes and tokens

Lexemes are the "chunks" (pieces) the scanner produces from segmenting the input source code (and typically dropping whitespace). Tokens are the result of /classifying those lexemes.

- token = token name $\times$ token value

A scanner classifies & does a bit more

- token data structure in *OO* settings
  - token themselves defined by classes (i.e., as instance of a class representing a specific token)
  - token values: as attribute (instance variable) in its values
- often: scanner does slightly *more* than just classification
  - store names in some *table* and store a corresponding index as attribute
  - store text constants in some *table*, and store corresponding index as attribute
  - even: *calculate* numeric constants and store value as attribute

name/identifier	`abc123`
integer constant	`42`
real number constant	`3.14E3`
text constant, string literal	`"this is a text constant"`
arithmetic op's	`+ - * /`
boolean/logical op's	`and or not` (alternatively `/\ \/`)
relational symbols	`<= < >= > = == !=`
all other tokens: every one it its own group	`{ } ( ) [ ] , ; := .` etc.

- this classification: not the only possible (and not necessarily complete)
- note: *overlap*:
  - `"."` is here a token, but also part of real number constant
  - `"<"` is part of `"<="`

```c
typedef struct {
  TokenType tokenval;
  char * stringval;
  int numval;
} TokenRecord;
```

If one only wants to store one attribute:

```c
typedef struct {
  Tokentype tokenval;
  union
  { char * stringval;
    int numval
  } attribute;
} TokenRecord;
```

How to define lexical analysis and implement a scanner?

- even for complex languages: lexical analysis (in principle) not hard to do
- "manual" implementation straightforwardly possible
- *specification* (e.g., of different token classes) may be given in "prosa"
- however: there are straightforward formalisms and efficient, rock-solid tools available:
  - easier to specify unambigously
  - easier to communicate the lexical definitions to others
  - easier to change and maintain
- often called parser generators typically not just generate a scanner, but code for the next phase (parser), as well.

Outline

1. regular expressions to describe language's *lexical* aspects
   - like whitespaces, comments, keywords, format of identifiers etc.
   - often: more "user friendly" variants of reg-expr are supported to specify that phase
2. *classify* the lexemes to tokens
3. translate the reg-expressions $\Rightarrow$ NFA.
4. turn the NFA into a *deterministic* FSA (= DFA)
5. the DFA can straightforwardly be implementated

- Above steps are done automatically by a "lexer generator"
- lexer generators help also in other user-friendly ways of specifying the lexer: defining *priorities*, assuring that the longest possible token is given back, repeat the processs to generate a sequence of tokens[8]
- Step 2 is actually *not* covered by the classical Reg-expr = DFA = NFA results, it's something extra.

[8] Maybe even prepare useful error messages if scanning (not scanner generation) fails.

Use of regular expressions

- regular languages: fundamental class of "languages"
- regular expressions: standard way to describe regular languages
- origin of regular expressions: one starting point is Kleene [Kleene, 1956] but there had been earlier works outside "computer science"
- Not just used in compilers
- often used for flexible " *searching* ": simple form of pattern matching
- e.g. input to search engine interfaces
- also supported by many editors and text processing or scripting languages (starting from classical ones like `awk` or `sed`)
- but also tools like `grep` or `find`

```
find . -name "*.tex"
```

- often *extended* regular expressions, for user-friendliness, not theoretical expressiveness.

Alphabets and languages

Definition (Alphabet Σ)

Finite set of elements called "letters" or "symbols" or "characters"

Definition (Words and languages over Σ)

Given alphabet Σ, a word over Σ is a finite sequence of letters from Σ. A language over alphabet Σ is a *set* of finite *words* over Σ.

- in this lecture: we avoid terminology "symbols" for now, as later we deal with e.g. symbol tables, where symbols means something slighly different (at least: at a different level).
- Sometimes Σ left "implicit" (as assumed to be understood from the context)
- practical examples of alphabets: ASCII, Norwegian letters (capital and non-capitals) etc.

- note: Σ is finite, and words are of *finite* length
- languages: in general infinite sets of words
- Simple examples: Assume $\Sigma = \{a, b\}$
- *words* as finite "sequences" of letters
  - ϵ: the empty word (= empty sequence)
  - ab means " first a then b "
- sample languages over Σ are
  1. $\{\}$ (also written as $\emptyset$) the empty set
  2. $\{a, b, ab\}$: language with 3 finite words
  3. $\{\epsilon\}$ ($\neq \emptyset$)
  4. $\{\epsilon, a, aa, aaa, \ldots\}$: infinite languages, all words using only a 's.
  5. $\{\epsilon, a, ab, aba, abab, \ldots\}$: alternating a's and b's
  6. $\{ab, bbab, aaaaa, bbabbabab, aabb, \ldots\}$: ?????

- language mostly here in the abstract sense just defined.
- the "dot-dot-dot" (...) is not a good way to describe to a computer (and many humans) what is meant
- enumerating explicitly all allowed words for an infinite language does not work either

Needed

A finite way of describing infinite languages (which is hopefully efficiently implementable & easily readable)

Beware

Is it apriori clear to expect that *all* infinite languages can even be captured in a finite manner?

- small metaphor

$$2.727272727\ldots \qquad 3.1415926\ldots \tag{1}$$

Definition (Regular expressions)

A *regular expression* is one of the following

1. a *basic* regular expression of the form **a** (with $a \in \Sigma$), or ϵ, or $\emptyset$
2. an expression of the form $r \mid s$, where r and s are regular expressions.
3. an expression of the form $r\,s$, where r and s are regular expressions.
4. an expression of the form r^*, where r is a regular expression.
5. an expression of the form $\mathbf{(}r\mathbf{)}$, where r is a regular expression.

Precedence (from high to low): $*$, concatenation, $|$

later introduced as (notation for) context-free grammars:

$$
\begin{aligned}
r &\rightarrow \mathbf{a} \\
r &\rightarrow \epsilon \\
r &\rightarrow \emptyset \\
r &\rightarrow r \mid r \\
r &\rightarrow r\ r \\
r &\rightarrow r^* \\
r &\rightarrow (r)
\end{aligned}
\tag{2}
$$

Notational conventions

Later, for CF grammars, we use capital letters to denote "variables" of the grammars (then called *non-terminals*). If we like to be consistent with that convention, the definition looks as follows:

$$
\begin{array}{lcl}
R & \rightarrow & \mathbf{a} \\
R & \rightarrow & \epsilon \\
R & \rightarrow & \emptyset \\
R & \rightarrow & R \mid R \\
R & \rightarrow & R\,R \\
R & \rightarrow & R^* \\
R & \rightarrow & (R)
\end{array}
\tag{3}
$$

Symbols, meta-symbols, meta-meta-symbols . . .

- regexps: notation or "language" to describe "languages" over a given alphabet Σ (i.e. subsets of Σ^*)
- language being described $\Leftrightarrow$ language used to describe the language

$\Rightarrow$ language $\Leftrightarrow$ meta-language

- here:
  - regular expressions: notation to describe regular languages
  - English resp. context-free notation:[9] notation to describe regular expression
- for now: carefully use *notational convention* for precision

[9]To be careful, we will (later) distinguish between context-free languages on the one hand and notations to denote context-free languages on the other, in the same manner that we *now* don't want to confuse regular languages as concept from particular notations (specifically, regular expressions) to write them down.

- notational conventions by *typographic* means (i.e., different fonts etc.)
- not easy discscernible, but: difference between
  - $\boldsymbol{a}$ and a
  - $\boldsymbol{\epsilon}$ and ϵ
  - $\boldsymbol{\emptyset}$ and $\emptyset$
  - $\boldsymbol{|}$ and $|$ (especially hard to see :-))
  - ...
- later (when gotten used to it) we may take a more "relaxed" attitude toward it, assuming things are clear, as do many textbooks
- Note: in compiler *implementations*, the distinction between language and meta-language etc. is very real (even if not done by typographic means ...)

$$
\begin{array}{rclr}
R & \rightarrow & \mathbf{a} \mid \epsilon \mid \emptyset & \text{basic reg. expr.} \\
 & \mid & R \,\boldsymbol{|}\, R \mid R\,R \mid R^{*} \mid \mathbf{(}R\mathbf{)} & \text{compound reg. expr.}
\end{array} \tag{4}
$$

Note:

- symbol $\boldsymbol{|}$: as symbol of regular expressions
- symbol $\mid$: meta-symbol of the CF grammar notation
- The meta-notation use here for regular expressions will be the subject of later chapters

Definition (Regular expression)

Given an alphabet Σ. The meaning of a regexp r (written $\mathcal{L}(r)$) over Σ is given by equation (5).

$$
\begin{array}{rcll}
\mathcal{L}(\emptyset) & = & \{\} & \text{empty language} \\
\mathcal{L}(\epsilon) & = & \epsilon & \text{empty word} \\
\mathcal{L}(\boldsymbol{a}) & = & \{a\} & \text{single ``letter'' from } \Sigma \\
\mathcal{L}(r \mid s) & = & \mathcal{L}(r) \cup \mathcal{L}(s) & \text{alternative} \\
\mathcal{L}(r^*) & = & \mathcal{L}(r)^* & \text{iteration}
\end{array} \tag{5}
$$

- conventional *precedences*: $*$, concatenation, $|$.
- Note: left of "$=$": reg-expr *syntax*, right of "$=$": semantics/meaning/math [10]

[10] Sometimes confusingly "the same" notation.

In the following:

- $\Sigma = \{a, b, c\}$.
- we don't bother to "boldface" the syntax

words with exactly one b	$(a \mid c)^* b(a \mid c)^*$
words with max. one b	$((a \mid c)^*) \mid ((a \mid c)^* b(a \mid c)^*)$ $(a \mid c)^* \ (b \mid \epsilon) \ (a \mid c)^*$
words of the form $a^n b a^n$, i.e., equal number of a's before and after 1 b	

words that do *not* contain two b's in a row.

$$
\begin{array}{lll}
(b\ (a \mid c))^* & & \text{not quite there yet} \\
((a \mid c)^* \mid (b\ (a \mid c))^*)^* & & \text{better, but still not there} \\
 & = & \text{(simplify)} \\
((a \mid c) \mid (b\ (a \mid c)))^* & = & \text{(simplifiy even more)} \\
(a \mid c \mid ba \mid bc)^* & & \\
(a \mid c \mid ba \mid bc)^*\ (b \mid \epsilon) & & \text{potential } b \text{ at the end} \\
(notb \mid notb\ b)^*(b \mid \epsilon) & & \text{where } notb \triangleq a \mid c
\end{array}
$$

Additional “user-friendly” notations

$$\begin{array}{rcl} r^{+} & = & rr^{*} \\ r? & = & r \mid \epsilon \end{array}$$

Special notations for *sets* of letters:

$$\begin{array}{rl} [0-9] & \text{range (for ordered alphabets)} \\ {\sim}a & \text{not } a \text{ (everything except } a\text{)} \\ . & \text{all of } \Sigma \end{array}$$

naming regular expressions (“regular definitions”)

$$\begin{array}{rcl} \mathit{digit} & = & [0-9] \\ \mathit{nat} & = & \mathit{digit}^{+} \\ \mathit{signedNat} & = & (+|-)\mathit{nat} \\ \mathit{number} & = & \mathit{signedNat}(\text{"."}\,\mathit{nat})?(\mathtt{E}\ \mathit{signedNat})? \end{array}$$

Outline

- simple "computational" machine
- (variations of) FSA's exist in many flavors and under different names
- other rather well-known names include finite-state machines, finite labelled transition systems,
- "state-and-transition" representations of programs or behaviors (finite state or else) are wide-spread as well
  - state diagrams
  - Kripke-structures
  - I/O automata
  - Moore & Mealy machines
- the logical behavior of certain classes of electronic circuitry with internal memory ("flip-flops") is described by finite-state automata.[11]

[11] Historically, design of electronic circuitry (not yet chip-based, though) was one of the early very important applications of finite-state machines.

Definition (FSA)

A FSA $\mathcal{A}$ over an alphabet Σ is a tuple $(\Sigma, Q, I, F, \delta)$

- Q: finite set of states
- $I \subseteq Q$, $F \subseteq Q$: initial and final states.
- $\delta \subseteq Q \times \Sigma \times Q$ transition relation

- final states: also called *accepting* states
- transition relation: can *equivalently* be seen as function $\delta : Q \times \Sigma \to 2^Q$: for each state and for each letter, give back the set of sucessor states (which may be empty)
- more suggestive notation: $q_1 \xrightarrow{a} q_2$ for $(q_1, a, q_2) \in \delta$
- We also use freely —self-evident, we hope— things like

$$q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3$$

FSA as scanning machine?

- FSA have slightly unpleasant properties when considering them as decribing an actual program (i.e., a scanner procedure/lexer)
- given the "theoretical definition" of acceptance:

Mental picture of a scanning automaton

The automaton eats one character after the other, and, when reading a letter, it moves to a successor state, if any, of the current state, depending on the character at hand.

- 2 problematic aspects of FSA
  - non-determinism: what if there is more than one possible successor state?
  - undefinedness: what happens if there's no next state for a given input
- the second one is *easily* repaired, the first one requires more thought

Definition (DFA)

A *deterministic, finite automaton* $\mathcal{A}$ (DFA for short) over an alphabet Σ is a tuple $(\Sigma, Q, I, F, \delta)$

- Q: finite set of states
- $I = \{i\} \subseteq Q$, $F \subseteq Q$: initial and final states.
- $\delta : Q \times \Sigma \to Q$ transition function

- transition function: special case of transition relation:
  - deterministic
  - left-total[12]

[12] That means, for each pair q, a from $Q \times \Sigma$, $\delta(q, a)$ is defined. Some people call an automaton where δ is not a left-total but a determinstic relation (or, equivalently, the function δ is not total, but partial) still a deterministic automaton. In that terminology, the DFA as defined here would be determinstic *and* total.

Semantics

The intended meaning of an FSA over an alphabet Σ is the set consisting of all the finite words, the automaton accepts.

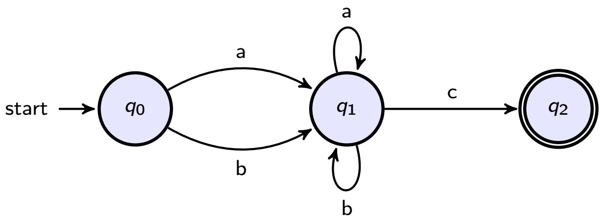
Definition (Accepting words and language of an automaton)

A word $c_1 c_2 \ldots c_n$ with $c_i \in \Sigma$ is accepted by automaton $\mathcal{A}$ over Σ, if there exists states $q_0, q_2, \ldots q_n$ all from Q such that

$$q_0 \xrightarrow{c_1} q_1 \xrightarrow{c_2} q_2 \xrightarrow{c_3} \quad \ldots \quad q_{n-1} \xrightarrow{c_n} q_n \ ,$$

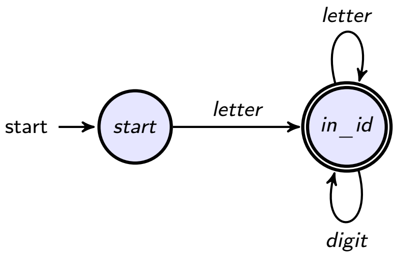
and were $q_0 \in I$ and $q_n \in F$. The language of an FSA $\mathcal{A}$, written $\mathcal{L}(\mathcal{A})$, is the set of all words $\mathcal{A}$ accepts

FSA example



Regular expression

$$identifier = letter(letter \mid digit)^* \tag{6}$$

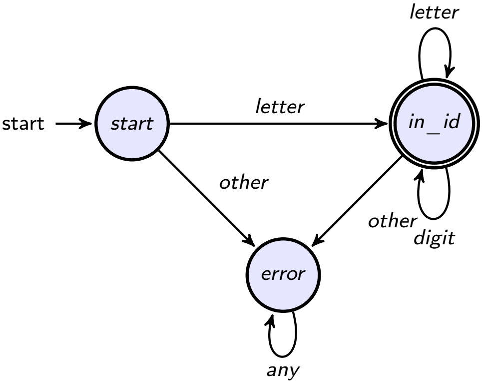


- transition *function*/relation δ *not* completely defined (= *partial* function)

Example: identifiers

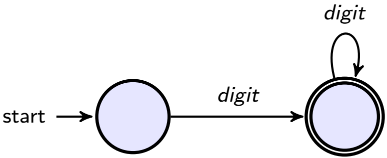
Regular expression

$$identifier = letter(letter \mid digit)^* \tag{6}$$

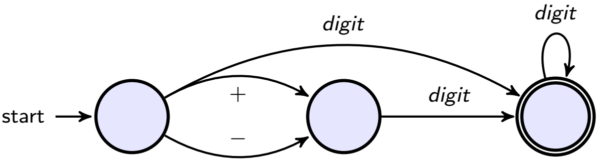


Automata for numbers: natural numbers

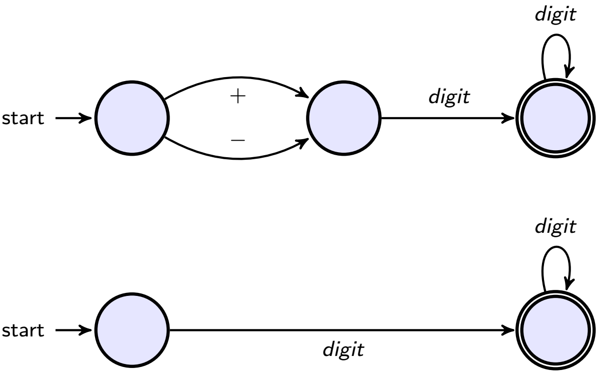
$$\begin{aligned} digit &= [0-9] \\ nat &= digit^{+} \end{aligned} \tag{7}$$



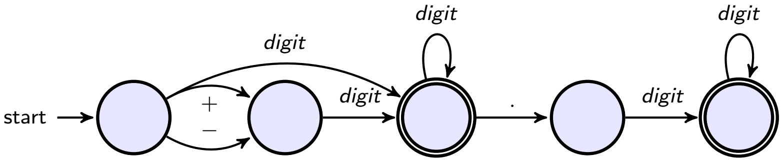
$$signednat \quad = \quad (+ \mid -)nat \mid nat \tag{8}$$



Signed natural numbers: non-deterministic



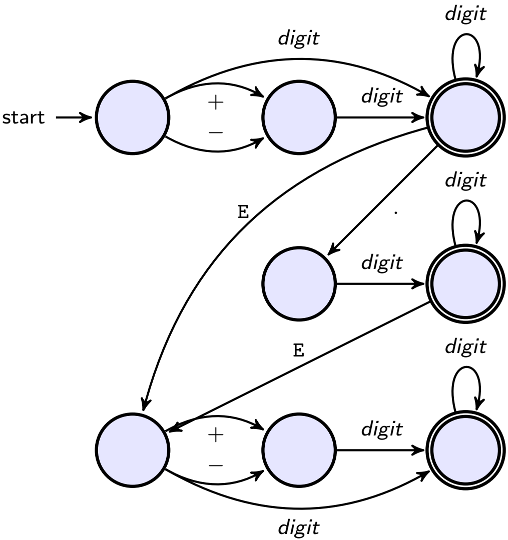
$$frac \quad = \quad signednat(''.''\,nat)? \tag{9}$$



$$\begin{aligned} digit &= [0-9] \\ nat &= digit^{+} \\ signednat &= (+ \mid -)nat \mid nat \\ frac &= signednat(''.''\,nat)? \\ float &= frac(\mathtt{E}\ signednat)? \end{aligned} \tag{10}$$

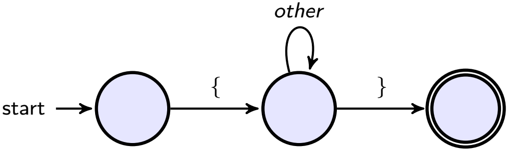
- Note: no (explicit) recursion in the definitions
- note also the treatment of *digit* in the automata.

DFA for floats

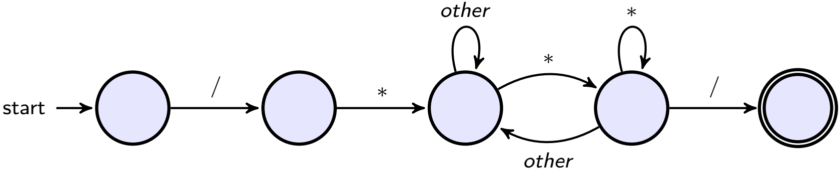


DFAs for comments

Pascal-style



C, C^{++}, Java



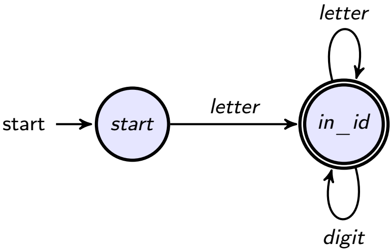
Outline

1. Scanning

Example: identifiers

Regular expression

$$identifier = letter(letter \mid digit)^* \tag{6}$$

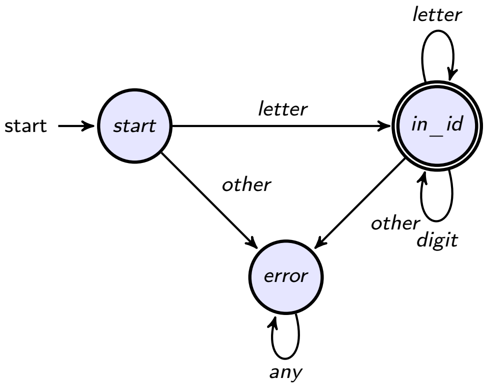


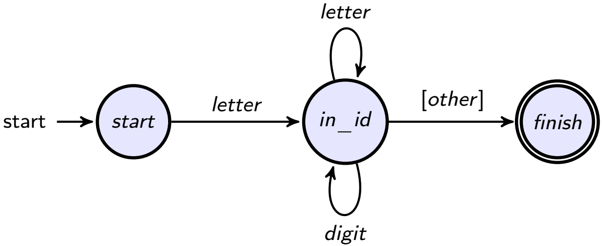
- transition *function*/relation δ *not* completely defined (= *partial* function)

Example: identifiers

Regular expression

$$identifier = letter(letter \mid digit)^* \qquad (6)$$





start
start
letter
in_id
letter
digit
[other]
finish

```
{ starting state }

if  the next character is a letter
then
  advance the input;
  { now in state 2 }
  while the next character is a letter or digit
  do
     advance the input;
     { stay in state 2 }
  end while;
  { go to state 3, without advancing input }
  accept;
else
  { error or other cases }
end
```

```
state := 1  { start }
while state = 1 or 2
do
  case state of
  1:  case input character of
      letter: advance the input;
              state := 2
      else state := .... { error or other };
      end case;
  2: case input character of
     letter,digit: advance the input;
                   state := 2; { actually unessessary }
     else          state := 3;
     end case;
  end case;
end while;
if state = 3 then accept else error;
```

Table representation of a DFA

state \ input char	letter	digit	other
1	2		
2	2	2	3
3			

state \ input char	letter	digit	other	accepting
1	2			no
2	2	2	[3]	no
3				yes

add info for

- accepting or not
- "non-advancing" transitions
  - here: 3 can be reached from 2 via such a transition

```
state := 1  { start }
ch := next input character;
while not Accept[state] and not error(state)
do

while state = 1 or 2
do
   newstate := T[state,ch];
   {if Advance[state,ch]
    then ch:=next input character};
   state := newstate
end while;
if Accept [state] then accept;
```

Outline

Definition (NFA (with ϵ transitions))

A non-deterministic finite-state automaton (NFA for short) $\mathcal{A}$ over an alphabet Σ is a tuple $(\Sigma, Q, I, F, \delta)$, where

- Q: finite set of states
- $I \subseteq Q$, $F \subseteq Q$: initial and final states.
- $\delta : Q \times \Sigma \to 2^Q$ transition function

In case, one uses the alphabet $\Sigma + \{\epsilon\}$, one speaks about an NFA with ϵ-transitions.

- in the following: NFA mostly means, allowing ϵ transitions[13]
- ϵ: treated *differently* than the "normal" letters from Σ.
- δ can *equivalently* be interpreted as *relation*: $\delta \subseteq Q \times \Sigma \times Q$ (transition relation labelled by elements from Σ).

[13] It does not matter much anyhow, as we will see.

- Remember $\mathcal{L}(\mathcal{A})$ (Definition 7 on page 41)
- applying definition directly to $\Sigma + \{\epsilon\}$: accepting words "containing" letters ϵ
- as said: special treatment for ϵ-transitions/ϵ-"letters". ϵ rather represents absence of input character/letter.

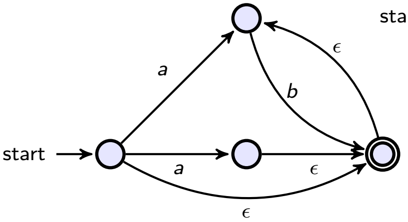
Definition (Acceptance with ϵ-transitions)

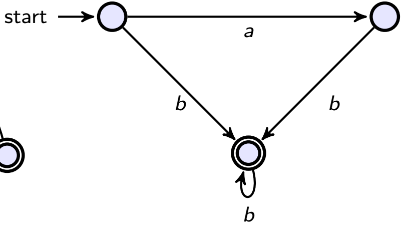
A word w over alphabet Σ is accepted by an NFA with ϵ-transitions, if there exists a word w' which is accepted by the NFA with alphabet $\Sigma + \{\epsilon\}$ according to Definition 7 and where w is w' with all occurrences of ϵ removed.

Alternative (but equivalent) intuition

$\mathcal{A}$ reads one character after the other (following its transition relation). If in a state with an outgoing ϵ-transition, $\mathcal{A}$ can move to a corresponding successor state *without* reading an input symbol.

- *NFA*: often easier (and smaller) to write down, esp. starting from a reg expression.
- Non-determinism: not *immediately* transferable to an *algo*

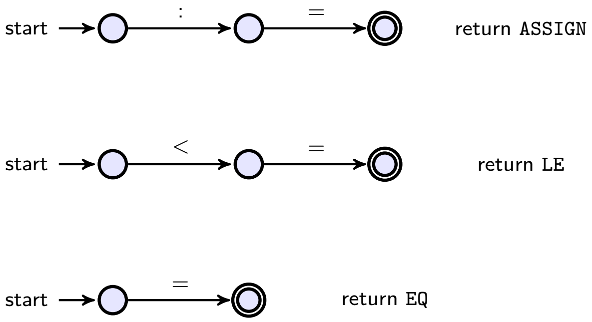


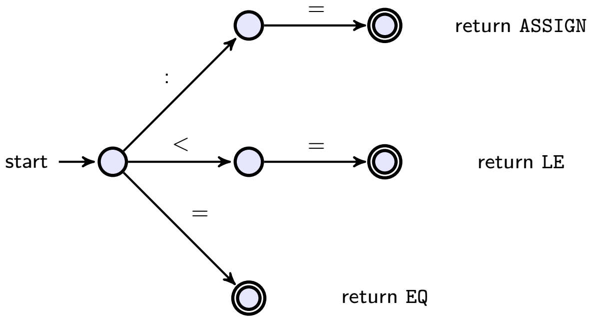


Outline

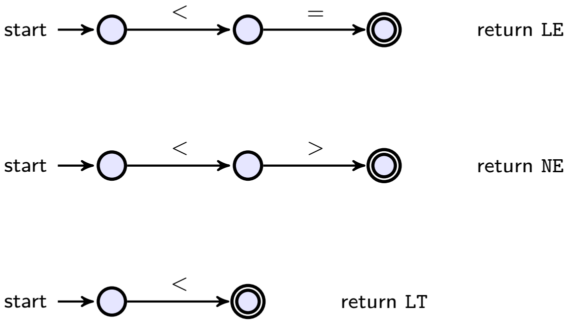
1. Scanning
   - Intro
   - Regular expressions
   - DFA
   - Implementation of DFA
   - NFA
   - From regular expressions to DFAs
   - Thompson's construction
   - Determinization
   - Minimization
   - Scanner generation tools

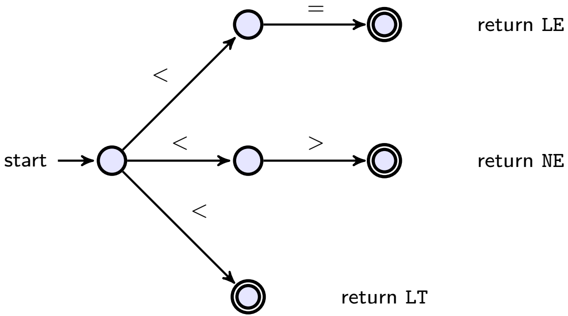
Task: recognize $:=$, $<=$, and $=$ as three different tokens:

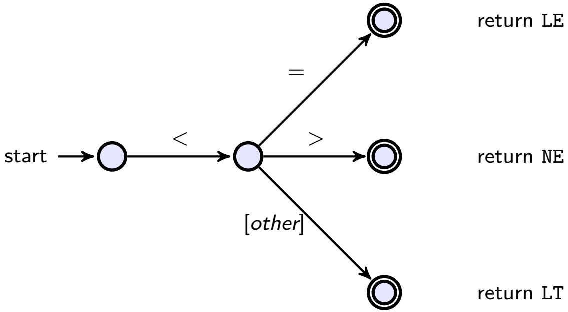



start
:
=
<
=
=
return ASSIGN
return LE
return EQ

What about the following 3 tokens?




start
<
=
<
>
<
return LE
return NE
return LT


start
<
=
>
[other]
return LE
return NE
return LT

- needed: a *systematic* translation
- conceptually easiest: translate to NFA (with ϵ-transitions)
  - postpone determinization for a second step
  - (postpone minimization for later, as well)

Compositional construction [Thompson, 1968]

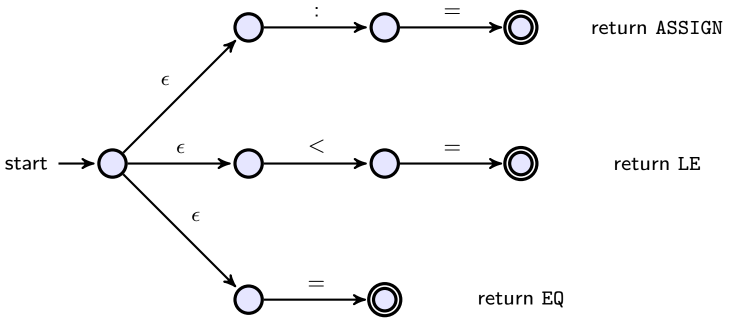
Design goal: The NFA of a compound regular expression is given by taking the NFA of the immediate subexpressions and connecting them appropriately.

- construction slightly[14] simpler, if one uses automata with one start and one accepting state

$\Rightarrow$ ample use of ϵ-transitions

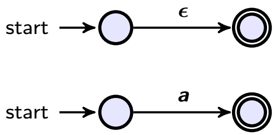
[14] does not matter much, though.

Illustration for ϵ-transitions

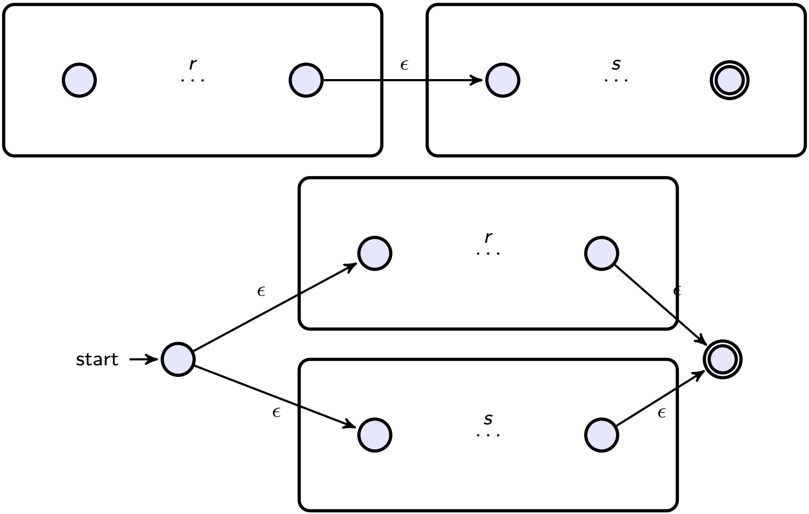


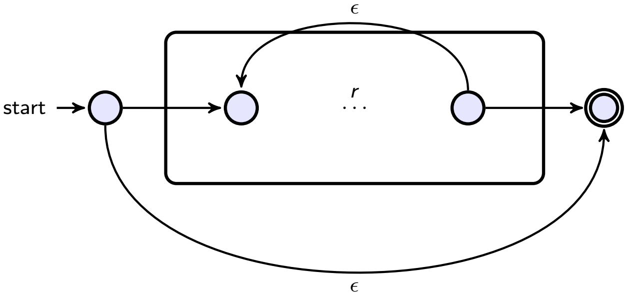
basic regular expressions

basic (= non-composed) regular expressions: ϵ, $\emptyset$, $\boldsymbol{a}$ (for all $a \in \Sigma$)

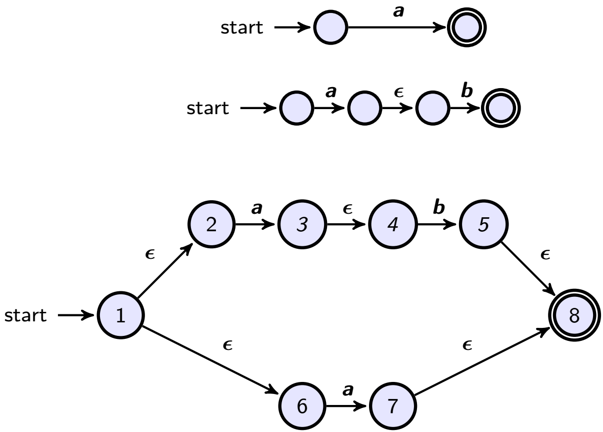


Thompson's construction: compound expressions





ϵ
start
r
. . .
ϵ



ab* | *a

Outline

1. Scanning

Main idea

- Given a non-det. automaton $\mathcal{A}$. To construct a DFA $\overline{\mathcal{A}}$: instead of *backtracking*: explore all successors "at the same time" $\Rightarrow$
- each state q' in $\overline{\mathcal{A}}$: represents a *subset* of states from $\mathcal{A}$
- Given a word w: "feeding" that to $\overline{\mathcal{A}}$ leads to *the* state representing *all* states of $\mathcal{A}$ *reachable* via w.

- side remark: this construction, known also as *powerset* construction, seems straightforward enough, but: analogous constructions works for some other kinds of automata, as well, but for others, the approach does *not* work.[15]
- Origin [Rabin and Scott, 1959]

[15] For some forms of automata, non-deterministic versions are strictly more expressive than the deterministic one.

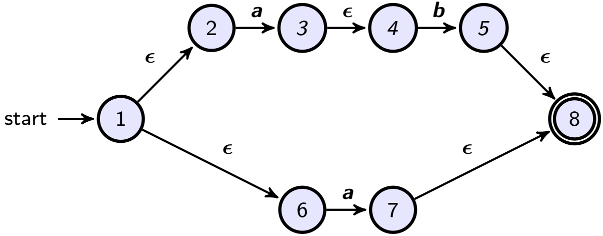
Definition (ϵ-closure, a-successors)

Given a state q, the ϵ-closure of q, written $close_\epsilon(a)$, is the set of states reachable via zero, one, or more ϵ-transitions. We write q_a for the set of states, reachable from q with one a-transition. Both definitions are used analogously for sets of states.

Input: NFA $\mathcal{A}$ over a given Σ
Output: DFA $\overline{\mathcal{A}}$

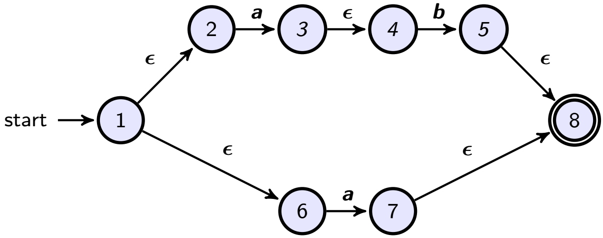
1. the *initial* state: $close_\epsilon(I)$, where I are the initial states of $\overline{\mathcal{A}}$
2. for a state Q' in $\overline{\mathcal{A}}$: the *a-sucessor* of Q is given by $close_\epsilon(Q_a)$, i.e.,
$$Q \xrightarrow{a} close_\epsilon(Q_a) \tag{11}$$
3. repeat step 2 for all states in $\overline{\mathcal{A}}$ and all $a \in \Sigma$, until no more states are being added
4. the *accepting* states in $\overline{\mathcal{A}}$: those containing *at least* one accepting states of $\mathcal{A}$.

Example $ab \mid a$

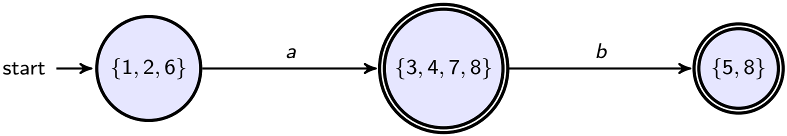


$ab \mid a$

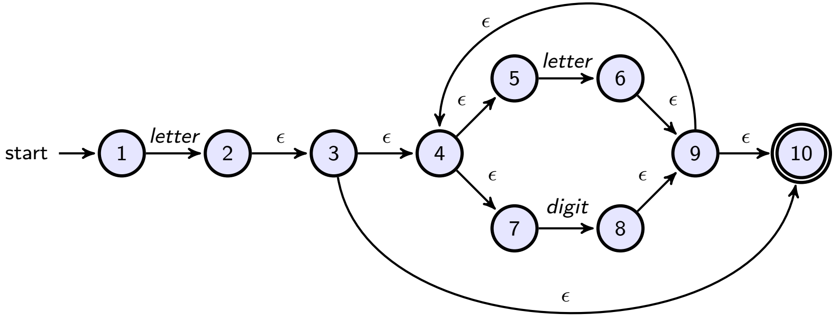
Example $ab \mid a$

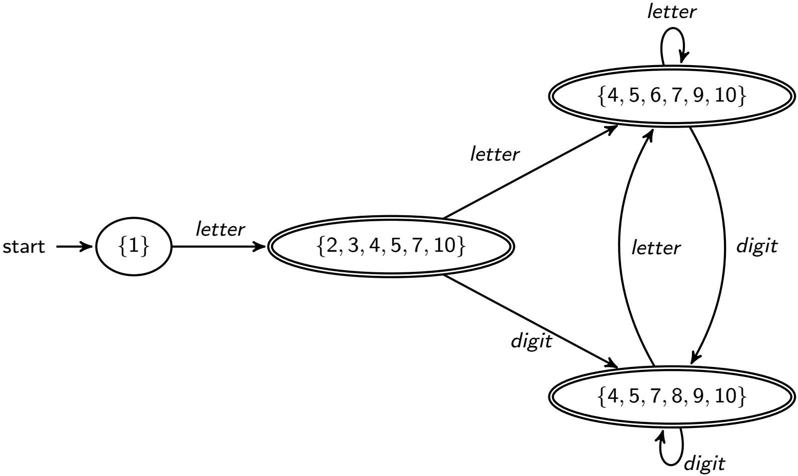


$ab \mid a$



Remember: regexpr for identifies from equation (6)




start
{1}
letter
{2, 3, 4, 5, 7, 10}
letter
{4, 5, 6, 7, 9, 10}
letter
digit
{4, 5, 7, 8, 9, 10}
letter
digit
digit

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- automatic construction of DFA (via e.g. Thompson): often many superfluous states
- goal: "combine" states of a DFA without changing the accepted language

Properties of the minimization algo

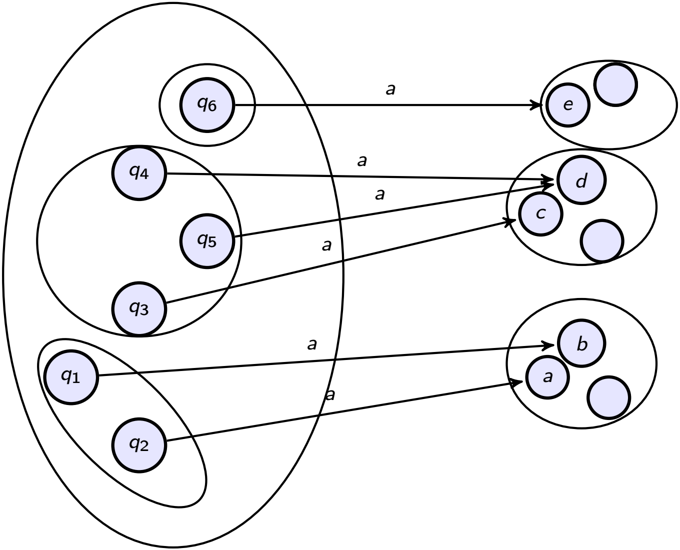
Canonicity: all DFA for the same language are transformed to the *same* DFA

Minimality: resulting DFA has *minimal* number of states

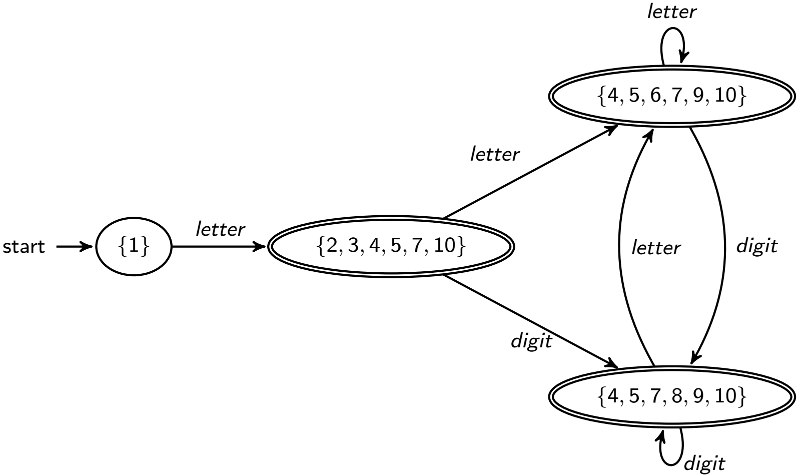
- "side effects": answers to *equivalence* problems
  - given 2 DFA: do they accept the same language?
  - given 2 regular expressions, do they describe the same language?
- modern version: [Hopcroft, 1971].

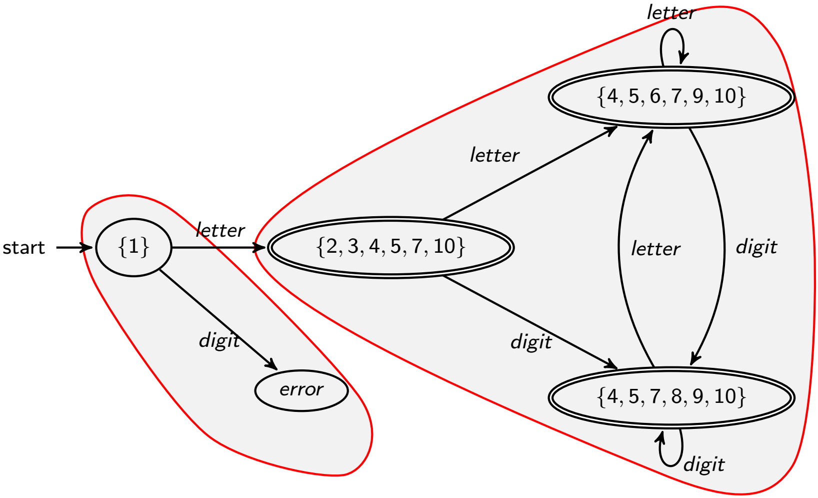
- starting point: *complete* DFA (i.e., *error*-state possibly needed)
- first idea: *equivalent* states in the given DFA may be *identified*
- equivalent: when used as starting point, accepting the same language
- partition refinement:
  - works "the other way around"
  - instead of collapsing equivalent states:
    - start by "collapsing as much as possible" and then,
    - iteratively, detect *non-equivalent* states, and then *split* a "collapsed" state
    - stop when no violations of "equivalence" are detected
- *partitioning* of a set (of states):
- *worklist*: data structure of to keep non-treated classes, termination if worklist is empty

- Initial partitioning: 2 partitions: set containing all *accepting* states F, set containing all *non-accepting* states $Q \backslash F$
- Loop do the following: pick a current equivalence class Q_i and a symbol a
  - if for all $q \in Q_i$, $\delta(q, a)$ is member of the *same* class $Q_j \Rightarrow$ consider Q_i as done (for now)
  - else:
    - split Q_i into $Q_i^1, \ldots Q_i^k$ s.t. the above situation is repaired for each Q_i^l (but don't split more than necessary).
    - be aware: a split may have a "cascading effect": other classes being fine before the split of Q_i need to be reconsidered $\Rightarrow$ *worklist* algo
- stop if the situation stabilizes, i.e., no more split happens (= worklist empty, at latest if back to the original DFA)



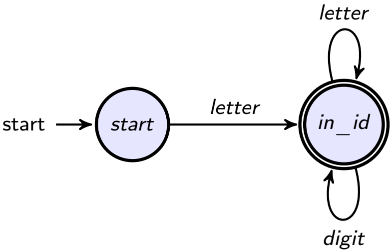
- before the split $\{q_1, q_2, \ldots, q_6\}$
- after the split on a: $\{q_1, q_2\}, \{q_3, q_4, q_5\}, \{q_6\}$


letter
{4, 5, 6, 7, 9, 10}
letter
start
{1}
letter
{2, 3, 4, 5, 7, 10}
letter
digit
digit
{4, 5, 7, 8, 9, 10}
digit

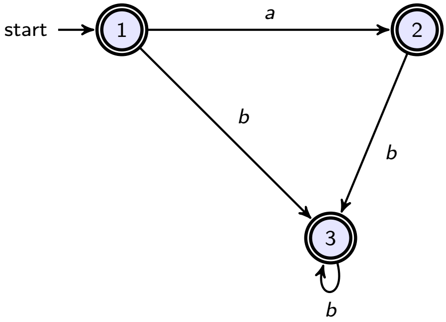


letter
start
$\{1\}$
letter
$\{2,3,4,5,7,10\}$
letter
$\{4,5,6,7,9,10\}$
letter
digit
digit
digit
error
digit
$\{4,5,7,8,9,10\}$
digit

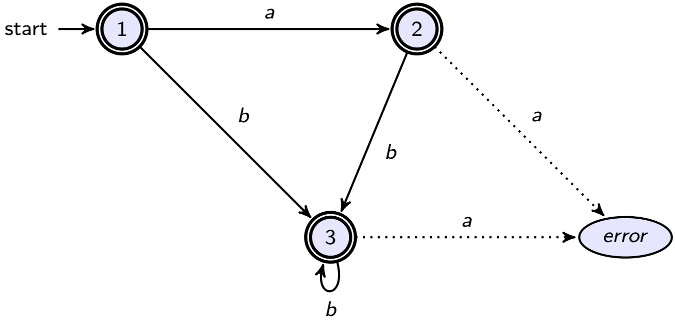
Minimized automaton (error state omitted)



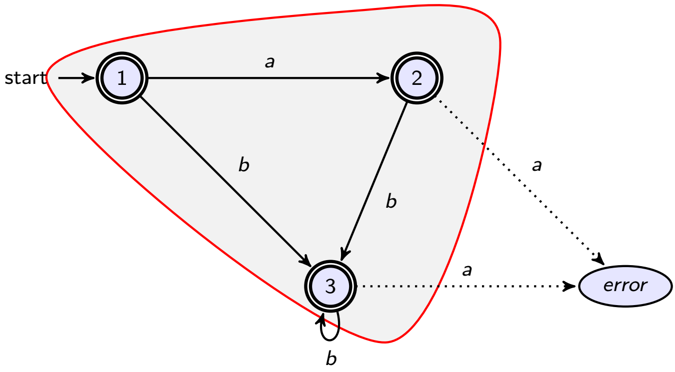
$$(a \mid \epsilon)b^* \tag{12}$$



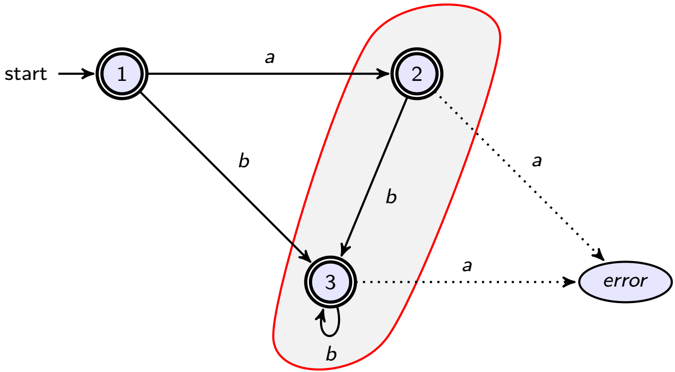
error state added



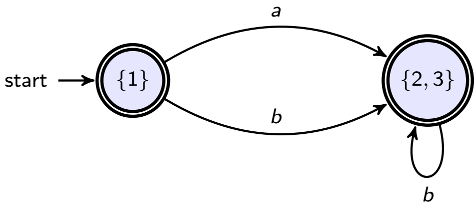
initial partitioning



split after a



End result (error state omitted again)



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Tools for generating scanners

- scanners: simple and well-understood part of compiler
- hand-coding possible
- mostly better off with: generated scanner
- standard tools lex / flex (also in combination with *parser* generators, like yacc / bison
- variants exist for many implementing languages
- based on the results of this section

Main idea of (f)lex and similar

- output of lexer/scanner = input for parser
- programmer specifies regular expressions for each token-class and corresponding actions[16] (and whitespace, comments etc.)
- the spec. language offers some conveniences (extended regexpr with priorities, associativities etc) to ease the task
- automatically translated to NFA (e.g. Thompson)
- then made into a deterministic DFA ("subset construction")
- minimized (with a little care to keep the token classes separate)
- implement the DFA (usually with the help of a *table* representation)

[16] Tokens and actions of a parser will be covered later. For example, identifiers and digits as described but the reg. expressions, would end up in two different token classes, where the actual string of characters (also known as *lexeme*) being the value of the token attribute.

[Hopcroft, 1971] Hopcroft, J. E. (1971).
An $n \log n$ algorithm for minimizing the states in a finite automaton.
In Kohavi, Z., editor, *The Theory of Machines and Computations*, pages 189–196. Academic Press, New York.

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Finite automata and their decision problems.
IBM Journal of Research Developments, 3:114–125.

[Thompson, 1968] Thompson, K. (1968).
Programming techniques: Regular expression search algorithm.
Communications of the ACM, 11(6):419.