Chapter 4

Parsing

Course “Compiler Construction”
Martin Steffen
Spring 2021

Section

Introduction to parsing

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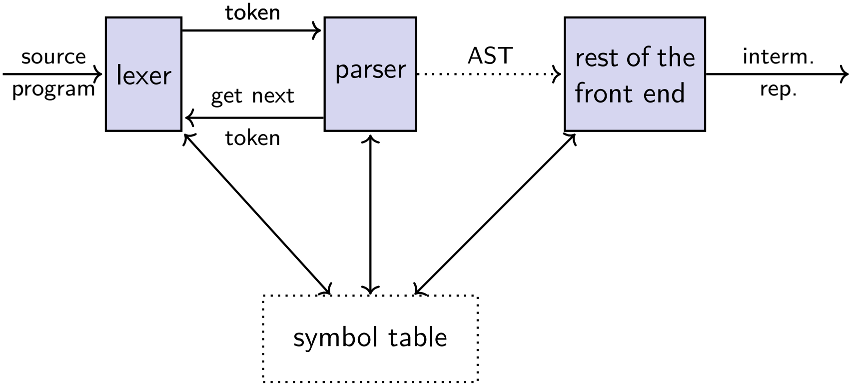
What's a parser generally doing

task of parser = syntax analysis

- input: stream of tokens from lexer
- output:
  - abstract syntax tree
  - or meaningful diagnosis of source of *syntax error*

- the full "power" (i.e., expressiveness) of CFGs not used
- thus:
  - consider *restrictions* of CFGs, i.e., a specific subclass, and/or
  - *represented* in specific ways (no left-recursion, left-factored . . .)


token
source
program
lexer
parser
AST
rest of the
front end
interm.
rep.
get next
token
symbol table

Top-down vs. bottom-up

- all parsers (together with lexers): *left-to-right*
- remember: parsers operate with *trees*
  - parse tree (concrete syntax tree): representing grammatical derivation
  - abstract syntax tree: data structure
- 2 fundamental classes
- while parser eats through the token stream, it grows, i.e., builds up (at least conceptually) the parse tree:

Bottom-up	**Top-down**
Parse tree is being grown from the leaves to the root.	Parse tree is being grown from the root to the leaves.

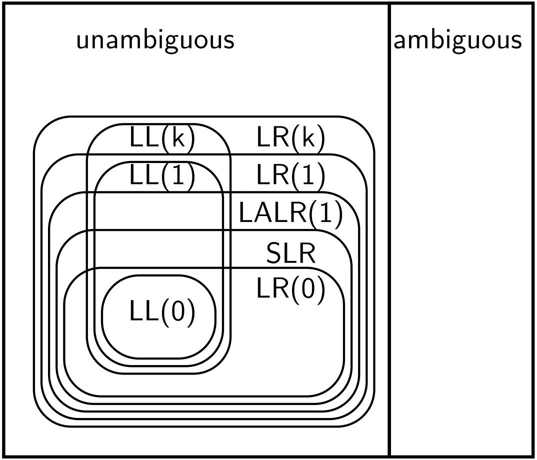
Parsing restricted classes of CFGs

- parser: better be "efficient"
- full complexity of CFLs: not really needed in practice
- classification of CF languages vs. CF grammars, e.g.:
    - left-recursion-freedom: condition on a grammar
    - ambiguous language vs. ambiguous grammar
- classification of grammars ⇒ classification of *languages*
    - a CF language is (inherently) ambiguous, if there's no unambiguous grammar for it
    - a CF language is top-down parseable, if there exists a grammar that allows top-down parsing . . .
- in practice: classification of parser generating tools:
    - based on accepted notation for grammars: (BNF or some form of EBNF etc.)

Classes of CFG grammars/languages

- *maaaany* have been proposed & studied, including their relationships
- lecture concentrates on
  - top-down parsing, in particular
    - LL(1)
    - recursive descent
  - bottom-up parsing
    - LR(1)
    - SLR
    - LALR(1) (the class covered by yacc-style tools)
- grammars typically written in *pure* BNF

Relationship of some grammar (not language) classes



taken from [1]

Section

Top-down parsing

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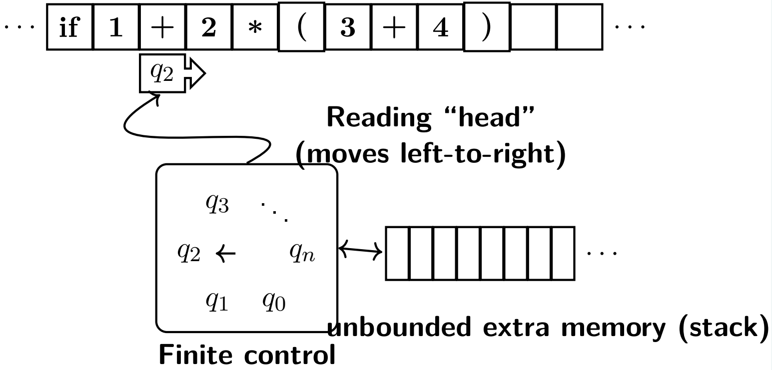
General task (once more)

- Given: a CFG (but appropriately restricted)
- Goal: “systematic method” s.t.
  1. for every given word w: check syntactic correctness
  2. [build AST/representation of the parse tree as side effect]
  3. [do reasonable error handling]

Schematic view on “parser machine”



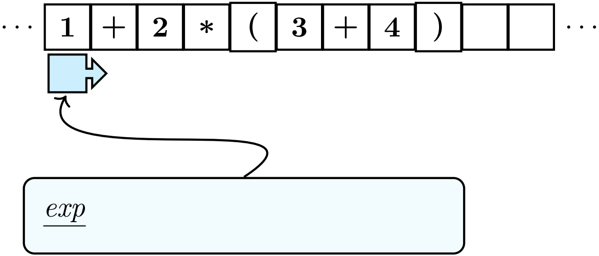
Reading “head” (moves left-to-right)

unbounded extra memory (stack)

Finite control

Note: sequence of *tokens* (not characters)

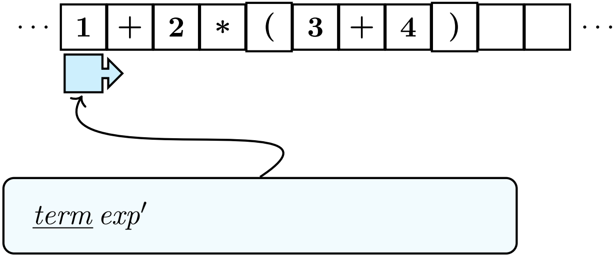
Derivation of an expression



factors and terms

$$
\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \rightarrow & + \ \mid\ - \\
term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\ exp\ ) \ \mid\ \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

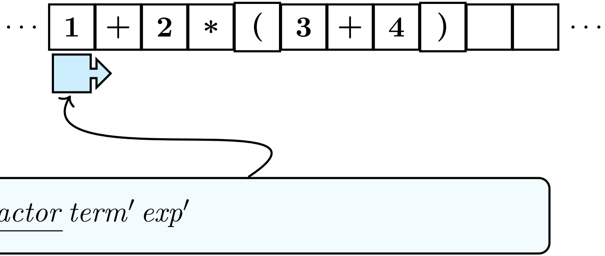


$\underline{term}\ exp'$

factors and terms

$$\begin{aligned} exp &\rightarrow term\ exp' \\ exp' &\rightarrow addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\ addop &\rightarrow \mathbf{+} \ \mid\ \mathbf{-} \\ term &\rightarrow factor\ term' \\ term' &\rightarrow mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\ mulop &\rightarrow \mathbf{*} \\ factor &\rightarrow \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} \end{aligned} \tag{1}$$

Derivation of an expression



$\underline{factor}\ term'\ exp'$

factors and terms

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\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \rightarrow & + \ \mid\ - \\
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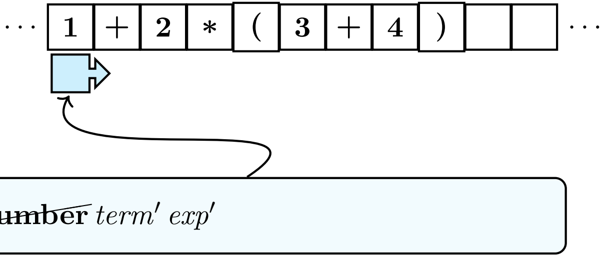
Derivation of an expression



~~**number**~~ *term′ exp′*

factors and terms

$$
\begin{array}{rcl}
exp & \to & term\ exp' \\
exp' & \to & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \to & \mathbf{+} \ \mid\ \mathbf{-} \\
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mulop & \to & \mathbf{*} \\
factor & \to & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n}
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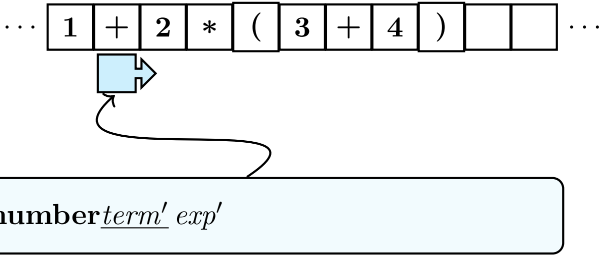
Derivation of an expression

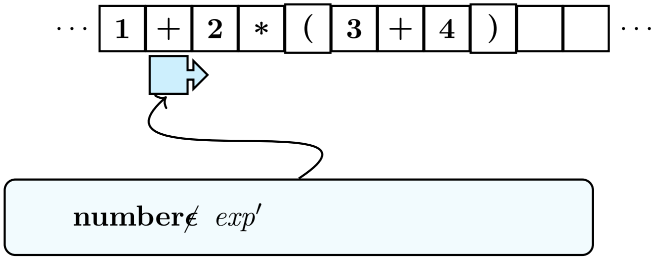


$\mathbf{number}\,\underline{term'}\;exp'$

factors and terms

$$
\begin{array}{rcl}
exp & \rightarrow & term\;exp' \\
exp' & \rightarrow & addop\;term\;exp' \;\mid\; \boldsymbol{\epsilon} \\
addop & \rightarrow & \mathbf{+} \;\mid\; \mathbf{-} \\
term & \rightarrow & factor\;term' \\
term' & \rightarrow & mulop\;factor\;term' \;\mid\; \boldsymbol{\epsilon} \\
mulop & \rightarrow & \mathbf{*} \\
factor & \rightarrow & \mathbf{(}\;exp\;\mathbf{)} \;\mid\; \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression



$\mathbf{number}\cancel{\epsilon}\;\;exp'$

factors and terms

$$\begin{aligned}
exp &\to term\ exp' \\
exp' &\to addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop &\to \mathbf{+} \ \mid\ \mathbf{-} \\
term &\to factor\ term' \\
term' &\to mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\
mulop &\to \mathbf{*} \\
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\end{aligned} \tag{1}$$

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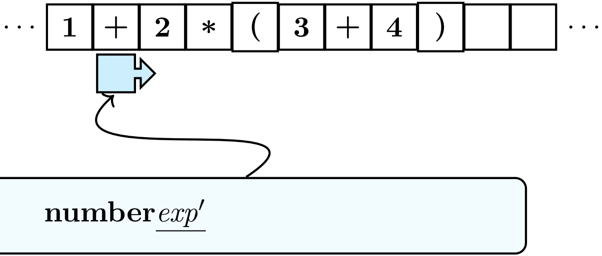

$\mathbf{number}\,\underline{exp'}$

factors and terms

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\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\
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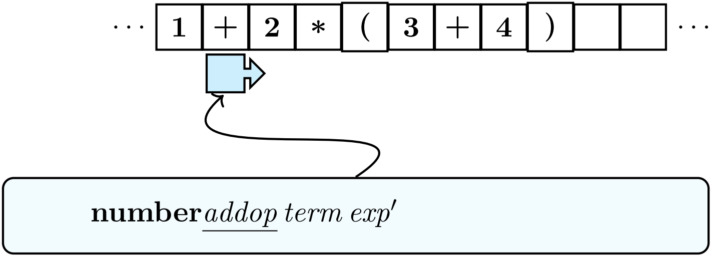
mulop & \rightarrow & \mathbf{*} \\
factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

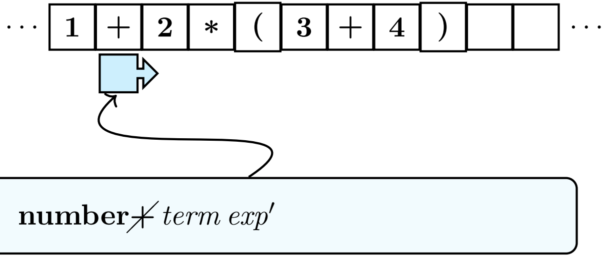


$\mathbf{number}\,\underline{addop}\; term\; exp'$

factors and terms

$$
\begin{aligned}
exp &\rightarrow term\; exp' \\
exp' &\rightarrow addop\; term\; exp' \mid \boldsymbol{\epsilon} \\
addop &\rightarrow \mathbf{+} \mid \mathbf{-} \\
term &\rightarrow factor\; term' \\
term' &\rightarrow mulop\; factor\; term' \mid \boldsymbol{\epsilon} \\
mulop &\rightarrow \mathbf{*} \\
factor &\rightarrow \mathbf{(}\; exp\; \mathbf{)} \mid \mathbf{n}
\end{aligned}
\tag{1}
$$

Derivation of an expression

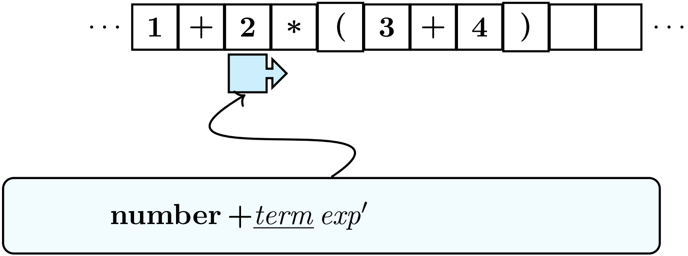


number $\cancel{+}$ *term exp′*

factors and terms

$$
\begin{array}{rcl}
\textit{exp} & \rightarrow & \textit{term}\ \textit{exp}' \\
\textit{exp}' & \rightarrow & \textit{addop}\ \textit{term}\ \textit{exp}' \mid \boldsymbol{\epsilon} \\
\textit{addop} & \rightarrow & \mathbf{+} \mid \mathbf{-} \\
\textit{term} & \rightarrow & \textit{factor}\ \textit{term}' \\
\textit{term}' & \rightarrow & \textit{mulop}\ \textit{factor}\ \textit{term}' \mid \boldsymbol{\epsilon} \\
\textit{mulop} & \rightarrow & \mathbf{*} \\
\textit{factor} & \rightarrow & \mathbf{(}\ \textit{exp}\ \mathbf{)} \mid \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression



$\mathbf{number} + \underline{term}\ exp'$

factors and terms

$$
\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \mid \boldsymbol{\epsilon} \\
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term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \mid \boldsymbol{\epsilon} \\
mulop & \rightarrow & \mathbf{*} \\
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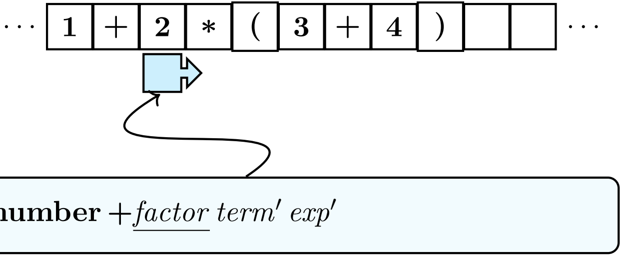
Derivation of an expression



$\mathbf{number} + \underline{factor}\ term'\ exp'$

factors and terms

$$
\begin{array}{rcl}
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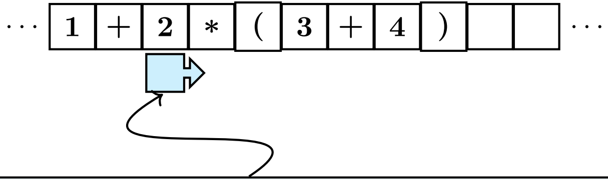
Derivation of an expression



number + ~~**number**~~ $\mathit{term}'\ \mathit{exp}'$

factors and terms

$$
\begin{array}{rcl}
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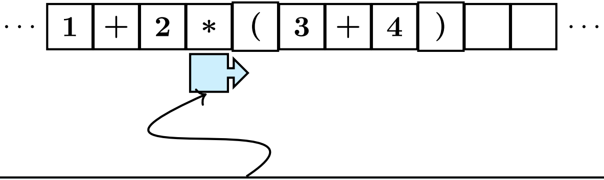
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array} \tag{1}
$$

Derivation of an expression

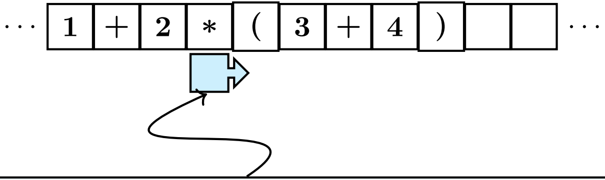


$$\mathbf{number} + \mathbf{number}\,\underline{term'}\ exp'$$

factors and terms

$$\begin{array}{rcl} exp & \to & term\ exp' \\ exp' & \to & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\ addop & \to & \mathbf{+} \ \mid\ \mathbf{-} \\ term & \to & factor\ term' \\ term' & \to & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\ mulop & \to & \mathbf{*} \\ factor & \to & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

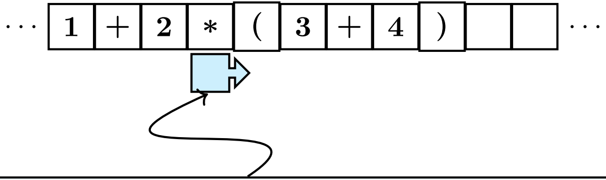


$$\mathbf{number} + \mathbf{number}\, \underline{mulop}\, factor\ term'\ exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\ exp' \\ exp' & \rightarrow & addop\ term\ exp' \mid \boldsymbol{\epsilon} \\ addop & \rightarrow & + \mid - \\ term & \rightarrow & factor\ term' \\ term' & \rightarrow & mulop\ factor\ term' \mid \boldsymbol{\epsilon} \\ mulop & \rightarrow & * \\ factor & \rightarrow & (\ exp\ ) \mid \mathbf{n} \end{array} \qquad (1)$$

Derivation of an expression

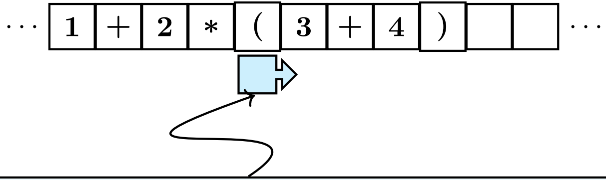


$$\mathbf{number} + \mathbf{number} \cancel{*}\ \underline{factor}\ term'\ exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\ exp' \\ exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\ addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\ term & \rightarrow & factor\ term' \\ term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\ mulop & \rightarrow & \mathbf{*} \\ factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

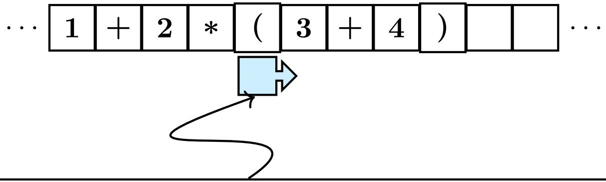


$$\textbf{number} + \textbf{number} * (\ \mathit{exp}\ )\ \mathit{term}'\ \mathit{exp}'$$

factors and terms

$$
\begin{array}{rcl}
\mathit{exp} & \to & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \to & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \mid \boldsymbol{\epsilon} \\
\mathit{addop} & \to & + \mid - \\
\mathit{term} & \to & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \to & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \mid \boldsymbol{\epsilon} \\
\mathit{mulop} & \to & * \\
\mathit{factor} & \to & (\ \mathit{exp}\ ) \mid \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression



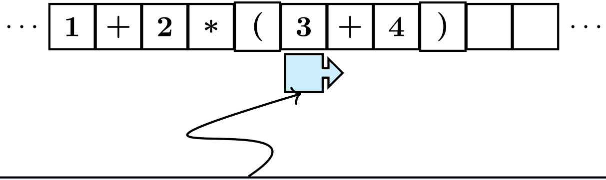
$\cdots$ **1** **+** **2** ***** **(** **3** **+** **4** **)** $\cdots$

$$\textbf{number} + \textbf{number} * \cancel{(}\; exp\;)\; term'\; exp'$$

factors and terms

$$\begin{array}{rcl}
exp & \rightarrow & term\; exp' \\
exp' & \rightarrow & addop\; term\; exp' \;\mid\; \boldsymbol{\epsilon} \\
addop & \rightarrow & + \;\mid\; - \\
term & \rightarrow & factor\; term' \\
term' & \rightarrow & mulop\; factor\; term' \;\mid\; \boldsymbol{\epsilon} \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\; exp\;) \;\mid\; \mathbf{n}
\end{array} \tag{1}$$

Derivation of an expression

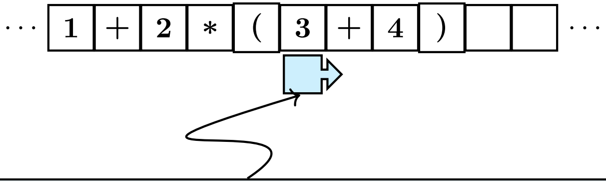


$$\mathbf{number} + \mathbf{number} * (\underline{exp}) \; term' \; exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\; exp' \\ exp' & \rightarrow & addop\; term\; exp' \;\mid\; \boldsymbol{\epsilon} \\ addop & \rightarrow & + \;\mid\; - \\ term & \rightarrow & factor\; term' \\ term' & \rightarrow & mulop\; factor\; term' \;\mid\; \boldsymbol{\epsilon} \\ mulop & \rightarrow & * \\ factor & \rightarrow & (\; exp\;) \;\mid\; \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

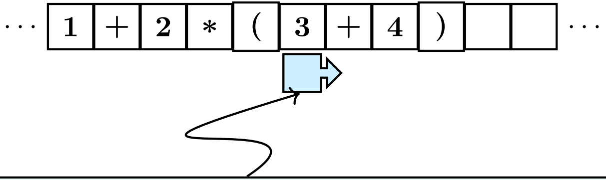


$$\mathbf{number} + \mathbf{number} * (\, \underline{term}\ exp'\,)\ term'\ exp'$$

factors and terms

$$\begin{aligned} exp &\rightarrow term\ exp' \\ exp' &\rightarrow addop\ term\ exp' \mid \boldsymbol{\epsilon} \\ addop &\rightarrow + \mid - \\ term &\rightarrow factor\ term' \\ term' &\rightarrow mulop\ factor\ term' \mid \boldsymbol{\epsilon} \\ mulop &\rightarrow * \\ factor &\rightarrow (\, exp\,) \mid \mathbf{n} \end{aligned} \tag{1}$$

Derivation of an expression

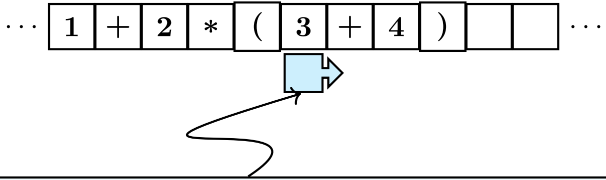


$$\mathbf{number} + \mathbf{number} * (\, \underline{factor}\ term'\ exp'\,)\ term'\ exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\ exp' \\ exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\ addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\ term & \rightarrow & factor\ term' \\ term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\ mulop & \rightarrow & \mathbf{*} \\ factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

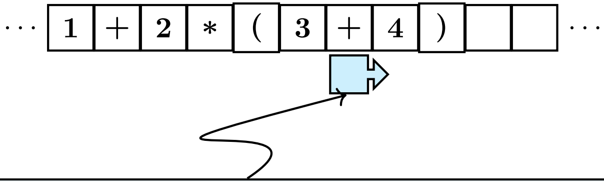


$$\mathbf{number} + \mathbf{number} * (\, \cancel{\mathbf{number}}\; term'\; exp'\,)\; term'\; exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\; exp' \\ exp' & \rightarrow & addop\; term\; exp' \;\mid\; \boldsymbol{\epsilon} \\ addop & \rightarrow & \mathbf{+} \;\mid\; \mathbf{-} \\ term & \rightarrow & factor\; term' \\ term' & \rightarrow & mulop\; factor\; term' \;\mid\; \boldsymbol{\epsilon} \\ mulop & \rightarrow & \mathbf{*} \\ factor & \rightarrow & (\, exp\,) \;\mid\; \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression



$$\textbf{number} + \textbf{number} * (\textbf{number} \, \underline{term'} \; exp' \,) \; term' \; exp'$$

factors and terms

$$\begin{array}{rcl}
exp & \rightarrow & term \; exp' \\
exp' & \rightarrow & addop \; term \; exp' \;\mid\; \boldsymbol{\epsilon} \\
addop & \rightarrow & + \;\mid\; - \\
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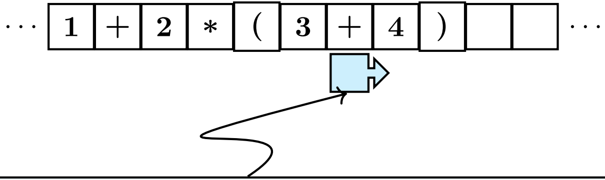
mulop & \rightarrow & * \\
factor & \rightarrow & (\, exp \,) \;\mid\; \mathbf{n}
\end{array} \tag{1}$$

Derivation of an expression

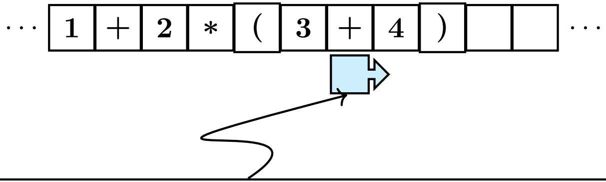


$$\mathbf{number} + \mathbf{number} * (\mathbf{number} \cancel{\epsilon}\ \mathit{exp}')\ \mathit{term}'\ \mathit{exp}'$$

factors and terms

$$\begin{array}{rcl} \mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\ \mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \mid \boldsymbol{\epsilon} \\ \mathit{addop} & \rightarrow & + \mid - \\ \mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\ \mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \mid \boldsymbol{\epsilon} \\ \mathit{mulop} & \rightarrow & * \\ \mathit{factor} & \rightarrow & (\ \mathit{exp}\ ) \mid \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

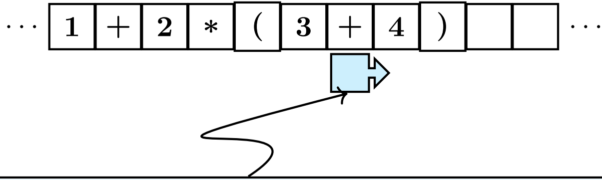


$$\mathbf{number} + \mathbf{number} * (\,\mathbf{number}\,\underline{exp'}\,)\; term'\; exp'$$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\; exp' \\ exp' & \rightarrow & addop\; term\; exp' \;\mid\; \boldsymbol{\epsilon} \\ addop & \rightarrow & \mathbf{+} \;\mid\; \mathbf{-} \\ term & \rightarrow & factor\; term' \\ term' & \rightarrow & mulop\; factor\; term' \;\mid\; \boldsymbol{\epsilon} \\ mulop & \rightarrow & \mathbf{*} \\ factor & \rightarrow & \mathbf{(}\; exp\; \mathbf{)} \;\mid\; \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

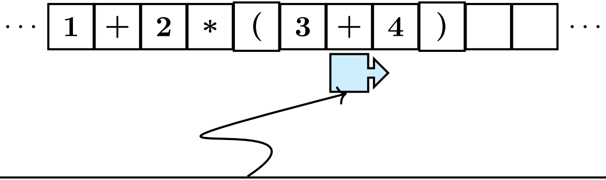


$$\mathbf{number} + \mathbf{number} * (\, \mathbf{number}\, \underline{\mathit{addop}}\ \mathit{term}\ \mathit{exp}' \,)\ \mathit{ter}$$

factors and terms

$$\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array} \tag{1}$$

Derivation of an expression

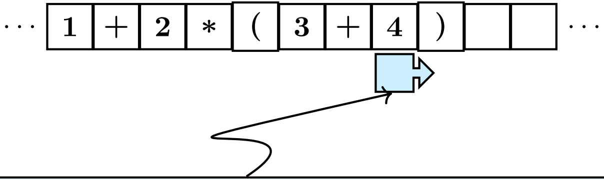


$\mathbf{number} + \mathbf{number} * (\mathbf{number} \cancel{+} \, \mathit{term} \; \mathit{exp}' \,) \; \mathit{term}' \; e$

factors and terms

$$
\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\; \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\; \mathit{term}\; \mathit{exp}' \;\mid\; \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \;\mid\; \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\; \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\; \mathit{factor}\; \mathit{term}' \;\mid\; \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\; \mathit{exp}\; \mathbf{)} \;\mid\; \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

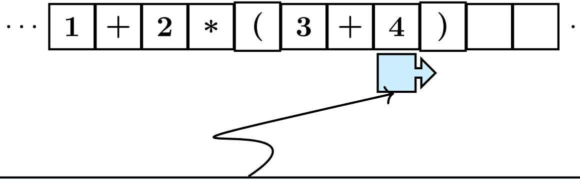


$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \underline{term}\ exp' \,)\ term'$

factors and terms

$$
\begin{aligned}
exp &\rightarrow term\ exp' \qquad (1)\\
exp' &\rightarrow addop\ term\ exp' \mid \boldsymbol{\epsilon}\\
addop &\rightarrow \mathbf{+} \mid \mathbf{-}\\
term &\rightarrow factor\ term'\\
term' &\rightarrow mulop\ factor\ term' \mid \boldsymbol{\epsilon}\\
mulop &\rightarrow \mathbf{*}\\
factor &\rightarrow \mathbf{(}\ exp\ \mathbf{)} \mid \mathbf{n}
\end{aligned}
$$

Derivation of an expression

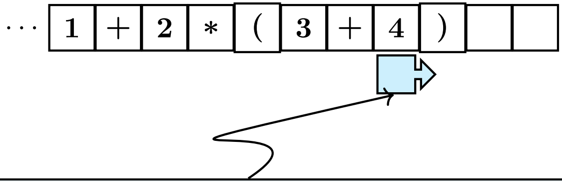


$$\textbf{number} + \textbf{number} * (\, \textbf{number} + \underline{\mathit{factor}}\ \mathit{term}'\ \mathit{exp}'$$

factors and terms

$$\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array} \tag{1}$$

Derivation of an expression

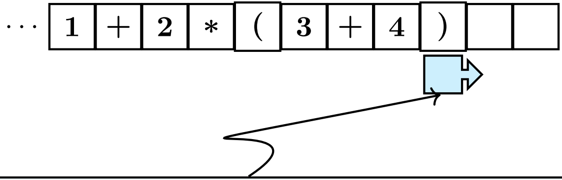


$$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \cancel{\mathbf{number}}\; \mathit{term}'$$

factors and terms

$$\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \mid \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \mid \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \mid \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \boldsymbol{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \mid \mathbf{n}
\end{array} \tag{1}$$

Derivation of an expression

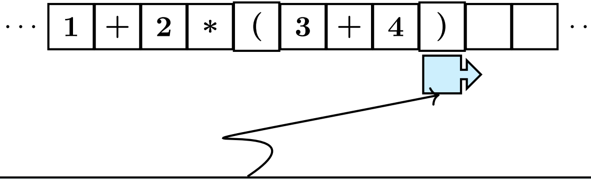


$\mathbf{number} + \mathbf{number} * (\mathbf{number} + \mathbf{number}\, \underline{\mathit{term}'}$

factors and terms

$$\begin{aligned} \mathit{exp} &\rightarrow \mathit{term}\ \mathit{exp}' \\ \mathit{exp}' &\rightarrow \mathit{addop}\ \mathit{term}\ \mathit{exp}' \mid \boldsymbol{\epsilon} \\ \mathit{addop} &\rightarrow + \mid - \\ \mathit{term} &\rightarrow \mathit{factor}\ \mathit{term}' \\ \mathit{term}' &\rightarrow \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \mid \boldsymbol{\epsilon} \\ \mathit{mulop} &\rightarrow \boldsymbol{*} \\ \mathit{factor} &\rightarrow \mathbf{(}\ \mathit{exp}\ \mathbf{)} \mid \mathbf{n} \end{aligned} \tag{1}$$

Derivation of an expression

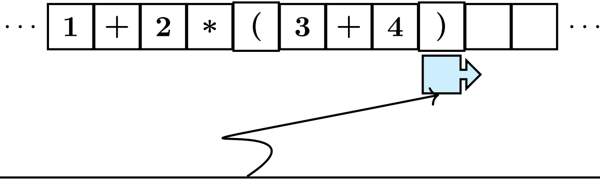


$\mathbf{number} + \mathbf{number} * (\mathbf{number} + \mathbf{number} \not\epsilon \; exp$

factors and terms

$$
\begin{aligned}
exp &\rightarrow term\; exp' \\
exp' &\rightarrow addop\; term\; exp' \;\mid\; \boldsymbol{\epsilon} \\
addop &\rightarrow \mathbf{+} \;\mid\; \mathbf{-} \\
term &\rightarrow factor\; term' \\
term' &\rightarrow mulop\; factor\; term' \;\mid\; \boldsymbol{\epsilon} \\
mulop &\rightarrow \mathbf{*} \\
factor &\rightarrow \mathbf{(}\; exp \;\mathbf{)} \;\mid\; \mathbf{n}
\end{aligned} \tag{1}
$$

Derivation of an expression

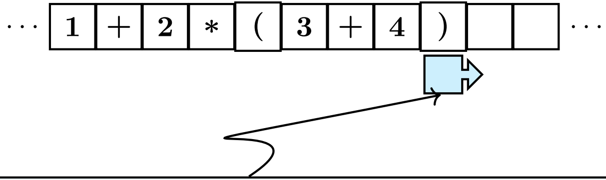


$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \mathbf{number}\, \underline{exp'}$

factors and terms

$$
\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \mid \boldsymbol{\epsilon} \\
addop & \rightarrow & \mathbf{+} \mid \mathbf{-} \\
term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \mid \boldsymbol{\epsilon} \\
mulop & \rightarrow & \mathbf{*} \\
factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \mid \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

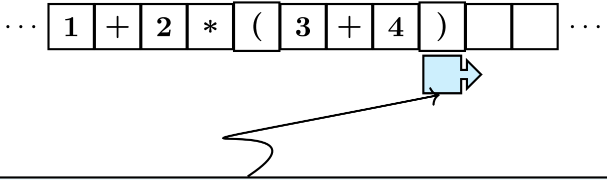


$$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \mathbf{number}\cancel{\boldsymbol{\epsilon}}\,)$$

factors and terms

$$
\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\
term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\
mulop & \rightarrow & \mathbf{*} \\
factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

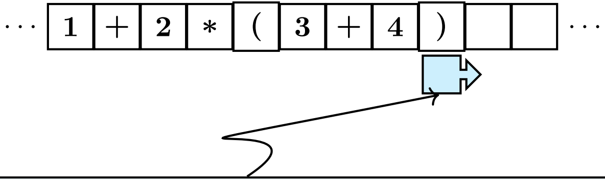


$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \mathbf{number} \cancel{)}\ t$

factors and terms

$$\begin{array}{rcl} exp & \rightarrow & term\ exp' \\ exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\ addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\ term & \rightarrow & factor\ term' \\ term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\ mulop & \rightarrow & \mathbf{*} \\ factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} \end{array} \tag{1}$$

Derivation of an expression

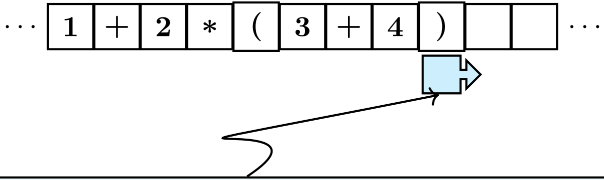


$$\mathbf{number} + \mathbf{number} * (\ \mathbf{number} + \mathbf{number}\ )$$

factors and terms

$$\begin{aligned}
\mathit{exp} &\rightarrow \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' &\rightarrow \mathit{addop}\ \mathit{term}\ \mathit{exp}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{addop} &\rightarrow \mathbf{+} \ \mid\ \mathbf{-} \\
\mathit{term} &\rightarrow \mathit{factor}\ \mathit{term}' \\
\mathit{term}' &\rightarrow \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} &\rightarrow \mathbf{*} \\
\mathit{factor} &\rightarrow \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{aligned} \tag{1}$$

Derivation of an expression

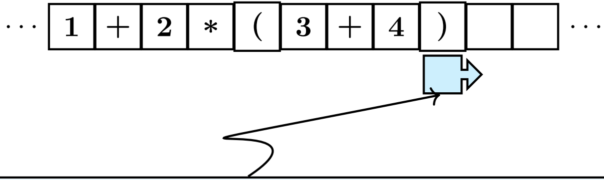


$\mathbf{number} + \mathbf{number} * (\, \mathbf{number} + \mathbf{number} \,)$

factors and terms

$$
\begin{array}{rcl}
\mathit{exp} & \to & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \to & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{addop} & \to & \mathbf{+} \ \mid\ \mathbf{-} \\
\mathit{term} & \to & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \to & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} & \to & \mathbf{*} \\
\mathit{factor} & \to & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

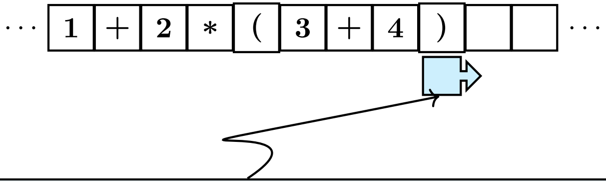


$\mathbf{number} + \mathbf{number} * (\mathbf{number} + \mathbf{number}$

factors and terms

$$
\begin{array}{rcl}
exp & \to & term\ exp' \\
exp' & \to & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} \\
addop & \to & + \ \mid\ - \\
term & \to & factor\ term' \\
term' & \to & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} \\
mulop & \to & * \\
factor & \to & (\ exp\ ) \ \mid\ \mathbf{n}
\end{array}
\tag{1}
$$

Derivation of an expression

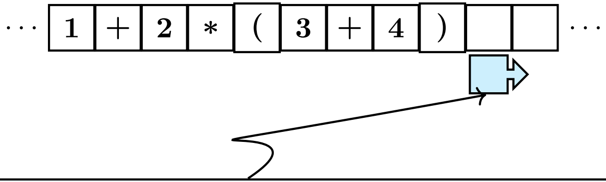


number + **number** * (**number** + **numbe**

factors and terms

$$
\begin{array}{rcll}
exp & \rightarrow & term\ exp' & (1)\\
exp' & \rightarrow & addop\ term\ exp' \ \mid\ \boldsymbol{\epsilon} & \\
addop & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} & \\
term & \rightarrow & factor\ term' & \\
term' & \rightarrow & mulop\ factor\ term' \ \mid\ \boldsymbol{\epsilon} & \\
mulop & \rightarrow & \mathbf{*} & \\
factor & \rightarrow & \mathbf{(}\ exp\ \mathbf{)} \ \mid\ \mathbf{n} &
\end{array}
$$

Derivation of an expression



number +number * (number + numbe

factors and terms

$$
\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\;\mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\;\mathit{term}\;\mathit{exp}' \;\mid\; \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \;\mid\; \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\;\mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\;\mathit{factor}\;\mathit{term}' \;\mid\; \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\;\mathit{exp}\;\mathbf{)} \;\mid\; \mathbf{n}
\end{array}
\tag{1}
$$

Remarks concerning the derivation

Note:

- input = stream of tokens
- there: $\mathbf{1}\dots$ stands for token class $\mathbf{number}$ (for readability/concreteness), in the grammar: just $\mathbf{number}$
- in full detail: pair of token class and token value $\langle \mathbf{number}, 1 \rangle$

Notation:

- underline: the *place* (occurrence of *non-terminal* where production is used)
- ~~*crossed out*~~:
  - *terminal* = *token* is considered treated
  - parser “moves on”
  - later implemented as `match` or `eat` procedure

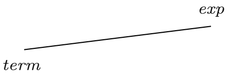
Not as a "film" but at a glance: reduction sequence

$$
\begin{array}{ll}
\underline{\mathit{exp}} & \Rightarrow \\
\underline{\mathit{term}}\ \mathit{exp}' & \Rightarrow \\
\underline{\mathit{factor}}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\cancel{\mathbf{number}}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number}\ \underline{\mathit{term}'}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number}\ \cancel{\epsilon}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number}\ \underline{\mathit{exp}'} & \Rightarrow \\
\mathbf{number}\ \underline{\mathit{addop}}\ \mathit{term}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number}\ \cancel{+}\ \mathit{term}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \underline{\mathit{term}}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \underline{\mathit{factor}}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \cancel{\mathbf{number}}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number}\ \underline{\mathit{term}'}\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number}\ \underline{\mathit{mulop}}\ \mathit{factor}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number}\ \cancel{*}\ \underline{\mathit{factor}}\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number} * (\ \mathit{exp}\ )\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number} * \cancel{(}\ \mathit{exp}\ )\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\mathbf{number} + \mathbf{number} * (\ \underline{\mathit{exp}}\ )\ \mathit{term}'\ \mathit{exp}' & \Rightarrow \\
\ldots &
\end{array}
$$

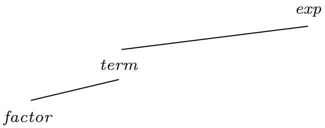
Best viewed as a tree

exp

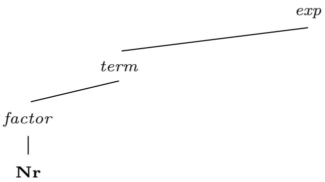
Best viewed as a tree



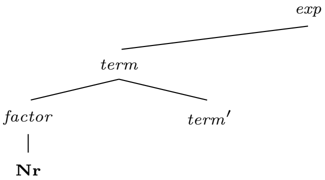
Best viewed as a tree



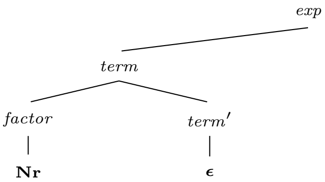
Best viewed as a tree



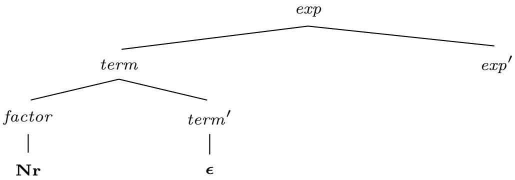
Best viewed as a tree



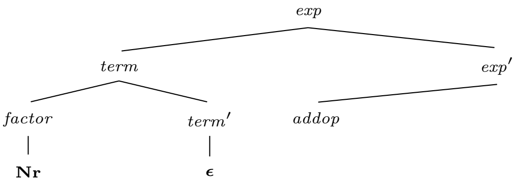
Best viewed as a tree



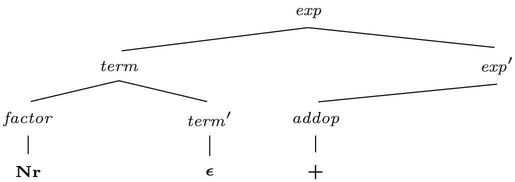
Best viewed as a tree



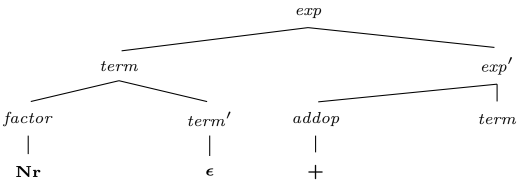
Best viewed as a tree



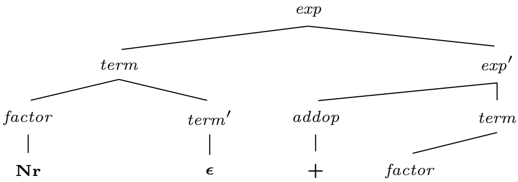
Best viewed as a tree



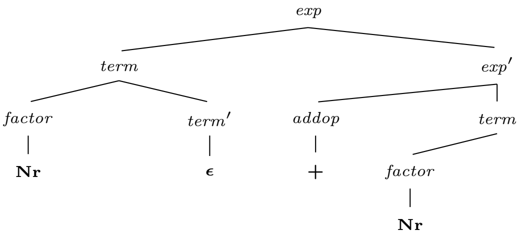
Best viewed as a tree



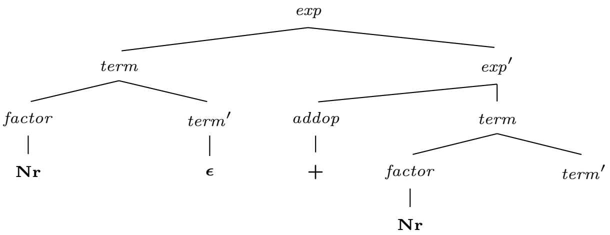
Best viewed as a tree



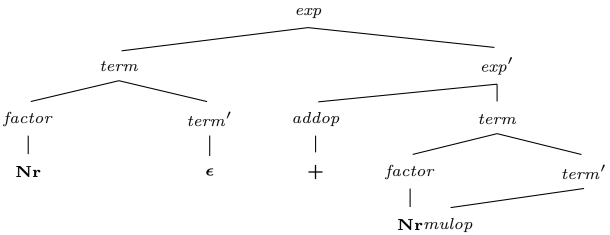
Best viewed as a tree



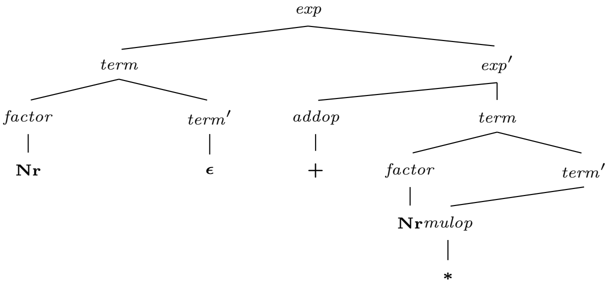
Best viewed as a tree



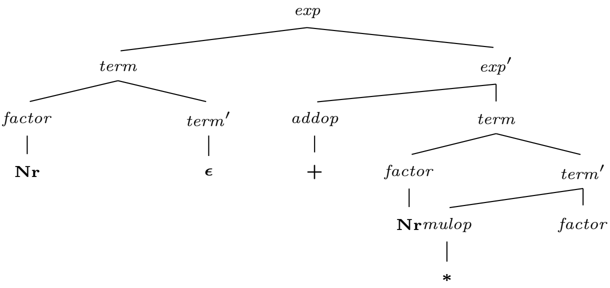
Best viewed as a tree



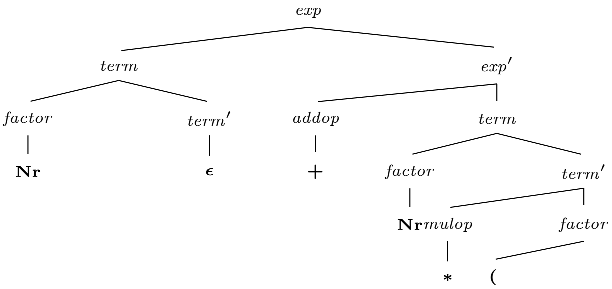
Best viewed as a tree



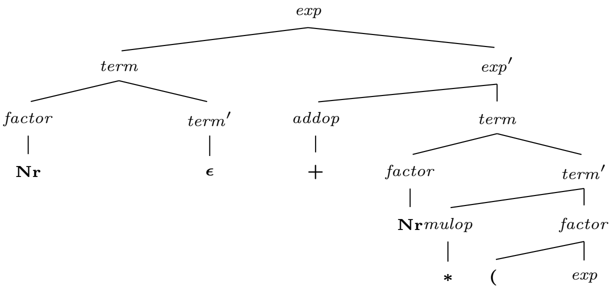
Best viewed as a tree



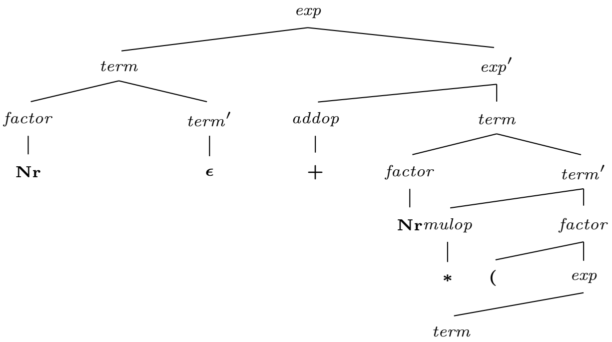
Best viewed as a tree



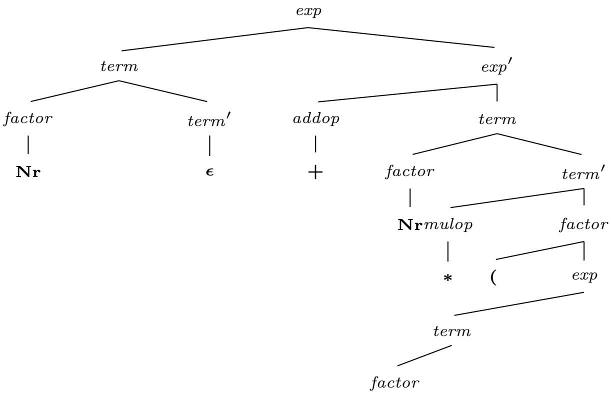
Best viewed as a tree



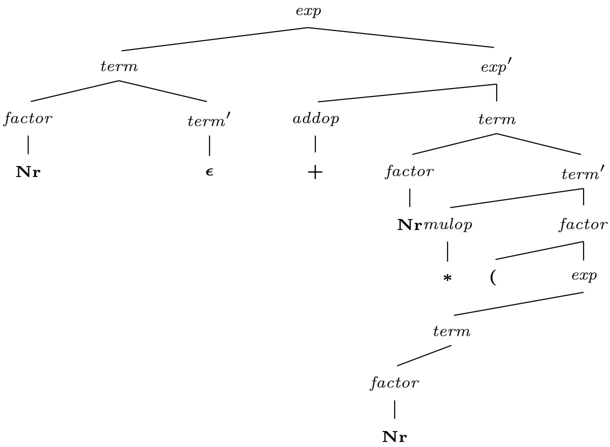
Best viewed as a tree



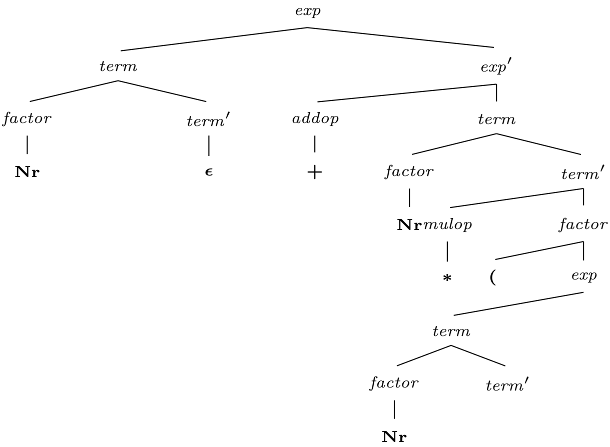
Best viewed as a tree



Best viewed as a tree



Best viewed as a tree



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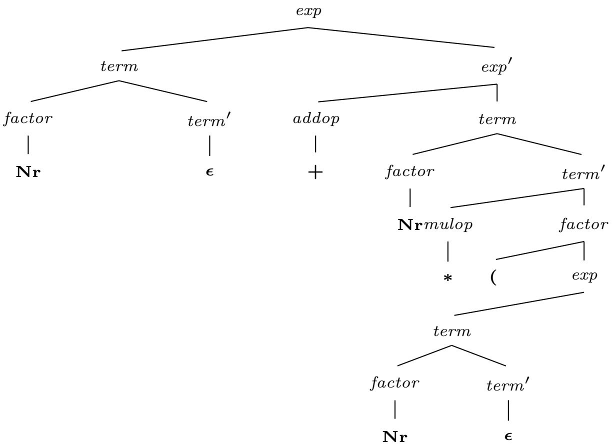

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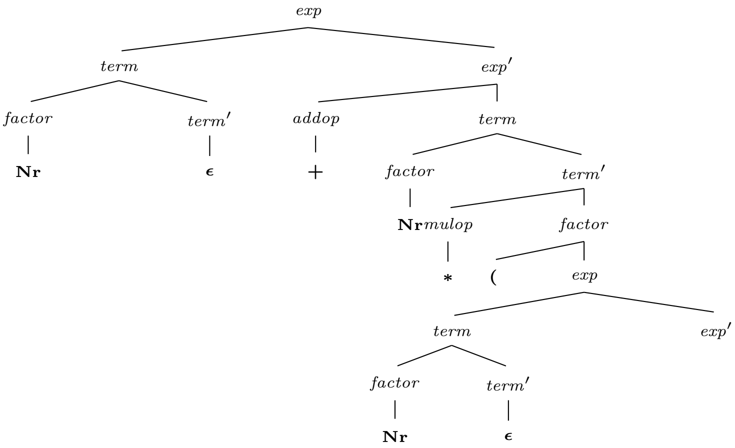

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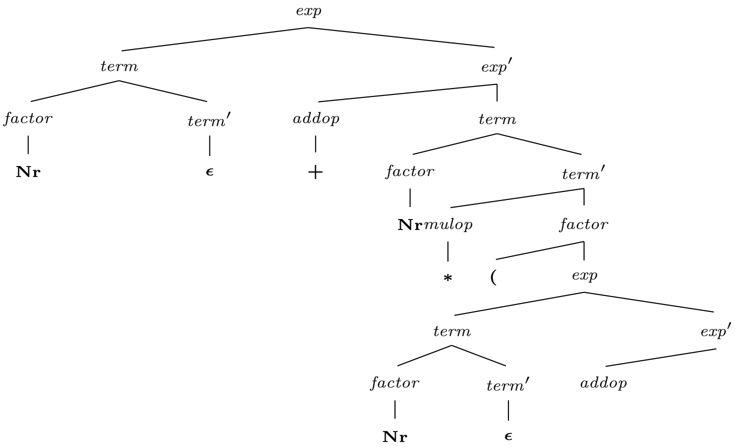

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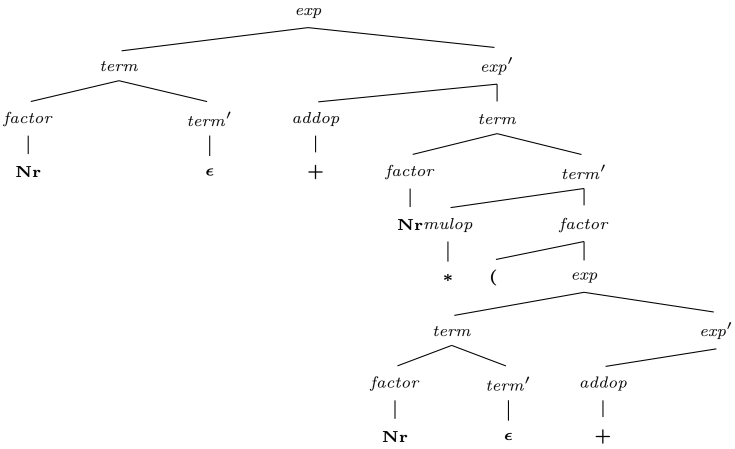

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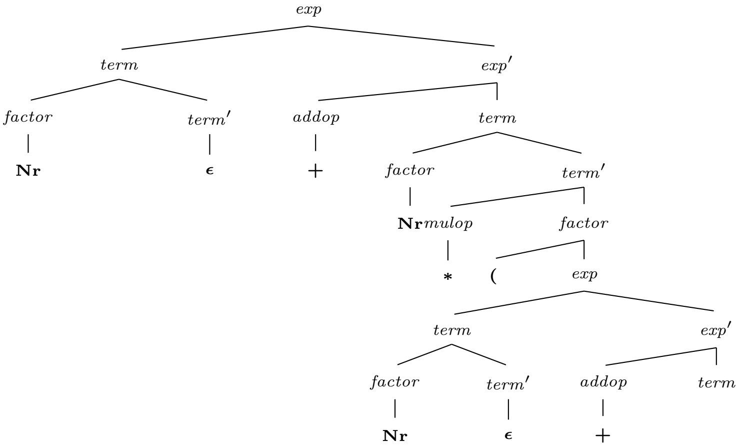

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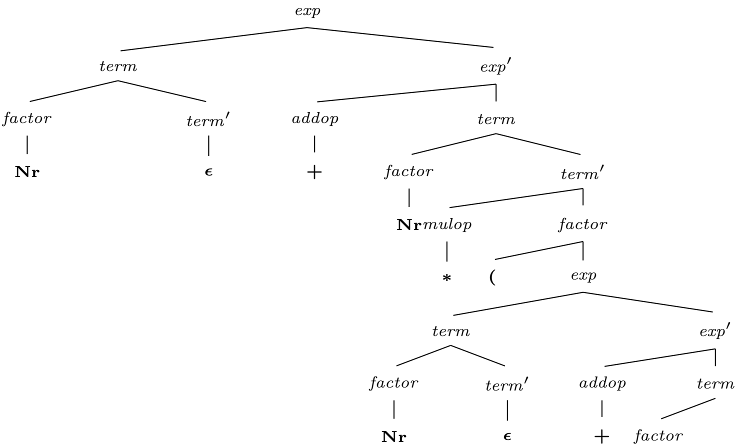

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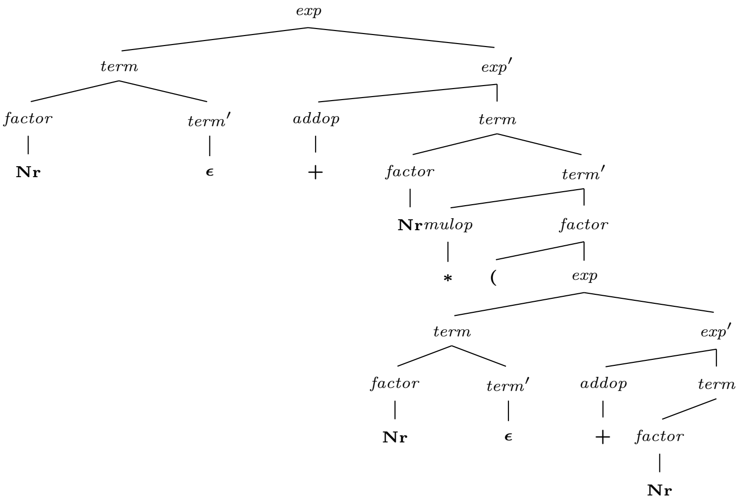

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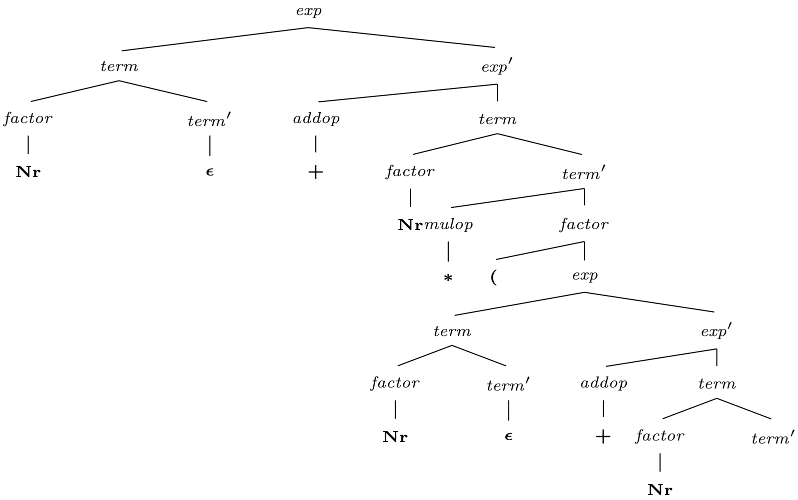

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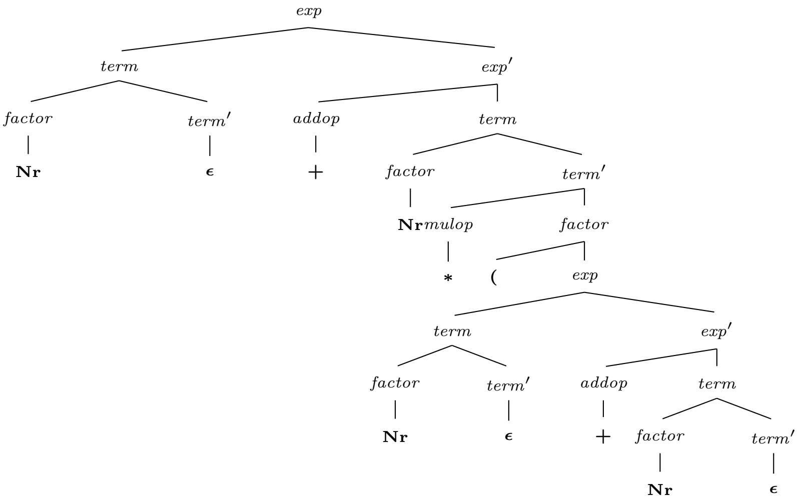

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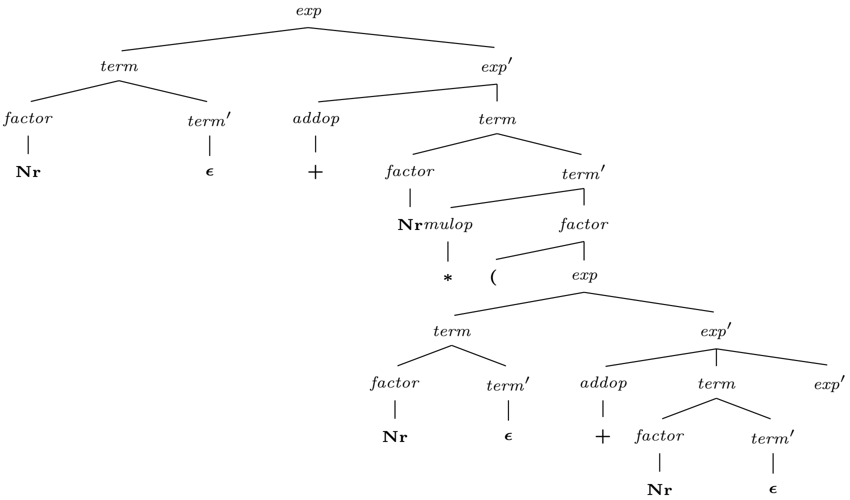

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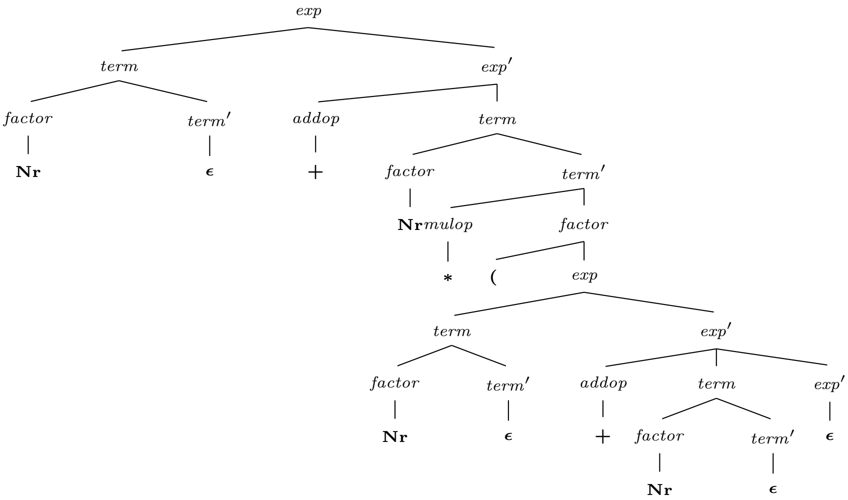

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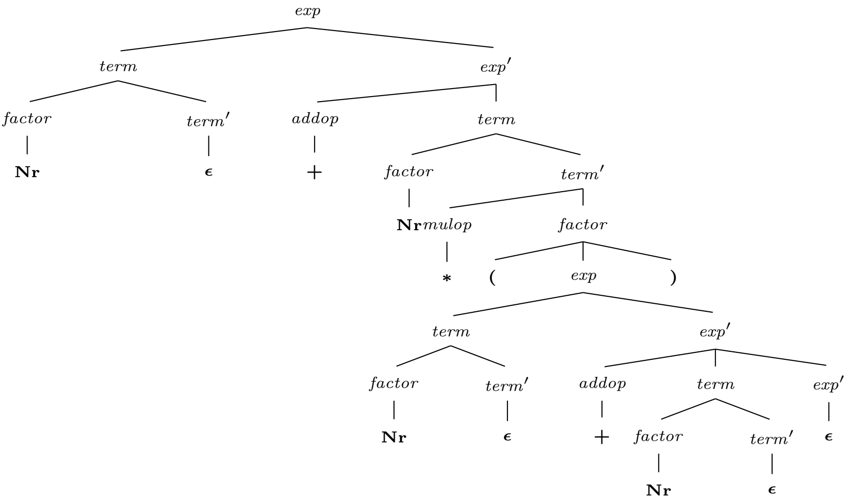

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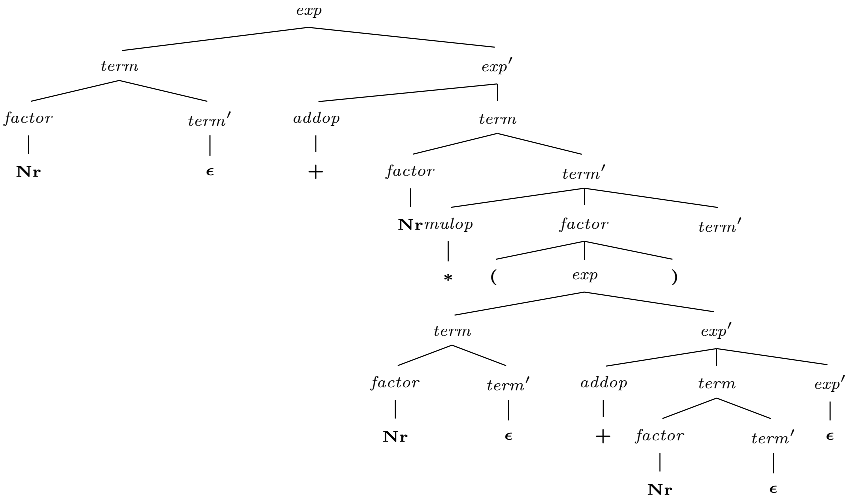

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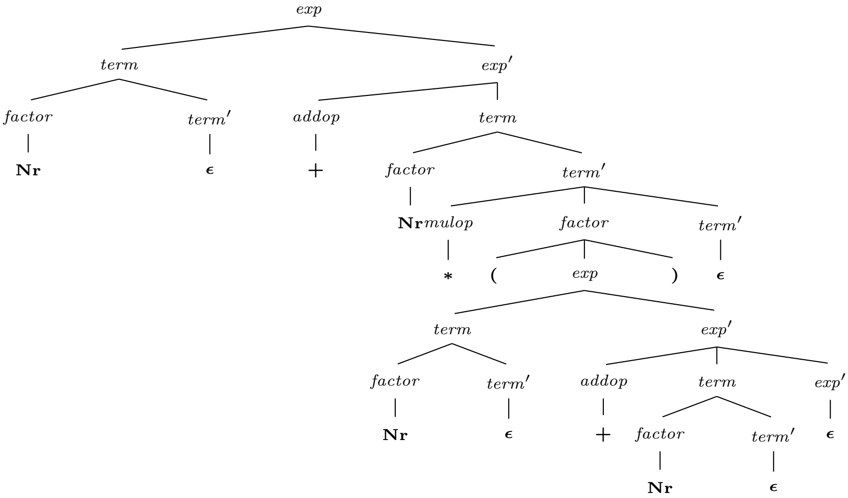

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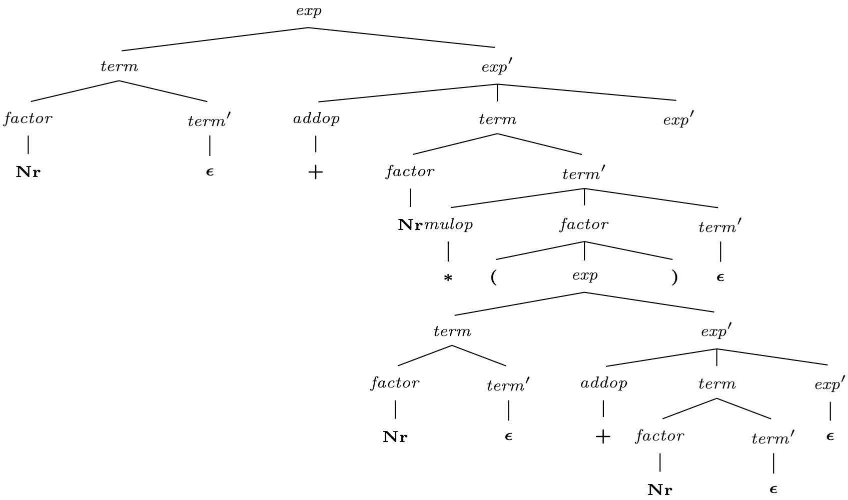

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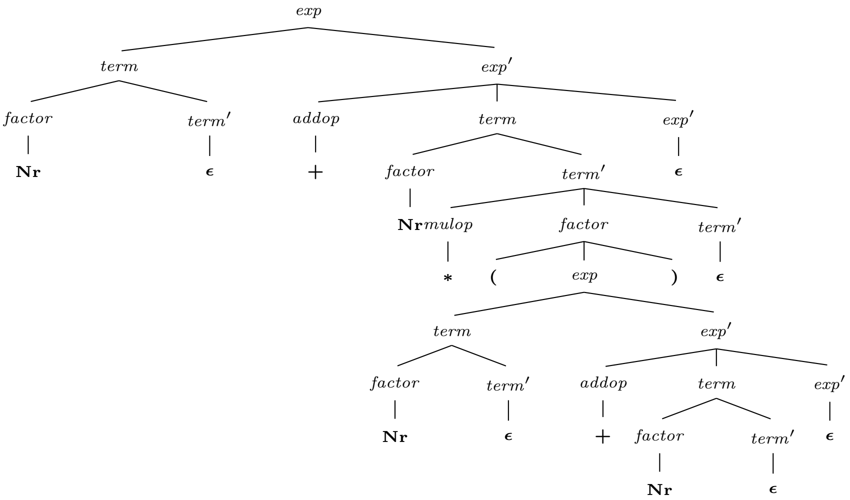


Non-determinism?

- not a "free" expansion/reduction/generation of some word, but
  - reduction of start symbol towards the *target word of terminals*

$$exp \Rightarrow^* \mathbf{1 + 2 * (3 + 4)}$$

  - i.e.: input stream of tokens "guides" the derivation process (at least it fixes the target)
- but: how much "guidance" does the target word (in general) gives?

Oracular derivation

$$\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{exp} + \mathit{term} \mid \mathit{exp} - \mathit{term} \mid \mathit{term} \\
\mathit{term} & \rightarrow & \mathit{term} * \mathit{factor} \mid \mathit{factor} \\
\mathit{factor} & \rightarrow & (\, \mathit{exp} \,) \mid \mathbf{number}
\end{array}$$

$$\begin{array}{lll}
\underline{\mathit{exp}} & \Rightarrow_1 & \downarrow 1 + 2 * 3 \\
\underline{\mathit{exp}} + \mathit{term} & \Rightarrow_3 & \downarrow 1 + 2 * 3 \\
\underline{\mathit{term}} + \mathit{term} & \Rightarrow_5 & \downarrow 1 + 2 * 3 \\
\underline{\mathit{factor}} + \mathit{term} & \Rightarrow_7 & \downarrow 1 + 2 * 3 \\
\mathbf{number} + \mathit{term} & & \downarrow 1 + 2 * 3 \\
\mathbf{number} + \mathit{term} & & 1 \downarrow +2 * 3 \\
\mathbf{number} + \underline{\mathit{term}} & \Rightarrow_4 & 1+ \downarrow 2 * 3 \\
\mathbf{number} + \underline{\mathit{term}} * \mathit{factor} & \Rightarrow_5 & 1+ \downarrow 2 * 3 \\
\mathbf{number} + \underline{\mathit{factor}} * \mathit{factor} & \Rightarrow_7 & 1+ \downarrow 2 * 3 \\
\mathbf{number} + \mathbf{number} * \mathit{factor} & & 1+ \downarrow 2 * 3 \\
\mathbf{number} + \mathbf{number} * \mathit{factor} & & 1 + 2 \downarrow *3 \\
\mathbf{number} + \mathbf{number} * \underline{\mathit{factor}} & \Rightarrow_7 & 1 + 2* \downarrow 3 \\
\mathbf{number} + \mathbf{number} * \mathbf{number} & & 1 + 2* \downarrow 3 \\
\mathbf{number} + \mathbf{number} * \mathbf{number} & & 1 + 2 * 3 \downarrow
\end{array}$$

Two principle sources of non-determinism

Using production $A \to \beta$

$$S \Rightarrow^* \alpha_1 \; A \; \alpha_2 \Rightarrow \alpha_1 \; \beta \; \alpha_2 \Rightarrow^* w$$

- $\alpha_1, \alpha_2, \beta$: word of terminals and nonterminals
- w: word of terminals, only
- A: one non-terminal

2 choices to make

1. where, i.e., on which occurrence of a non-terminal in $\alpha_1 A \alpha_2$ to apply a production
2. which production to apply (for the chosen non-terminal).

Left-most derivation

- that's the *easy* part of non-determinism
- taking care of "where-to-reduce" non-determinism: *left-most* derivation
- notation $\Rightarrow_l$
- some of the example derivations earlier used that

Non-determinism vs. ambiguity

- Note: the "where-to-reduce"-non-determinism $\neq$ ambiguitiy of a grammar
- in a way ("theoretically"): where to reduce next is *irrelevant*:
  - the order in the sequence of derivations *does not matter*
  - what does matter: the derivation tree (aka the parse tree)

Lemma (Left or right, who cares)

$S \Rightarrow_l^* w$ *iff* $S \Rightarrow_r^* w$ *iff* $S \Rightarrow^* w$.

- however ("practically"): a (deterministic) parser implementation: must make a *choice*

Using production $A \to \beta$

$$S \Rightarrow^* \alpha_1 \; A \; \alpha_2 \Rightarrow \alpha_1 \; \beta \; \alpha_2 \Rightarrow^* w$$

Non-determinism vs. ambiguity

- Note: the “where-to-reduce”-non-determinism $\neq$ ambiguitiy of a grammar
- in a way (“theoretically”): where to reduce next is *irrelevant*:
  - the order in the sequence of derivations *does not matter*
  - what does matter: the derivation tree (aka the parse tree)

Lemma (Left or right, who cares)

$S \Rightarrow_l^* w$ *iff* $S \Rightarrow_r^* w$ *iff* $S \Rightarrow^* w$.

- however (“practically”): a (deterministic) parser implementation: must make a *choice*

Using production $A \to \beta$

$$S \Rightarrow_l^* w_1 \; A \; \alpha_2 \Rightarrow w_1 \; \beta \; \alpha_2 \Rightarrow_l^* w$$

What about the "which-right-hand side" non-determinism?

$$A \to \beta \ \mid \ \gamma$$

Is that the correct choice?

$$S \Rightarrow_l^* w_1 \ A \ \alpha_2 \Rightarrow w_1 \ \beta \ \alpha_2 \Rightarrow_l^* w$$

- reduction with "guidance": don't loose sight of the target w
  - "past" is fixed: $w = w_1 w_2$
  - "future" is not:

$A\alpha_2 \Rightarrow_l \beta\alpha_2 \Rightarrow_l^* w_2$ or else $A\alpha_2 \Rightarrow_l \gamma\alpha_2 \Rightarrow_l^* w_2$?

Needed (minimal requirement):

In such a situation, "future target" w_2 must *determine* which of the rules to take!

Deterministic, yes, but still impractical

$$A\alpha_2 \Rightarrow_l \beta\alpha_2 \Rightarrow_l^* w_2 \quad \text{or else} \quad A\alpha_2 \Rightarrow_l \gamma\alpha_2 \Rightarrow_l^* w_2 \ ?$$

- the "target" w_2 is of *unbounded length*!

$\Rightarrow$ impractical, therefore:

Look-ahead of length k

resolve the "which-right-hand-side" non-determinism inspecting only fixed-length prefix of w_2 (for *all* situations as above)

LL(k) grammars

CF-grammars which *can* be parsed doing that.

Section

First and follow sets

Chapter 4 “Parsing”
Course “Compiler Construction”
Martin Steffen
Spring 2021

First and Follow sets

- general concept for grammars
- certain types of analyses (e.g. parsing):
  - info needed about possible "forms" of *derivable* words,

First-set of A

which terminal symbols can appear at the start of strings *derived from* a given nonterminal A

Follow-set of A

Which terminals can follow A in some *sentential form*.

- sentential form: word *derived from* grammar's starting symbol
- later: different algos for first and follow sets, for all non-terminals of a given grammar
- mostly straightforward
- one complication: *nullable* symbols (non-terminals)
- Note: those sets depend on grammar, not the language

First sets

Definition (First set)

Given a grammar G and a non-terminal A. The *first-set* of A, written $First_G(A)$ is defined as

$$First_G(A) = \{a \mid A \Rightarrow_G^* a\alpha, \quad a \in \Sigma_T\} + \{\epsilon \mid A \Rightarrow_G^* \epsilon\} . \tag{2}$$

Definition (Nullable)

Given a grammar G. A non-terminal $A \in \Sigma_N$ is *nullable*, if $A \Rightarrow^* \epsilon$.

Examples

- Cf. the Tiny grammar
- in Tiny, as in most languages

$$First(\textit{if-stmt}) = \{\text{"}\mathbf{if}\text{"}\}$$

- in many languages:

$$First(\textit{assign-stmt}) = \{\mathbf{identifier}, \text{"(")}\}$$

- typical *Follow* (see later) for statements:

$$Follow(stmt) = \{\text{";"}, \text{"}\mathbf{end}\text{"}, \text{"}\mathbf{else}\text{"}, \text{"}\mathbf{until}\text{"}\}$$

Remarks

- note: special treatment of the empty word ϵ
- in the following: if grammar G clear from the context
  - $\Rightarrow^*$ for $\Rightarrow^*_G$
  - $First$ for $First_G$
  - ...
- definition so far: "top-level" for start-symbol, only
- next: a more general definition
  - definition of First set of arbitrary symbols (and even words)
  - and also: definition of First for a symbol *in terms of* First for "other symbols" (connected by *productions*)

$\Rightarrow$ recursive definition

A more algorithmic/recursive definition

- grammar *symbol* X: terminal or non-terminal or ϵ

Definition (First set of a symbol)

Given a grammar G and grammar symbol X. The *first-set* of X, written $\mathit{First}(X)$, is defined as follows:

1. If $X \in \Sigma_T + \{\epsilon\}$, then $\mathit{First}(X)$ contains X.
2. If $X \in \Sigma_N$: For each production

$$X \to X_1 X_2 \ldots X_n$$

   2.1 $\mathit{First}(X)$ contains $\mathit{First}(X_1) \setminus \{\epsilon\}$
   2.2 If, for some $i < n$, *all* $\mathit{First}(X_1), \ldots, \mathit{First}(X_i)$ contain ϵ, then $\mathit{First}(X)$ contains $\mathit{First}(X_{i+1}) \setminus \{\epsilon\}$.
   2.3 If all $\mathit{First}(X_1), \ldots, \mathit{First}(X_n)$ contain ϵ, then $\mathit{First}(X)$ contains $\{\epsilon\}$.

For words

Definition (First set of a word)

Given a grammar G and word α. The *first-set* of

$$\alpha = X_1 \dots X_n \ ,$$

written $First(\alpha)$ is defined inductively as follows:

1. $First(\alpha)$ contains $First(X_1) \setminus \{\epsilon\}$
2. for each $i = 2, \dots n$, if $First(X_k)$ contains ϵ for *all* $k = 1, \dots, i-1$, then $First(\alpha)$ contains $First(X_i) \setminus \{\epsilon\}$
3. If all $First(X_1), \dots, First(X_n)$ contain ϵ, then $First(X)$ contains $\{\epsilon\}$.

Pseudo code

```
for all $X \in A \cup \{\epsilon\}$ do
   First[X] := X
end;

for all non-terminals $A$ do
  First[A] := {}
end
while there are changes to any First[A] do
  for each production $A \to X_1 \ldots X_n$ do
    k := 1;
    continue := true
    while continue = true and $k \leq n$ do
      First[A] := First[A] ∪ First[$X_k$] \ {$\epsilon$}
      if $\epsilon \notin$ First[$X_k$] then continue := false
      k := k + 1
    end;
    if   continue = true
    then First[A] := First[A] ∪ {$\epsilon$}
  end;
end
```

If only we could do away with special cases for the empty words . . .

for a grammar without *ϵ-productions*.[1]

```
for  all non-terminals $A$  do
   First[A] := {}            // counts as change
end
while  there are changes to any  First[A]  do
   for  each production $A \to X_1 \dots X_n$  do
        First[A]  :=  First[A] ∪ First[$X_1$]
   end;
end
```

---

[1] A production of the form $A \to \epsilon$.

Example expression grammar (from before)

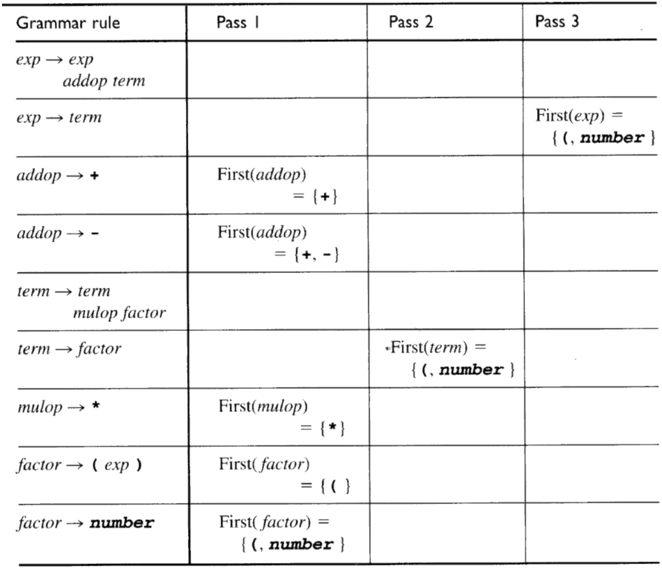
$$
\begin{array}{rcl}
exp & \rightarrow & exp \;\; addop \;\; term \mid term \\
addop & \rightarrow & + \mid - \\
term & \rightarrow & term \;\; mulop \;\; factor \mid factor \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\, exp\,) \mid \mathbf{number}
\end{array}
\tag{3}
$$

Example expression grammar (expanded)

$$
\begin{array}{rcl}
exp & \rightarrow & exp\ \ addop\ \ term \\
exp & \rightarrow & term \\
addop & \rightarrow & + \\
addop & \rightarrow & - \\
term & \rightarrow & term\ \ mulop\ \ factor \\
term & \rightarrow & factor \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\, exp\,) \\
factor & \rightarrow & \mathbf{n}
\end{array} \tag{4}
$$

nr		pass 1	pass 2	pass 3
1	$exp \to exp\ addop\ term$			
2	$exp \to term$			
3	$addop \to \mathbf{+}$			
4	$addop \to \mathbf{-}$			
5	$term \to term\ mulop\ factor$			
6	$term \to factor$			
7	$mulop \to \boldsymbol{*}$			
8	$factor \to \mathbf{(}\ exp\ \mathbf{)}$			
9	$factor \to \mathbf{n}$			

“Run” of the algo



Grammar rule	Pass 1	Pass 2	Pass 3
exp → *exp* *addop* *term*			
exp → *term*			First(*exp*) = { **(**, **number** }
addop → **+**	First(*addop*) = { **+** }		
addop → **-**	First(*addop*) = { **+**, **-** }		
term → *term* *mulop* *factor*			
term → *factor*		First(*term*) = { **(**, **number** }	
mulop → *****	First(*mulop*) = { ***** }		
factor → **(** *exp* **)**	First(*factor*) = { **(** }		
factor → **number**	First(*factor*) = { **(**, **number** }		

Collapsing the rows & final result

- results per pass:

	1	2	3
exp			$\{(, \mathbf{n}\}$
$addop$	$\{+, -\}$		
$term$		$\{(, \mathbf{n}\}$	
$mulop$	$\{*\}$		
$factor$	$\{(, \mathbf{n}\}$		

- final results (at the end of pass 3):

	$First[\_]$
exp	$\{(, \mathbf{n}\}$
$addop$	$\{+, -\}$
$term$	$\{(, \mathbf{n}\}$
$mulop$	$\{*\}$
$factor$	$\{(, \mathbf{n}\}$

Work-list formulation

```
for  all non-terminals $A$  do
   First[A] := {}
  WL         := $P$    // all productions
end
while WL $\neq \emptyset$ do
  remove one $(A \to X_1 \ldots X_n)$ from WL
  if     First[A] $\neq$ First[A] $\cup$ First[$X_1$]
  then   First[A] := First[A] $\cup$ First[$X_1$]
      add all productions $(A \to X_1' \ldots X_m')$ to WL
  else   skip
end
```

- no ϵ-productions
- worklist here: “collection” of productions
- alternatively, with slight reformulation: “collection” of non-terminals instead also possible

Follow sets

Definition (Follow set)

Given a grammar G with start symbol S, and a non-terminal A.

The *follow-set* of A, written $Follow_G(A)$, is

$$Follow_G(A) = \{a \mid S\,\$ \Rightarrow_G^* \alpha_1 A a \alpha_2, \quad a \in \Sigma_T + \{\,\$\,\}\} \,. \tag{5}$$

- $ as special end-marker
- typically: start symbol *not* on the right-hand side of a production

Follow sets, recursively

Definition (Follow set of a non-terminal)

Given a grammar G and nonterminal A. The *Follow-set* of A, written $Follow(A)$ is defined as follows:

1. If A is the start symbol, then $Follow(A)$ contains $\$$.
2. If there is a production $B \rightarrow \alpha A \beta$, then $Follow(A)$ contains $First(\beta) \setminus \{\epsilon\}$.
3. If there is a production $B \rightarrow \alpha A \beta$ such that $\epsilon \in First(\beta)$, then $Follow(A)$ contains $Follow(B)$.

- $\$$: "end marker" special symbol, only to be contained in the follow set

More imperative representation in pseudo code

```
Follow[$S$] := $\{\$\}$
for all non-terminals $A \neq S$ do
  Follow[$A$] := $\{\}$
end
while there are changes to any Follow-set do
  for each production $A \rightarrow X_1 \ldots X_n$ do
    for each $X_i$ which is a non-terminal do
      Follow[$X_i$] := Follow[$X_i$] $\cup$ (First($X_{i+1} \ldots X_n$) $\setminus \{\epsilon\}$)
      if $\epsilon \in$ First($X_{i+1} X_{i+2} \ldots X_n$)
      then Follow[$X_i$] := Follow[$X_i$] $\cup$ Follow[$A$]
    end
  end
end
```

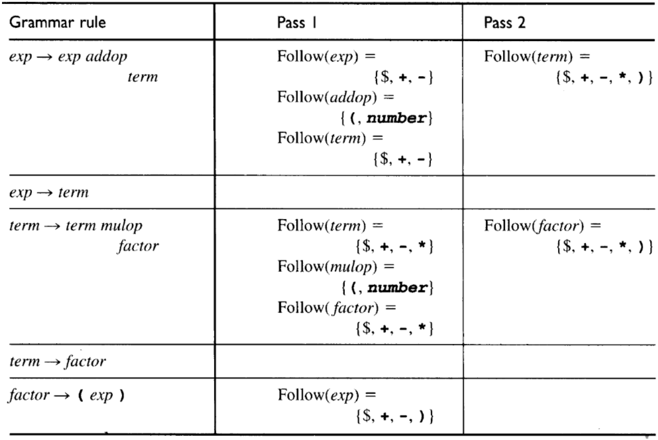
Note! $First() = \{\epsilon\}$

Expression grammar once more

$$
\begin{array}{rcl}
exp & \rightarrow & exp\ \ addop\ \ term \\
exp & \rightarrow & term \\
addop & \rightarrow & + \\
addop & \rightarrow & - \\
term & \rightarrow & term\ \ mulop\ \ factor \\
term & \rightarrow & factor \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\, exp\,) \\
factor & \rightarrow & \mathbf{n}
\end{array}
\tag{6}
$$

nr		pass 1	pass 2
1	$exp \rightarrow exp\ addop\ term$		
2	$exp \rightarrow term$		
5	$term \rightarrow term\ mulop\ factor$		
6	$term \rightarrow factor$		
8	$factor \rightarrow (\ exp\ )$		

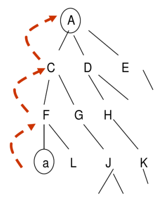
“Run” of the algo



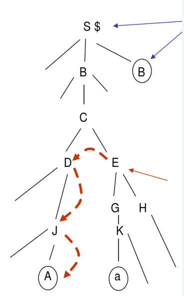
Grammar rule	Pass 1	Pass 2
$exp \to exp\ addop\ term$	Follow(exp) = {\$, **+**, **-**} Follow($addop$) = { **(**, **number**} Follow($term$) = {\$, **+**, **-**}	Follow($term$) = {\$, **+**, **-**, *****, **)** }
$exp \to term$		
$term \to term\ mulop\ factor$	Follow($term$) = {\$, **+**, **-**, *****} Follow($mulop$) = { **(**, **number**} Follow($factor$) = {\$, **+**, **-**, *****}	Follow($factor$) = {\$, **+**, **-**, *****, **)** }
$term \to factor$		
$factor \to$ **(** exp **)**	Follow(exp) = {\$, **+**, **-**, **)** }	

Illustration of first/follow sets

$a \in First(A)$



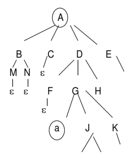
$a \in Follow(A)$



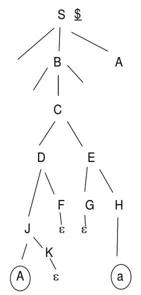
- red arrows: illustration of *information flow* in the algos
- run of $Follow$:
  - relies on $First$
  - in particular $a \in First(E)$ (right tree)
- $\$ \in Follow(B)$

More complex situation (nullability)

$a \in First(A)$



$a \in Follow(A)$



Section

Massaging grammars

Chapter 4 “Parsing”
Course “Compiler Construction”
Martin Steffen
Spring 2021

Some simple examples for both

- left-recursion

$$exp \to exp + term$$

- classical example for common left factor: rules for conditionals

$$\begin{array}{rcl} \mathit{if\text{-}stmt} & \to & \textbf{if} \; (\, exp \,) \; stmt \; \textbf{end} \\ & | & \textbf{if} \; (\, exp \,) \; stmt \; \textbf{else} \; stmt \; \textbf{end} \end{array}$$

Transforming the expression grammar

$$\begin{array}{rcl} exp & \rightarrow & exp\ \ addop\ \ term \mid term \\ addop & \rightarrow & + \mid - \\ term & \rightarrow & term\ \ mulop\ \ factor \mid factor \\ mulop & \rightarrow & * \\ factor & \rightarrow & (\, exp\,) \mid \textbf{number} \end{array}$$

- obviously left-recursive
- remember: this variant used for proper associativity!

After removing left recursion

$$\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \mid \epsilon \\
addop & \rightarrow & + \mid - \\
term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \mid \epsilon \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\ exp\ ) \mid \mathbf{n}
\end{array}$$

- still *unambiguous*
- unfortunate: *associativity* now different!
- note also: ϵ-productions & nullability

Left-recursion removal

Left-recursion removal

A transformation process to turn a CFG into one without left recursion

- price: ϵ-productions
- *3 cases* to consider
  - immediate (or direct) recursion
    - simple
    - general
  - *indirect* (or mutual) recursion

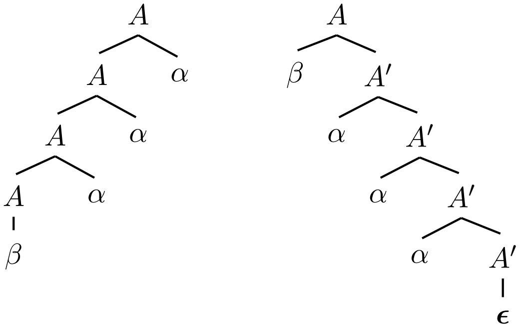
Left-recursion removal: simplest case

$$A \rightarrow A\alpha \mid \beta$$

$$\begin{aligned} A &\rightarrow \beta A' \\ A' &\rightarrow \alpha A' \mid \epsilon \end{aligned}$$

Schematic representation

$$A \rightarrow A\alpha \mid \beta \qquad \begin{array}{lll} A & \rightarrow & \beta A' \\ A' & \rightarrow & \alpha A' \mid \epsilon \end{array}$$



Remarks

- both grammars generate the same (context-free) language (= set of words over terminals)
- in EBNF:

$$A \to \beta\{\alpha\}$$

- two *negative* aspects of the transformation
  1. generated language unchanged, but: change in resulting structure (parse-tree), i.a.w. change in associativity, which may result in change of *meaning*
  2. introduction of ϵ-productions
- more concrete example for such a production: grammar for expressions

Left-recursion removal: immediate recursion (multiple)

Before

$$\begin{array}{rcl} A & \rightarrow & A\alpha_1 \mid \ldots \mid A\alpha_n \\ & \mid & \beta_1 \mid \ldots \mid \beta_m \end{array}$$

After

$$\begin{array}{rcl} A & \rightarrow & \beta_1 A' \mid \ldots \mid \beta_m A' \\ A' & \rightarrow & \alpha_1 A' \mid \ldots \mid \alpha_n A' \\ & \mid & \epsilon \end{array}$$

Note: can be written in *EBNF* as:

$$A \rightarrow (\beta_1 \mid \ldots \mid \beta_m)(\alpha_1 \mid \ldots \mid \alpha_n)^*$$

Removal of: general left recursion

Assume non-terminals $A_1, \ldots, A_m$

```
for i := 1 to m do
  for j := 1 to i-1 do
    replace each grammar rule of the form $A_i \to A_j\beta$ by  // $i < j$
    rule $A_i \to \alpha_1\beta \mid \alpha_2\beta \mid \ldots \mid \alpha_k\beta$
        where $A_j \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_k$
        is the current rule(s) for $A_j$  // current
  end
  { corresponds to $i = j$ }
  remove, if necessary, immediate left recursion for $A_i$
end
```

“current” = rule in the current stage of algo

Example (for the general case)

$$\begin{array}{lcl} A & \rightarrow & B\mathbf{a} \mid A\mathbf{a} \mid \mathbf{c} \\ B & \rightarrow & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

Example (for the general case)

$$\begin{array}{lcl} A & \rightarrow & B\mathbf{a} \mid A\mathbf{a} \mid \mathbf{c} \\ B & \rightarrow & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

$$\begin{array}{lcl} A & \rightarrow & B\mathbf{a}A' \mid \mathbf{c}A' \\ A' & \rightarrow & \mathbf{a}A' \mid \epsilon \\ B & \rightarrow & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

Example (for the general case)

$$\begin{array}{lcl} A & \rightarrow & B\mathbf{a} \mid A\mathbf{a} \mid \mathbf{c} \\ B & \rightarrow & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

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$$\begin{array}{lcl} A & \rightarrow & B\mathbf{a}A' \mid \mathbf{c}A' \\ A' & \rightarrow & \mathbf{a}A' \mid \epsilon \\ B & \rightarrow & B\mathbf{b} \mid B\mathbf{a}A'\mathbf{b} \mid \mathbf{c}A'\mathbf{b} \mid \mathbf{d} \end{array}$$

Example (for the general case)

$$\begin{array}{rcl} A & \to & B\mathbf{a} \mid A\mathbf{a} \mid \mathbf{c} \\ B & \to & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

$$\begin{array}{rcl} A & \to & B\mathbf{a}A' \mid \mathbf{c}A' \\ A' & \to & \mathbf{a}A' \mid \epsilon \\ B & \to & B\mathbf{b} \mid A\mathbf{b} \mid \mathbf{d} \end{array}$$

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$$\begin{array}{rcl} A & \to & B\mathbf{a}A' \mid \mathbf{c}A' \\ A' & \to & \mathbf{a}A' \mid \epsilon \\ B & \to & \mathbf{c}A'\mathbf{b}B' \mid \mathbf{d}B' \\ B' & \to & \mathbf{b}B' \mid \mathbf{a}A'\mathbf{b}B' \mid \epsilon \end{array}$$

Left factor removal

- CFG: not just describe a context-free languages
- also: intended (indirect) description of a parser for that language

⇒ common left factor undesirable

- cf.: *determinization* of automata for the lexer

Simple situation

$$A \to \alpha\beta \mid \alpha\gamma \mid \dots$$

$$\begin{array}{rcl} A & \to & \alpha A' \mid \dots \\ A' & \to & \beta \mid \gamma \end{array}$$

Example: sequence of statements

Before

$$\begin{array}{rcl} \mathit{stmt\text{-}seq} & \rightarrow & \mathit{stmt}\;\mathbf{;}\;\mathit{stmt\text{-}seq} \\ & \mid & \mathit{stmt} \end{array}$$

After

$$\begin{array}{rcl} \mathit{stmt\text{-}seq} & \rightarrow & \mathit{stmt}\;\mathit{stmt\text{-}seq}' \\ \mathit{stmt\text{-}seq}' & \rightarrow & \mathbf{;}\;\mathit{stmt\text{-}seq} \;\mid\; \epsilon \end{array}$$

Example: conditionals

Before

$$\begin{array}{rcl} \mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt\text{-}seq}\ \textbf{end} \\ & \mid & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt\text{-}seq}\ \textbf{else}\ \mathit{stmt\text{-}seq}\ \textbf{end} \end{array}$$

After

$$\begin{array}{rcl} \mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt\text{-}seq}\ \mathit{else\text{-}or\text{-}end} \\ \mathit{else\text{-}or\text{-}end} & \rightarrow & \textbf{else}\ \mathit{stmt\text{-}seq}\ \textbf{end}\ \mid\ \textbf{end} \end{array}$$

Example: conditionals (without else)

Before

$$\begin{array}{rcl} \textit{if-stmt} & \rightarrow & \textbf{if}\ (\ \textit{exp}\ )\ \textit{stmt-seq} \\ & \mid & \textbf{if}\ (\ \textit{exp}\ )\ \textit{stmt-seq}\ \textbf{else}\ \textit{stmt-seq} \end{array}$$

After

$$\begin{array}{rcl} \textit{if-stmt} & \rightarrow & \textbf{if}\ (\ \textit{exp}\ )\ \textit{stmt-seq}\ \textit{else-or-empty} \\ \textit{else-or-empty} & \rightarrow & \textbf{else}\ \textit{stmt-seq} \mid \epsilon \end{array}$$

Not all factorization doable in "one step"

Starting point

$$A \rightarrow \mathbf{abc}B \mid \mathbf{ab}C \mid \mathbf{a}E$$

After 1 step

$$\begin{array}{rcl} A & \rightarrow & \mathbf{ab}A' \mid \mathbf{a}E \\ A' & \rightarrow & \mathbf{c}B \mid C \end{array}$$

After 2 steps

$$\begin{array}{rcl} A & \rightarrow & \mathbf{a}A'' \\ A'' & \rightarrow & \mathbf{b}A' \mid E \\ A' & \rightarrow & \mathbf{c}B \mid C \end{array}$$

- note: we choose the *longest* common prefix (= longest

Left factorization

```
while there are changes to the grammar do
  for each nonterminal $A$ do
    let $\alpha$ be a prefix of max. length that is shared
                    by two or more productions for $A$
    if $\alpha \neq \epsilon$
    then
       let $A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ be all
               prod. for $A$ and suppose that $\alpha_1, \ldots, \alpha_k$ share $\alpha$
               so that $A \rightarrow \alpha\beta_1 \mid \ldots \mid \alpha\beta_k \mid \alpha_{k+1} \mid \ldots \mid \alpha_n$ ,
               that the $\beta_j$'s share no common prefix, and
               that the $\alpha_{k+1}, \ldots, \alpha_n$ do not share $\alpha$.
       replace rule $A \rightarrow \alpha_1 \mid \ldots \mid \alpha_n$ by the rules
       $A \rightarrow \alpha A' \mid \alpha_{k+1} \mid \ldots \mid \alpha_n$
       $A' \rightarrow \beta_1 \mid \ldots \mid \beta_k$
    end
  end
end
```

Section

LL-parsing (mostly LL(1))

Chapter 4 “Parsing”
Course “Compiler Construction”
Martin Steffen
Spring 2021

Parsing LL(1) grammars

- *this lecture*: we don't do LL(k) with $k > 1$
- LL(1): particularly easy to understand and to implement (efficiently)
- not as expressive than LR(1) (see later), but still kind of decent

LL(1) parsing principle

Parse from 1) left-to-right (as always anyway), do a 2) left-most derivation and resolve the "which-right-hand-side" non-determinism by 3) looking 1 symbol ahead.

- two flavors for LL(1) parsing here (both are top-down parsers)
  - *recursive descent*
  - *table-based* LL(1) parser
- *predictive* parsers

Sample expression grammar again

factors and terms

$$
\begin{aligned}
exp &\rightarrow term\ exp' \\
exp' &\rightarrow addop\ term\ exp' \mid \epsilon \\
addop &\rightarrow + \mid - \\
term &\rightarrow factor\ term' \\
term' &\rightarrow mulop\ factor\ term' \mid \epsilon \\
mulop &\rightarrow * \\
factor &\rightarrow (\ exp\ ) \mid \mathbf{n}
\end{aligned}
\tag{7}
$$

Look-ahead of 1: straightforward, but not trivial

- look-ahead of 1:
  - not much of a look-ahead, anyhow
  - just the “current token”

⇒ read the next token, and, based on that, decide

- but: what if there’s *no more symbols*?

⇒ read the next token if there is, and decide based on the token *or else* the fact that there’s none left[2]

Example: 2 productions for non-terminal $factor$

$$factor \rightarrow (\ exp\ )\ \mid\ \mathbf{number}$$

That situation here is more or less *trivial*, but that’s not all to LL(1) . . .

---

[2] Sometimes “special terminal” **\$** used to mark the end (as mentioned).

Recursive descent: general set-up

1. global variable, say `tok`, representing the "current token" (or pointer to current token)
2. parser has a way to *advance* that to the next token (if there's one)

Idea

For each *non-terminal* $nonterm$, write one procedure which:

- succeeds, if starting at the current token position, the "rest" of the token stream starts with a syntactically correct word of terminals representing $nonterm$
- fail otherwise

- ignored (for now): when doing the above successfully, build the *AST* for the accepted nonterminal.

Recursive descent (in C-like)

method `factor` for nonterminal *factor*

```
    final int LPAREN=1,RPAREN=2,NUMBER=3,
     PLUS=4,MINUS=5,TIMES=6;
```

```
    void factor () {
        switch (tok) {
        case LPAREN: eat(LPAREN); expr(); eat(RPAREN);
        case NUMBER: eat(NUMBER);
        }
    }
```

Recursive descent (in ocaml)

```
type token = LPAREN | RPAREN |  NUMBER
 |   PLUS | MINUS | TIMES
```

```
let factor () =    (* function for factors *)
  match !tok with
    LPAREN ->  eat(LPAREN); expr(); eat(RPAREN)
  | NUMBER ->  eat(NUMBER)
```

Slightly more complex

- previous 2 rules for $factor$: situation not always as immediate as that

LL(1) principle (again)

given a non-terminal, the next *token* must determine the choice of right-hand side.

$\Rightarrow$ definition of the $First$ set

Lemma (LL(1) (without nullable symbols))

A reduced context-free grammar without nullable non-terminals is an LL(1)-grammar iff for all non-terminals A *and for all pairs of productions* $A \to \alpha_1$ *and* $A \to \alpha_2$ *with* $\alpha_1 \neq \alpha_2$*:*

$$First_1(\alpha_1) \cap First_1(\alpha_2) = \emptyset \ .$$

Common problematic situation

- often: common *left factors* problematic

$$
\begin{array}{rcl}
\mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt} \\
 & \mid & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt}\ \textbf{else}\ \mathit{stmt}
\end{array}
$$

- requires a look-ahead of (at least) 2
- ⇒ try to rearrange the grammar
  1. *Extended* BNF ([2] suggests that)

     $$\mathit{if\text{-}stmt} \rightarrow \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt}[\textbf{else}\ \mathit{stmt}]$$

  1. *left-factoring*:

$$
\begin{array}{rcl}
\mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt}\ \mathit{else{-}part} \\
\mathit{else{-}part} & \rightarrow & \epsilon \mid \textbf{else}\ \mathit{stmt}
\end{array}
$$

Recursive descent for left-factored *if-stmt*

```
procedure ifstmt()
  begin
    match ("if");
    match  ("(");
    exp();
    match (")");
    stmt();
    if   token = "else"
    then match ("else");
         stmt()
    end
  end;
```

Left recursion is a no-go

factors and terms

$$
\begin{aligned}
exp &\rightarrow exp \;\; addop \;\; term \mid term \qquad (8)\\
addop &\rightarrow + \mid -\\
term &\rightarrow term \;\; mulop \;\; factor \mid factor\\
mulop &\rightarrow *\\
factor &\rightarrow (\, exp \,) \mid \mathbf{number}
\end{aligned}
$$

- consider treatment of exp: $First(exp)$?
- whatever is in $First(term)$, is in $First(exp)^3$ recursion.

Left-recursion

Left-recursive grammar *never* works for recursive descent.

---

[3] And it would not help to *look-ahead* more than 1 token either.

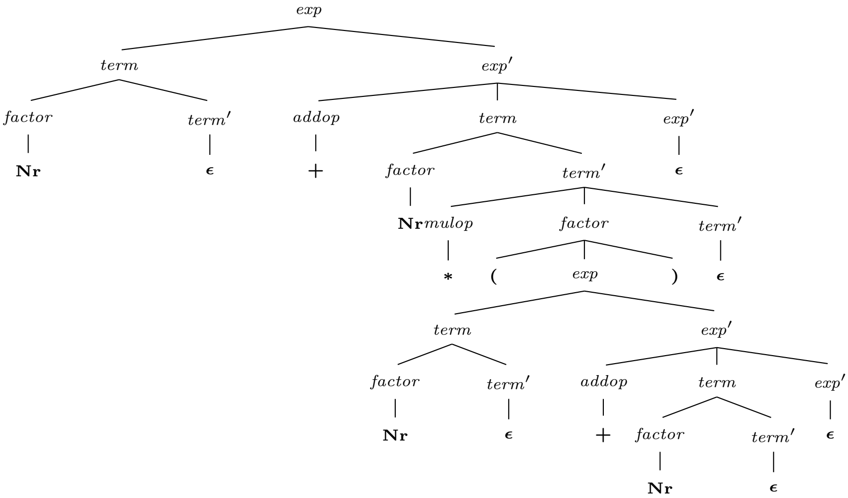
Removing left recursion may help

$$\begin{array}{rcl} exp & \to & term\ exp' \\ exp' & \to & addop\ term\ exp' \mid \epsilon \\ addop & \to & + \mid - \\ term & \to & factor\ term' \\ term' & \to & mulop\ factor\ term' \mid \epsilon \\ mulop & \to & * \\ factor & \to & (\ exp\ ) \mid \mathbf{n} \end{array}$$

```
procedure exp()
begin
     term();
     exp′()
end
```

```
procedure exp′()
begin
  case token of
    "+": match("+");
         term();
         exp′()
    "−": match("−");
         term();
         exp′()
   end
end
```

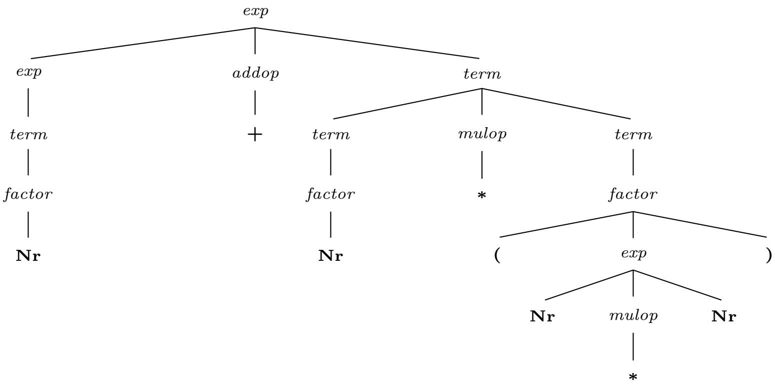
Recursive descent works, alright, but . . .



. . . who wants this form of trees?

Left-recursive grammar with nicer parse trees

$$1 + 2 * (3 + 4)$$

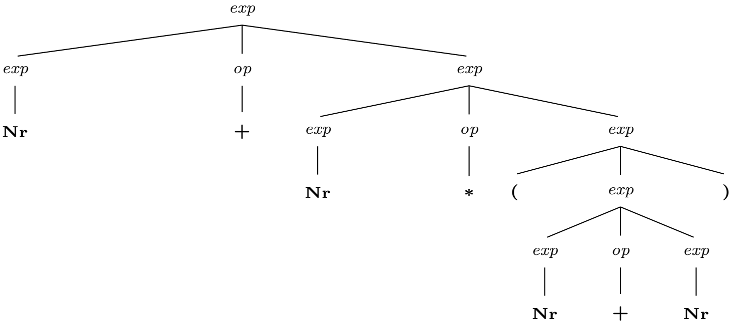


The simple “original” expression grammar (even nicer)

Flat expression grammar

$$\begin{array}{rcl} exp & \rightarrow & exp \;\; op \;\; exp \;\mid\; (\, exp \,) \;\mid\; \mathbf{number} \\ op & \rightarrow & + \;\mid\; - \;\mid\; * \end{array}$$

$$1 + 2 * (3 + 4)$$



Associtivity problematic

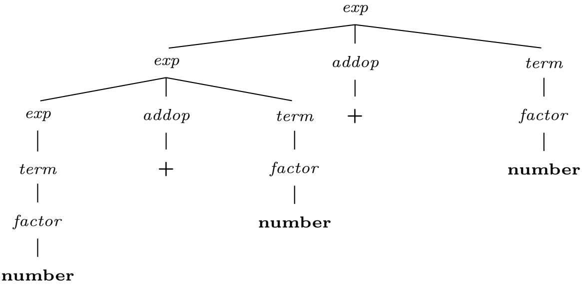
Precedence & assoc.

$$\begin{array}{rcl} exp & \rightarrow & exp \;\; addop \;\; term \;\mid\; term \\ addop & \rightarrow & + \;\mid\; - \\ term & \rightarrow & term \;\; mulop \;\; factor \;\mid\; factor \\ mulop & \rightarrow & * \\ factor & \rightarrow & \textbf{(}\, exp \,\textbf{)} \;\mid\; \textbf{number} \end{array}$$

$3+4+5$

parsed “as”

$(3+4)+5$



Associtivity problematic

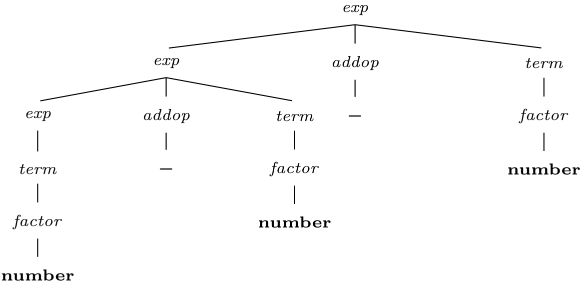
Precedence & assoc.

$$
\begin{array}{rcl}
exp & \rightarrow & exp \;\; addop \;\; term \;\mid\; term \\
addop & \rightarrow & + \mid - \\
term & \rightarrow & term \;\; mulop \;\; factor \;\mid\; factor \\
mulop & \rightarrow & * \\
factor & \rightarrow & \textbf{(}\, exp \,\textbf{)} \;\mid\; \textbf{number}
\end{array}
$$

$$3 - 4 - 5$$

parsed "as"

$$(3 - 4) - 5$$

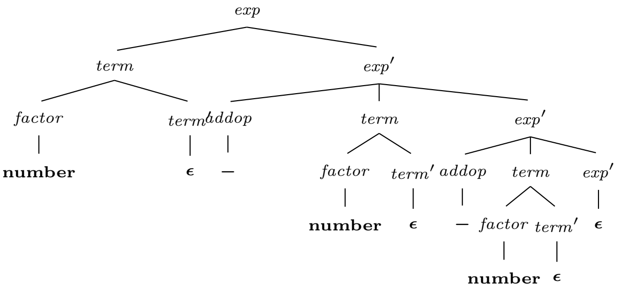


Now use the grammar without left-rec (but right-rec instead)

No left-rec.

$$
\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & \mathbf{+} \ \mid\ \mathbf{-} \\
\mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \ \mid\ \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & \mathbf{*} \\
\mathit{factor} & \rightarrow & \mathbf{(}\ \mathit{exp}\ \mathbf{)} \ \mid\ \mathbf{n}
\end{array}
$$

$$3 - 4 - 5$$



Now use the grammar without left-rec (but right-rec instead)

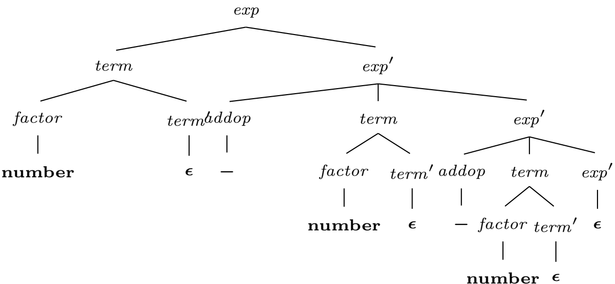
No left-rec.

$$
\begin{array}{rcl}
exp & \rightarrow & term\ exp' \\
exp' & \rightarrow & addop\ term\ exp' \mid \boldsymbol{\epsilon} \\
addop & \rightarrow & + \mid - \\
term & \rightarrow & factor\ term' \\
term' & \rightarrow & mulop\ factor\ term' \mid \boldsymbol{\epsilon} \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\ exp\ ) \mid \mathbf{n}
\end{array}
$$

$3-4-5$

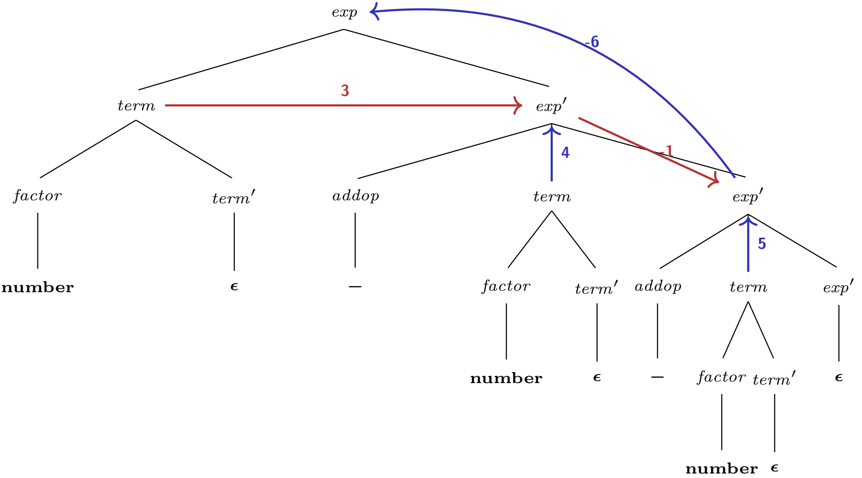
parsed “as”

$3-(4-5)$



But if we need a “left-associative” AST?

- we want $(3-4)-5$, *not* $3-(4-5)$



Code to “evaluate” ill-associated such trees correctly

```
function exp' (valsofar: int): int;
begin
  if token = '+' or token = '-'
  then
    case token of
      '+': match ('+');
           valsofar := valsofar + term;
      '-': match ('-');
           valsofar := valsofar - term;
    end case;
    return exp'(valsofar);
  else return valsofar
end;
```

- extra “accumulator” argument `valsofar`
- instead of evaluating the expression, one could build the AST with the appropriate associativity instead:
- instead of `valueSoFar`, one had `rootOfTreeSoFar`

"Designing" the syntax, its parsing, & its AST

trade offs:

1. starting from: design of the language, how much of the syntax is left "implicit"[4]
2. which language class? Is LL(1) good enough, or something stronger wanted?
3. how to parse? (top-down, bottom-up, etc.)
4. parse-tree/concrete syntax trees vs. ASTs

---

[4] Lisp is famous/notorious in that its surface syntax is more or less an explicit notation for the ASTs. Not that it was originally planned like this . . .

AST vs. CST

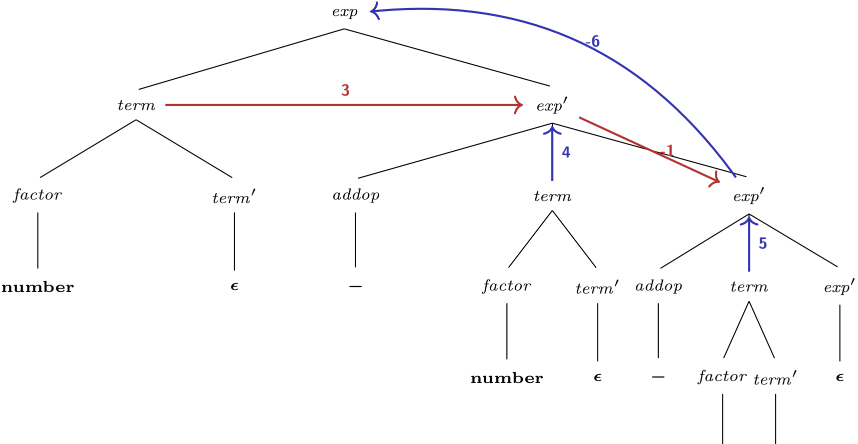
- once steps 1.–3. are fixed: *parse-trees* fixed!
- parse-trees = *essence* of grammatical derivation process
- often: parse trees only “conceptually” present in a parser
- AST:
  - *abstractions* of the parse trees
  - *essence* of the parse tree
  - actual tree data structure, as output of the parser
  - typically on-the fly: AST built while the parser parses, i.e. while it executes a derivation in the grammar

AST vs. CST/parse tree

Parser "builds" the AST data structure while "doing" the parse tree

AST: How "far away" from the CST?

- AST: only thing relevant for later phases ⇒ better be *clean* . . .
- AST "=" CST?
  - building AST becomes straightforward
  - possible choice, if the grammar is not designed "weirdly",



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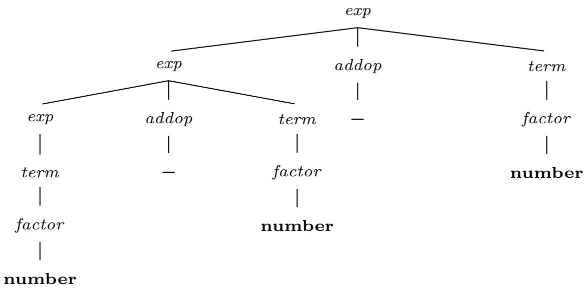

slightly more reasonably looking as AST (but underlying grammar not directly useful for recursive descent)

AST: How “far away” from the CST?

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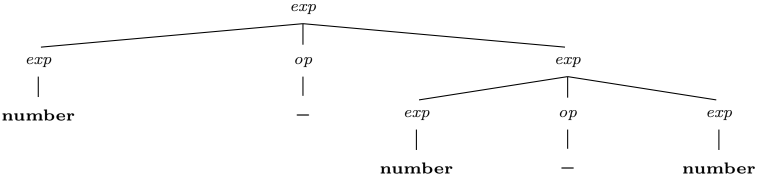

That parse tree looks reasonable clear and intuitive

AST: How “far away” from the CST?

- AST: only thing relevant for later phases ⇒ better be *clean* . . .
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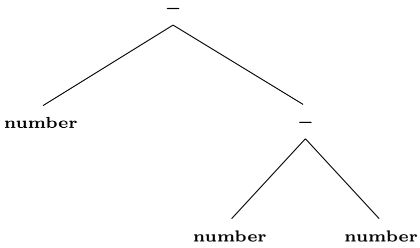

Wouldn’t that be the best AST here?

AST: How “far away” from the CST?

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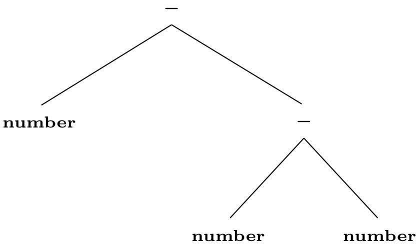
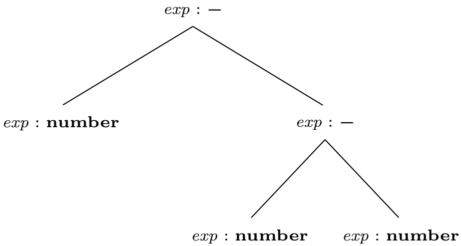


Wouldn’t that be the best AST here?

Certainly minimal amount of nodes, which is nice as such. However, what is missing (which might be interesting) is the fact that the 2 nodes labelled “−” are *expressions!*

AST: How “far away” from the CST?

- AST: only thing relevant for later phases ⇒ better be *clean* . . .
- AST “=” CST?
  - building AST becomes straightforward
  - possible choice, if the grammar is not designed “weirdly”,



Wouldn’t that be the best AST here?

Certainly minimal amount of nodes, which is nice as such. However, what is missing (which might be interesting) is the fact that the 2 nodes labelled “−” are *expressions!*

This is how it's done (a recipe)

Assume, one has a "non-weird" grammar

$$\begin{array}{lcl} exp & \rightarrow & exp \;\; op \;\; exp \mid (\, exp \,) \mid \textbf{number} \\ op & \rightarrow & + \mid - \mid * \end{array}$$

- typically that means: assoc. and precedences etc. are fixed *outside* the non-weird grammar
  - by massaging it to an equivalent one (no left recursion etc.)
  - or (better): use parser-generator that allows to *specify* assoc . . . , without cluttering the grammar.
- if grammar for *parsing* is not as clear: do a second one describing the ASTs

Remember (independent from parsing)

BNF describe trees

This is how it's done (recipe for OO data structures)

Recipe

- turn each non-terminal to an abstract class
- turn each right-hand side of a given non-terminal as (non-abstract) subclass of the class for considered non-terminal
- chose fields & constructors of concrete classes appropriately
- terminal: concrete class as well, field/constructor for token's *value*

Example in Java

$$
\begin{array}{rcl}
exp & \rightarrow & exp\ \ op\ \ exp \mid (\, exp\,) \mid \textbf{number} \\
op & \rightarrow & + \mid - \mid *
\end{array}
$$

```java
abstract public class Exp {
}
```

```java
public class BinExp extends Exp {  // exp -> exp op exp
    public Exp left, right;
    public Op  op;
    public BinExp(Exp l, Op o, Exp r) {
        left=l; op=o; right=r;}
}
```

```java
public class ParentheticExp extends Exp {  // exp -> ( op )
    public Exp exp;
    public ParentheticExp(Exp e) {exp = l;}
}
```

```java
public class NumberExp extends Exp {  // exp -> NUMBER
     public  number;                  // token value
     public Number(int i) {number = i;}
}
```

Example in Java

$$\begin{aligned} exp &\rightarrow exp \;\; op \;\; exp \mid (\, exp \,) \mid \textbf{number} \\ op &\rightarrow + \mid - \mid * \end{aligned}$$

```java
abstract public class Op {    // non-terminal = abstract
}
```

```java
public class Plus  extends Op {  // op -> "+"
}
```

```java
public class Minus  extends Op {  // op -> "-"
}
```

```java
public class Times extends Op {  // op -> "*"
}
```

$3 - (4 - 5)$

```
Exp e =  new BinExp(
           new NumberExp(3),
           new Minus(),
           new BinExp(new ParentheticExpr(
                new NumberExp(4),
                new Minus(),
                new NumberExp(5))))
```

Pragmatic deviations from the recipe

- it's nice to have a guiding principle, but no need to carry it too far . . .
- To the very least: the `ParentheticExpr` is completely without purpose: grouping is captured by the tree structure

⇒ that class is *not* needed

- some might prefer an implementation of

$$op \to + \mid - \mid *$$

as simply integers, for instance arranged like

```
public class BinExp extends Exp {  // exp -> exp op exp
   public Exp left, right;
   public int  op;
   public BinExp(Exp l, int o, Exp r) {
     pos=p; left=l; oper=o; right=r;}
   public final static int PLUS=0, MINUS=1, TIMES=2;
}
```

and used as `BinExpr.PLUS` etc.

Recipe for ASTs, final words:

- space considerations for AST representations are irrelevant in most cases
- clarity and cleanness trumps “quick hacks” and “squeezing bits”
- some deviation from the recipe or not, the advice still holds:

Do it systematically

A clean grammar is the specification of the syntax of the language and thus the parser. It is also a means of communicating with humans what the syntax of the language is, at least communicating with pros, like participants of a compiler course, who of course can read BNF . . . A clean grammar is a very systematic and structured thing which consequently *can* and *should* be systematically and cleanly represented in an AST, including judicious and systematic choice of names and conventions (nonterminal exp represented by class `Exp`, non-terminal $stmt$ by class `Stmt` etc).

Extended BNF may help alleviate the pain

BNF

$$\begin{array}{rcl} exp & \rightarrow & exp\ addop\ term \mid term \\ term & \rightarrow & term\ mulop\ factor \mid fa \end{array}$$

EBNF

$$\begin{array}{rcl} exp & \rightarrow & term\{\ addop\ term\ \} \\ term & \rightarrow & factor\{\ mulop\ factor\ \} \end{array}$$

but remember:

- EBNF just a notation, just because we do not see (left or right) recursion in $\{\ \ldots\ \}$, does not mean there is no recursion.
- not all parser generators support EBNF
- however: often easy to translate into loops- [5]
- does not offer a *general* solution if associativity etc. is problematic

---

[5] That results in a parser which is somehow not "pure recursive descent". It's "recursive descent, but sometimes, let's use a while-loop, if more convenient concerning, for instance, associativity"

Pseudo-code representing the EBNF productions

```
procedure exp;
begin
  term ;          { recursive call }
  while token = "+" or token = "-"
  do
    match(token);
    term;         // recursive call
  end
end
```

```
procedure term;
begin
  factor;          { recursive call }
  while token = "*"
  do
    match(token);
    factor;        // recursive call
  end
end
```

How to produce “something” during RD parsing?

Recursive descent

So far (mostly): RD = top-down (parse-)tree traversal via recursive procedure.[6] Possible outcome: termination or failure.

- Now: instead of returning “nothing” (return type `void` or similar), return some meaningful, and build that up during traversal
- for illustration: procedure for expressions:
    - return type `int`,
    - while traversing: *evaluate* the expression

---

[6] Modulo the fact that the tree being traversed is “conceptual” and not the input of the traversal procedure; instead, the traversal is “steered” by stream of tokens.

Evaluating an exp during RD parsing

```
function exp() : int;
var temp: int
begin
  temp := term ();        { recursive call }
  while token = "+" or token = "−"
    case token  of
      "+": match ("+");
           temp := temp + term();
      "−": match ("−")
           temp := temp − term();
    end
  end
  return temp;
end
```

Building an AST: expression

```
function exp() : syntaxTree;
var temp, newtemp: syntaxTree
begin
  temp := term ();          { recursive call }
  while token = "+" or token = "−"
    case token  of
      "+": match ("+");
           newtemp := makeOpNode("+");
           leftChild(newtemp)  := temp;
           rightChild(newtemp) := term();
           temp  := newtemp;
      "−": match ("−")
           newtemp := makeOpNode("−");
           leftChild(newtemp)  := temp;
           rightChild(newtemp) := term();
           temp  := newtemp;
    end
  end
  return temp;
end
```

- note: the use of `temp` and the `while` loop

Building an AST: factor

$$factor \rightarrow (\ exp\ )\ \mid\ \mathbf{number}$$

```
function factor() : syntaxTree;
var fact: syntaxTree
begin
  case token  of
    "(": match ("(");
         fact := exp();
         match (")");
    number:
        match (number)
        fact := makeNumberNode(number);
     else : error ...   // fall through
  end
  return fact;
end
```

Building an AST: conditionals

$$\textit{if-stmt} \rightarrow \textbf{if}\ (\ \textit{exp}\ )\ \textit{stmt}\ [\textbf{else}\ \textit{stmt}]$$

```
function ifStmt() : syntaxTree;
var temp: syntaxTree
begin
  match ("if");
  match ("(");
  temp := makeStmtNode("if")
  testChild(temp) := exp();
  match (")");
  thenChild(temp) := stmt();
  if   token = "else"
  then match "else";
       elseChild(temp) := stmt();
  else elseChild(temp) := nil;
  end
  return temp;
end
```

Building an AST: remarks and “invariant”

- LL(1) requirement: each procedure/function/method (covering one specific non-terminal) decides on alternatives, looking only at the current token
- call of function `A` for non-terminal A:
    - upon entry: first terminal symbol for A in `token`
    - upon exit: first terminal symbol *after* the unit derived from A in `token`
- `match("a")` : checks for `"a"` in `token` *and eats* the token (if matched).

LL(1) parsing

- remember LL(1) grammars & LL(1) parsing principle:

LL(1) parsing principle

1 look-ahead enough to resolve "which-right-hand-side" non-determinism.

- instead of recursion (as in RD): *explicit stack*
- decision making: collated into the LL(1) parsing table
- LL(1) parsing table:
  - finite data structure M (for instance, a 2 dimensional array)

$$M : \Sigma_N \times \Sigma_T \to ((\Sigma_N \times \Sigma^*) + \texttt{error})$$

  - $M[A, a] = w$
- we assume: pure BNF

Construction of the parsing table

Table recipe

1. If $A \to \alpha \in P$ and $\alpha \Rightarrow^* \mathbf{a}\beta$, then add $A \to \alpha$ to table entry $M[A, \mathbf{a}]$
2. If $A \to \alpha \in P$ and $\alpha \Rightarrow^* \boldsymbol{\epsilon}$ and $S\,\$ \Rightarrow^* \beta A \mathbf{a} \gamma$ (where $\mathbf{a}$ is a token (=non-terminal) *or* $\$$), then add $A \to \alpha$ to table entry $M[A, \mathbf{a}]$

Table recipe (again, now using our old friends First and Follow)

Assume $A \to \alpha \in P$.

1. If $\mathbf{a} \in \mathit{First}(\alpha)$, then add $A \to \alpha$ to $M[A, \mathbf{a}]$.
2. If α is *nullable* and $\mathbf{a} \in \mathit{Follow}(A)$, then add $A \to \alpha$ to $M[A, \mathbf{a}]$.

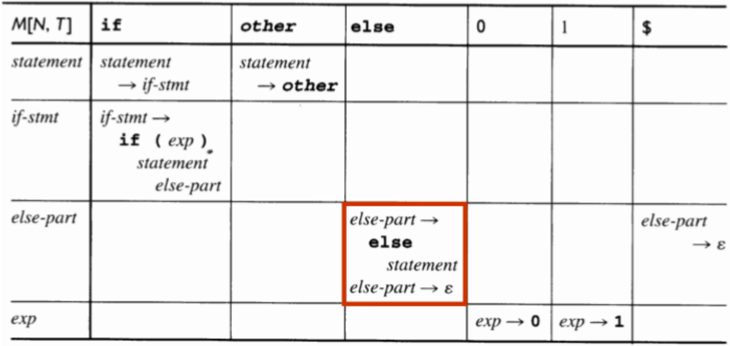
Example: if-statements

- grammars is left-factored and not left recursive

$$\begin{array}{rcl} \textit{stmt} & \rightarrow & \textit{if-stmt} \mid \textbf{other} \\ \textit{if-stmt} & \rightarrow & \textbf{if}\ \textbf{(}\ \textit{exp}\ \textbf{)}\ \textit{stmt}\ \textit{else}-\textit{part} \\ \textit{else}-\textit{part} & \rightarrow & \textbf{else}\ \textit{stmt} \mid \epsilon \\ \textit{exp} & \rightarrow & \textbf{0} \mid \textbf{1} \end{array}$$

	First	*Follow*
stmt	**other, if**	**$, else**
if-stmt	**if**	**$, else**
else−part	**else**, ϵ	**$, else**
exp	**0, 1**	**)**

Example: if statement: “LL(1) parse table”



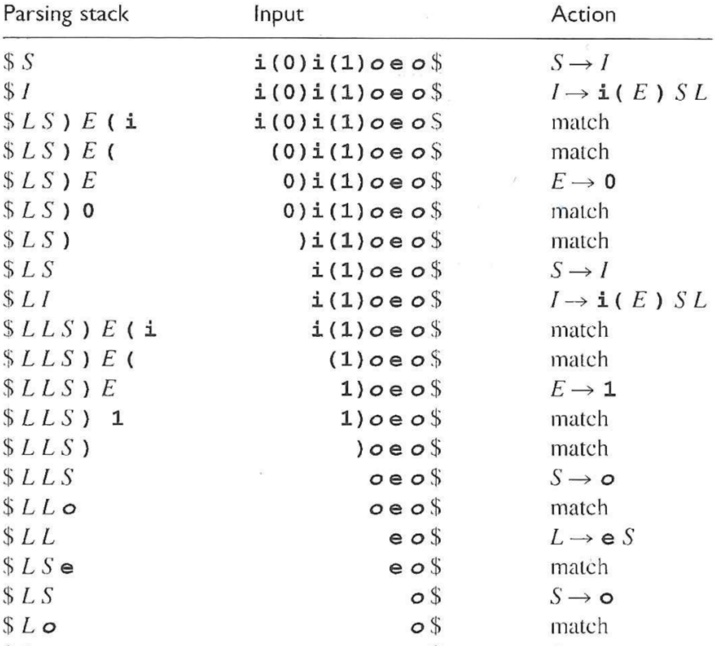
M[N, T]	**if**	**other**	**else**	0	1	$
statement	*statement* → *if-stmt*	*statement* → **other**				
if-stmt	*if-stmt* → **if** (*exp*) *statement* *else-part*					
else-part			*else-part* → **else** *statement* *else-part* → ε			*else-part* → ε
exp				*exp* → **0**	*exp* → **1**	

- 2 productions in the “red table entry”
- thus: it’s technically *not* an LL(1) table (and it’s not an LL(1) grammar)
- note: removing left-recursion and left-factoring did not help!

LL(1) table-based algo

```
while the top of the parsing stack $\neq \mathbf{\$}$
   if the top of the parsing stack is terminal $\mathbf{a}$
       and the next input token $= \mathbf{a}$
   then
       pop the parsing stack;
       advance the input; // ``match''
   else if   the top the parsing is non-terminal $A$
          and the next input token is $\mathbf{a}$ terminal or $\mathbf{\$}$
          and parsing table $M[A, \mathbf{a}]$ contains
              production $A \to X_1 X_2 \ldots X_n$
          then (* generate *)
              pop the parsing stack
              for $i := n$ to 1 do
              push $X_i$ onto the stack;
         else error
   if the top of the stack $= \mathbf{\$}$
   then accept
end
```

LL(1): illustration of a run of the algo



Parsing stack	Input	Action
\$ S	i(0)i(1)o e o \$	S → I
\$ I	i(0)i(1)o e o \$	I → i (E) S L
\$ L S) E (i	i(0)i(1)o e o \$	match
\$ L S) E (	(0)i(1)o e o \$	match
\$ L S) E	0)i(1)o e o \$	E → 0
\$ L S) 0	0)i(1)o e o \$	match
\$ L S)	)i(1)o e o \$	match
\$ L S	i(1)o e o \$	S → I
\$ L I	i(1)o e o \$	I → i (E) S L
\$ L L S) E (i	i(1)o e o \$	match
\$ L L S) E (	(1)o e o \$	match
\$ L L S) E	1)o e o \$	E → 1
\$ L L S) 1	1)o e o \$	match
\$ L L S)	)o e o \$	match
\$ L L S	o e o \$	S → o
\$ L L o	o e o \$	match
\$ L L	e o \$	L → e S
\$ L S e	e o \$	match
\$ L S	o \$	S → o
\$ L o	o \$	match

Expressions

Original grammar

$$\begin{array}{rcl} exp & \rightarrow & exp\ \ addop\ \ term \ \mid\ term \\ addop & \rightarrow & + \mid - \\ term & \rightarrow & term\ \ mulop\ \ factor \ \mid\ factor \\ mulop & \rightarrow & * \\ factor & \rightarrow & (\, exp\,) \ \mid\ \mathbf{number} \end{array}$$

	First	*Follow*
exp	$($, **number**	$\$,)$
exp'	$+, -, \epsilon$	$\$,)$
$addop$	$+, -$	$($, **number**
$term$	$($, **number**	$\$,), +, -$
$term'$	$*, \epsilon$	$\$,), +, -$
$mulop$	$*$	$($, **number**
$factor$	$($, **number**	$\$,), +, -, *$

Expressions

Original grammar

$$
\begin{array}{rcl}
exp & \rightarrow & exp \;\; addop \;\; term \;\mid\; term \\
addop & \rightarrow & + \mid - \\
term & \rightarrow & term \;\; mulop \;\; factor \;\mid\; factor \\
mulop & \rightarrow & * \\
factor & \rightarrow & (\, exp \,) \;\mid\; \mathbf{number}
\end{array}
$$

left-recursive ⇒ not LL(k)

	First	*Follow*
exp	$(, \mathbf{number}$	$\$,)$
exp'	$+, -, \epsilon$	$\$,)$
$addop$	$+, -$	$(, \mathbf{number}$
$term$	$(, \mathbf{number}$	$\$,), +, -$
$term'$	$*, \epsilon$	$\$,), +, -$
$mulop$	$*$	$(, \mathbf{number}$

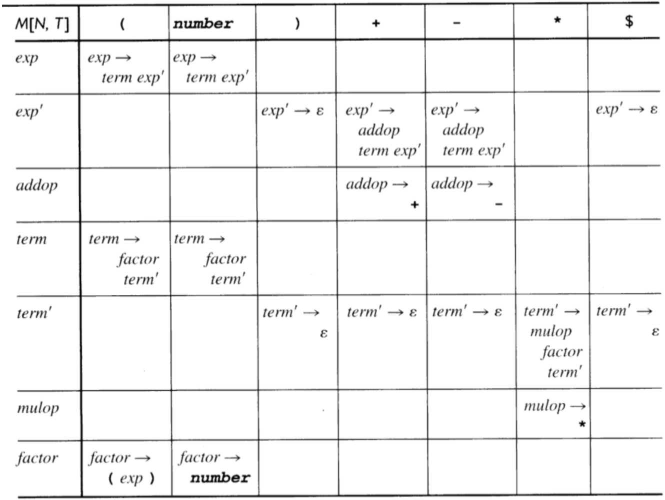
Expressions

Left-rec removed

$$
\begin{array}{rcl}
\mathit{exp} & \rightarrow & \mathit{term}\ \mathit{exp}' \\
\mathit{exp}' & \rightarrow & \mathit{addop}\ \mathit{term}\ \mathit{exp}' \mid \boldsymbol{\epsilon} \\
\mathit{addop} & \rightarrow & + \mid - \\
\mathit{term} & \rightarrow & \mathit{factor}\ \mathit{term}' \\
\mathit{term}' & \rightarrow & \mathit{mulop}\ \mathit{factor}\ \mathit{term}' \mid \boldsymbol{\epsilon} \\
\mathit{mulop} & \rightarrow & * \\
\mathit{factor} & \rightarrow & (\ \mathit{exp}\ ) \mid \mathbf{n}
\end{array}
$$

	First	*Follow*
exp	$(, \mathbf{number}$	$\$,)$
exp'	$+, -, \boldsymbol{\epsilon}$	$\$,)$
addop	$+, -$	$(, \mathbf{number}$
term	$(, \mathbf{number}$	$\$,), +, -$
term'	$*, \boldsymbol{\epsilon}$	$\$,), +, -$
mulop	$*$	$(, \mathbf{number}$
factor	$(, \mathbf{number}$	$\$,), +, -, *$

Expressions: LL(1) parse table



M[N, T]	(	**number**	)	+	-	*	$
exp	*exp* → *term exp′*	*exp* → *term exp′*					
exp′			*exp′* → ε	*exp′* → *addop term exp′*	*exp′* → *addop term exp′*		*exp′* → ε
addop				*addop* → +	*addop* → -		
term	*term* → *factor term′*	*term* → *factor term′*					
term′			*term′* → ε	*term′* → ε	*term′* → ε	*term′* → *mulop factor term′*	*term′* → ε
mulop						*mulop* → *	
factor	*factor* → (*exp*)	*factor* → **number**					

Section

Error handling

Chapter 4 “Parsing”
Course “Compiler Construction”
Martin Steffen
Spring 2021

Error handling

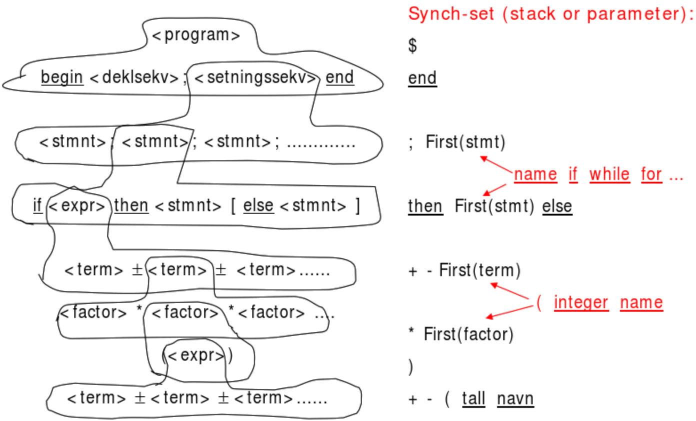
- at the least: do an understandable error message
- give indication of line / character or region responsible for the error in the source file
- potentially *stop* the parsing
- some compilers do *error recovery*
    - give an understandable error message (as minimum)
    - continue reading, until it's plausible to resume parsing ⇒ find more errors
    - however: when finding at least 1 error: no code generation
    - observation: resuming after syntax error is not easy

Error messages

- important:
  - try to avoid error messages that only occur because of an already reported error!
  - report error as early as possible, if possible at the first point where the program cannot be extended to a correct program.
  - make sure that, after an error, one doesn't end up in a infinite loop without reading any input symbols.
- What's a good error message?
  - assume: that the method `factor()` chooses the alternative (exp) but that it, when control returns from method `exp()`, does not find a)
  - one could report : `right paranthesis missing`
  - But this may often be confusing, e.g. if what the program text is: `( a + b c )`
  - here the `exp()` method will terminate after `( a + b`, as `c` cannot extend the expression). You should therefore rather give the message `error in expression or right paranthesis missing`.

Handling of syntax errors using recursive descent

Method: «Panic mode» with use of «Synchronizing set»



Synch-set (stack or parameter):

$

end

; First(stmt)

name if while for ...

then First(stmt) else

\+ - First(term)

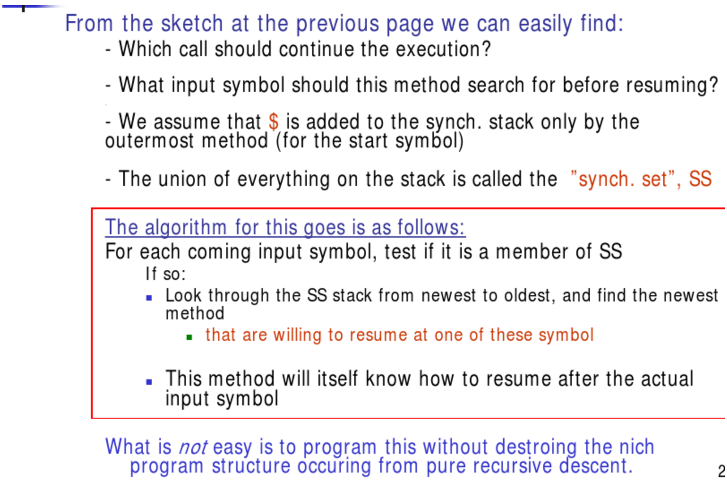
(integer name

* First(factor)

)

\+ - (tall navn

Syntax errors with sync stack



From the sketch at the previous page we can easily find:

- Which call should continue the execution?

- What input symbol should this method search for before resuming?

- We assume that $ is added to the synch. stack only by the outermost method (for the start symbol)

- The union of everything on the stack is called the "synch. set", SS

The algorithm for this goes is as follows:

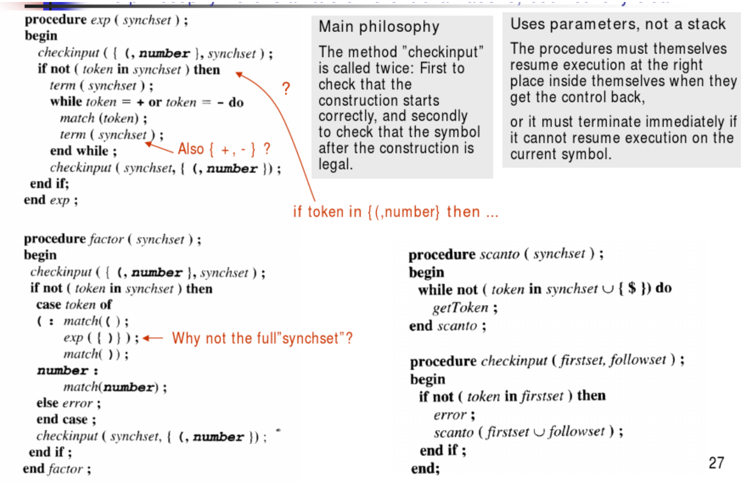
For each coming input symbol, test if it is a member of SS

If so:

- Look through the SS stack from newest to oldest, and find the newest method
  - that are willing to resume at one of these symbol
- This method will itself know how to resume after the actual input symbol

What is *not* easy is to program this without destroing the nich program structure occuring from pure recursive descent.

Procedures for expression with "error recovery"



```
procedure exp ( synchset ) ;
begin
  checkinput ( { (, number }, synchset ) ;
  if not ( token in synchset ) then
    term ( synchset ) ;
    while token = + or token = - do
      match (token) ;
      term ( synchset ) ;
    end while ;
    checkinput ( synchset, { (, number }) ;
  end if;
end exp ;
```

?

Also { + , - } ?

if token in {(,number} then ...

Main philosophy

The method "checkinput" is called twice: First to check that the construction starts correctly, and secondly to check that the symbol after the construction is legal.

Uses parameters, not a stack

The procedures must themselves resume execution at the right place inside themselves when they get the control back,

or it must terminate immediately if it cannot resume execution on the current symbol.

```
procedure factor ( synchset ) ;
begin
  checkinput ( { (, number }, synchset ) ;
  if not ( token in synchset ) then
    case token of
    ( :  match( ( ) ;
         exp ( { ) } ) ;
         match( ) ) ;
    number :
         match(number) ;
    else error ;
    end case ;
    checkinput ( synchset, { (, number }) ;
  end if ;
end factor ;
```

Why not the full"synchset"?

```
procedure scanto ( synchset ) ;
begin
  while not ( token in synchset ∪ { $ }) do
    getToken ;
end scanto ;

procedure checkinput ( firstset, followset ) ;
begin
  if not ( token in firstset ) then
    error ;
    scanto ( firstset ∪ followset ) ;
  end if ;
end;
```

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Section

Bottom-up parsing

Chapter 4 “Parsing”
Course “Compiler Construction”
Martin Steffen
Spring 2021

Bottom-up parsing: intro

"R" stands for *right-most* derivation.

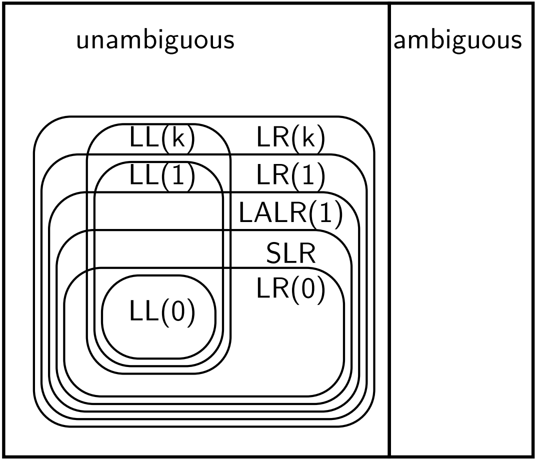
LR(0)
- only for very simple grammars
- approx. 300 states for standard programming languages
- only as warm-up for SLR(1) and LALR(1)

SLR(1)
- expressive enough for most grammars for standard PLs
- same number of states as LR(0)
- main focus here

LALR(1)
- slightly more expressive than SLR(1)
- same number of states as LR(0)
- we look at ideas behind that method as well

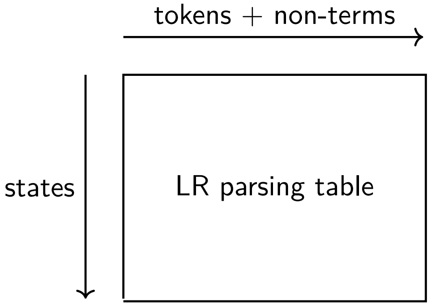
LR(1) covers all grammars, which can in principle be parsed by looking at the next token

Grammar classes overview (again)



LR-parsing and its subclasses

- *right-most* derivation (but left-to-right parsing)
- in general: bottom-up: more powerful than top-down
- typically: tool-supported (unlike recursive descent, which may well be hand-coded)
- based on *parsing tables* + explicit *stack*
- thankfully: *left-recursion* no longer problematic
- typical tools: yacc and friends (like bison, CUP, etc.)
- another name: *shift-reduce* parser

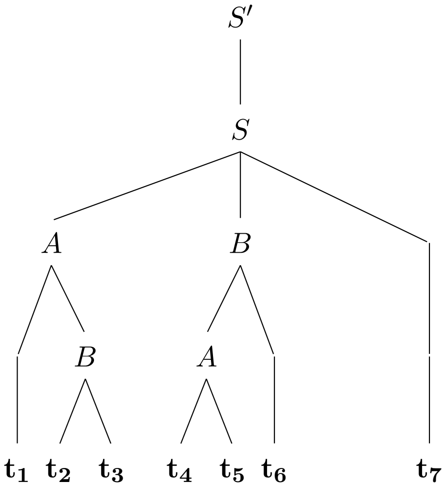


Example grammar

$$
\begin{array}{rcl}
S' & \rightarrow & S \\
S & \rightarrow & AB\mathbf{t_7} \mid \ldots \\
A & \rightarrow & \mathbf{t_4 t_5} \mid \mathbf{t_1} B \mid \ldots \\
B & \rightarrow & \mathbf{t_2 t_3} \mid A\mathbf{t_6} \mid \ldots
\end{array}
$$

- assume: grammar unambiguous
- assume word of terminals $\mathbf{t_1 t_2 \ldots t_7}$ and its (unique) parse-tree
- general agreement for bottom-up parsing:
  - start symbol *never* on the right-hand side of a production
  - routinely add another "extra" start-symbol (here S')

Parse tree for $\mathbf{t}_1 \ldots \mathbf{t}_7$



LR: left-to right scan, right-most derivation?

Potentially puzzling question at first sight:

what?: *right*-most derivation, when parsing *left*-to-right?

- short answer: parser builds the parse tree bottom-up
- derivation:
    - replacement of nonterminals by right-hand sides
    - *derivation*: builds (implicitly) a parse-tree *top-down*

Right-sentential form: right-most derivation

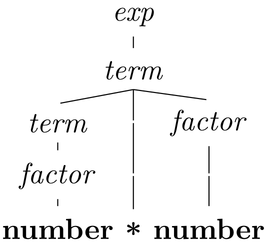
$$S \Rightarrow_r^* \alpha$$

Slighly longer answer

LR parser parses from left-to-right and builds the parse tree bottom-up. When doing the parse, the parser (implicitly) builds a *right-most* derivation in reverse (because of bottom-up).

Example expression grammar (from before)

$$
\begin{aligned}
exp &\rightarrow exp \ addop \ term \mid term \\
addop &\rightarrow + \mid - \\
term &\rightarrow term \ mulop \ factor \mid factor \\
mulop &\rightarrow * \\
factor &\rightarrow (\, exp \,) \mid \mathbf{number}
\end{aligned}
\tag{9}
$$

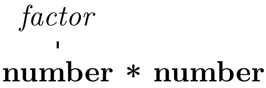


Bottom-up parse: Growing the parse tree

$$\mathbf{number} * \mathbf{number}$$

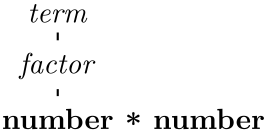
$$\underline{\mathbf{number}} * \mathbf{number}$$

Bottom-up parse: Growing the parse tree



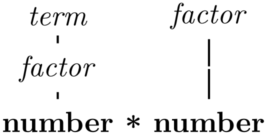
$$\underline{\mathbf{number}} * \mathbf{number} \quad \hookrightarrow \quad \underline{factor} * \mathbf{number}$$

Bottom-up parse: Growing the parse tree



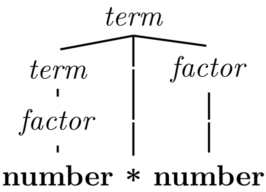
$$\begin{array}{lcl}\underline{\mathbf{number}} * \mathbf{number} & \hookrightarrow & \underline{\mathit{factor}} * \mathbf{number} \\ & \hookrightarrow & \mathit{term} * \underline{\mathbf{number}}\end{array}$$

Bottom-up parse: Growing the parse tree



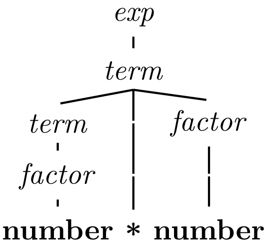
$$\begin{aligned} \underline{\textbf{number}} * \textbf{number} &\hookrightarrow \underline{factor} * \textbf{number} \\ &\hookrightarrow term * \underline{\textbf{number}} \\ &\hookrightarrow \underline{term * factor} \end{aligned}$$

Bottom-up parse: Growing the parse tree



$$\begin{array}{lll}\underline{\mathbf{number}} * \mathbf{number} & \hookrightarrow & \underline{\mathit{factor}} * \mathbf{number} \\ & \hookrightarrow & \mathit{term} * \underline{\mathbf{number}} \\ & \hookrightarrow & \underline{\mathit{term} * \mathit{factor}} \\ & \hookrightarrow & \underline{\mathit{term}}\end{array}$$

Bottom-up parse: Growing the parse tree



exp

term

term *factor*

factor

number * **number**

$$\begin{array}{rcl} \underline{\textbf{number}} * \textbf{number} & \hookrightarrow & \underline{\mathit{factor}} * \textbf{number} \\ & \hookrightarrow & \mathit{term} * \underline{\textbf{number}} \\ & \hookrightarrow & \underline{\mathit{term} * \mathit{factor}} \\ & \hookrightarrow & \underline{\mathit{term}} \\ & \hookrightarrow & \mathit{exp} \end{array}$$

Reduction in reverse = right derivation

Reduction

$$\begin{array}{lll} \underline{\mathbf{n}} * \mathbf{n} & \hookrightarrow & \underline{factor} * \mathbf{n} \\ & \hookrightarrow & term * \underline{\mathbf{n}} \\ & \hookrightarrow & \underline{term * factor} \\ & \hookrightarrow & \underline{term} \\ & \hookrightarrow & exp \end{array}$$

Right derivation

$$\begin{array}{lll} \mathbf{n} * \mathbf{n} & \Leftarrow_r & \underline{factor} * \mathbf{n} \\ & \Leftarrow_r & \underline{term} * \mathbf{n} \\ & \Leftarrow_r & term * \underline{factor} \\ & \Leftarrow_r & \underline{term} \\ & \Leftarrow_r & \underline{exp} \end{array}$$

- underlined part:
  - *different* in reduction vs. derivation
  - represents the "part being replaced"
    - for derivation: right-most non-terminal
    - for reduction: indicates the so-called handle (or part of it)
- consequently: all intermediate words are *right-sentential forms*

Handle

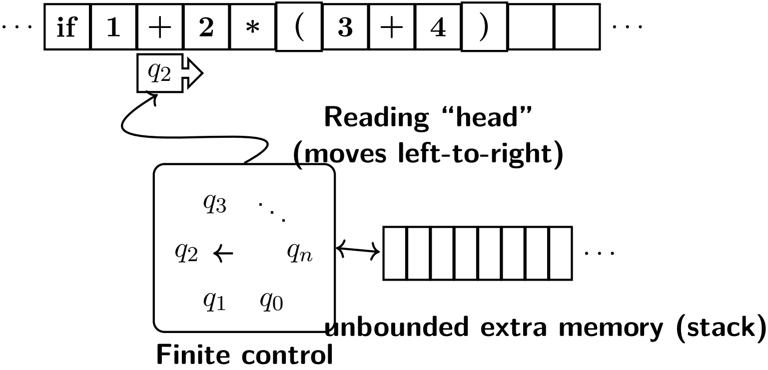
Definition (Handle)

Assume $S \Rightarrow_r^* \alpha A w \Rightarrow_r \alpha\beta w$. A production $A \to \beta$ at position k following α is a *handle of $\alpha\beta w$*. We write $\langle A \to \beta, k\rangle$ for such a handle.

Note:

- w (right of a handle) contains only terminals
- w: corresponds to the future input still to be parsed!
- $\alpha\beta$ will correspond to the stack content (β the part touched by reduction step).
- the $\Rightarrow_r$ -derivation-step *in reverse*:
  - one reduce-step in the LR-parser-machine
  - adding (implicitly in the LR-machine) a new parent to children β (= bottom-up!)
- "handle"-part β can be *empty* ($= \epsilon$)

Schematic picture of parser machine (again)



General LR "parser machine" configuration

- *stack*:
  - contains: terminals + non-terminals (+ $)
  - containing: what has been read already but not yet "processed"
- *position* on the "tape" (= token stream)
  - represented here as word of terminals *not yet read*
  - end of "rest of token stream": $, as usual
- *state* of the machine
  - in the following schematic illustrations: *not* yet part of the discussion
  - *later*: part of the parser table, currently we explain *without* referring to the state of the parser-engine
  - currently we assume: tree and rest of the input given
  - the trick ultimately will be: how do achieve the same *without that tree already given* (just parsing left-to-right)

Schematic run (reduction: from top to bottom)

$\$$	$\mathbf{t}_1\mathbf{t}_2\mathbf{t}_3\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,\mathbf{t}_1$	$\mathbf{t}_2\mathbf{t}_3\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,\mathbf{t}_1\mathbf{t}_2$	$\mathbf{t}_3\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,\mathbf{t}_1\mathbf{t}_2\mathbf{t}_3$	$\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,\mathbf{t}_1 B$	$\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,A$	$\mathbf{t}_4\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,A\mathbf{t}_4$	$\mathbf{t}_5\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,A\mathbf{t}_4\mathbf{t}_5$	$\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,AA$	$\mathbf{t}_6\mathbf{t}_7\,\$$
$\$\,AA\mathbf{t}_6$	$\mathbf{t}_7\,\$$
$\$\,AB$	$\mathbf{t}_7\,\$$
$\$\,AB\mathbf{t}_7$	$\$$
$\$\,S$	$\$$
$\$\,S'$	$\$$

2 basic steps: shift and reduce

- parsers reads input and uses stack as intermediate storage
- so far: no mention of look-ahead (i.e., action depending on the value of the next token(s)), but that may play a role, as well

Shift

Move the next input symbol (terminal) over to the top of the stack ("push")

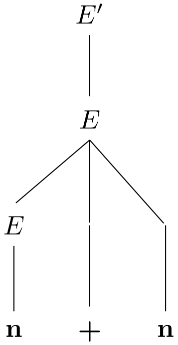
Reduce

Remove the symbols of the *right-most* subtree from the stack and replace it by the non-terminal at the root of the subtree (replace = "pop + push").

- decision *easy* to do if one has the parse tree already!
- *reduce* step: popped resp. pushed part = right- resp. left-hand side of handle

Example: LR parse for "+" (given the tree)

$$
\begin{array}{rcl}
E' & \rightarrow & E \\
E & \rightarrow & E + \mathbf{n} \mid \mathbf{n}
\end{array}
$$



	parse stack	input	action
1	$\$$	$\mathbf{n} + \mathbf{n}\,\$$	shift
2	$\$\,\mathbf{n}$	$+\,\mathbf{n}\,\$$	red:. $E \rightarrow \mathbf{n}$
3	$\$\,E$	$+\,\mathbf{n}\,\$$	shift
4	$\$\,E +$	$\mathbf{n}\,\$$	shift
5	$\$\,E + \mathbf{n}$	$\$$	reduce $E \rightarrow E + \mathbf{n}$
6	$\$\,E$	$\$$	red.: $E' \rightarrow E$
7	$\$\,E'$	$\$$	accept

note: line 3 vs line 6!; both contain E on top of stack

(right) derivation: reduce-steps "in reverse"

$$\underline{E'} \Rightarrow \underline{E} \Rightarrow \underline{E} + \mathbf{n} \Rightarrow \mathbf{n} + \mathbf{n}$$

Example with ϵ-transitions: parentheses

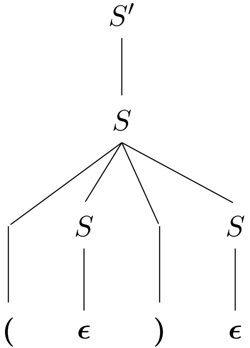
$$
\begin{array}{rcl}
S' & \rightarrow & S \\
S & \rightarrow & (\,S\,)\,S \;\mid\; \epsilon
\end{array}
$$

side remark: unlike previous grammar, here:

- production with *two* non-terminals on the right

⇒ difference between left-most and right-most derivations (and mixed ones)

Parentheses: run and right-most derivation



	parse stack	input	action
1	\$	( ) \$	shift
2	\$ (	) \$	reduce $S \rightarrow \epsilon$
3	\$ (S	) \$	shift
4	\$ (S)	\$	reduce $S \rightarrow \epsilon$
5	\$ (S) S	\$	reduce $S \rightarrow (\,S\,)\,S$
6	\$ S	\$	reduce $S' \rightarrow S$
7	\$ S'	\$	accept

Note: the 2 reduction steps for the ϵ productions

Right-most derivation and right-sentential forms

$$\underline{S'} \Rightarrow_r \underline{S} \Rightarrow_r (\,S\,)\,\underline{S} \Rightarrow_r (\,\underline{S}\,) \Rightarrow_r (\ )$$

Right-sentential forms & the stack

Right-sentential form: right-most derivation

$$S \Rightarrow_r^* \alpha$$

- right-sentential forms:
  - part of the “run”
  - but: split between *stack* and *input*

	parse stack	input	action
1	$\$$	$\mathbf{n}+\mathbf{n}\,\$$	shift
2	$\$\,\mathbf{n}$	$+\mathbf{n}\,\$$	red:. $E \to \mathbf{n}$
3	$\$\,E$	$+\mathbf{n}\,\$$	shift
4	$\$\,E+$	$\mathbf{n}\,\$$	shift
5	$\$\,E+\mathbf{n}$	$\$$	reduce $E \to E+\mathbf{n}$
6	$\$\,E$	$\$$	red.: $E' \to E$
7	$\$\,E'$	$\$$	accept

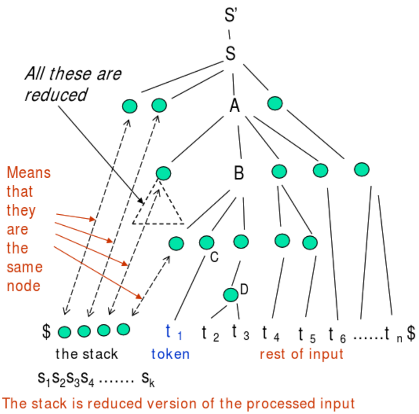
$$\underline{E'} \Rightarrow_r \underline{E} \Rightarrow_r \underline{E+\mathbf{n}} \Rightarrow_r \mathbf{n}+\mathbf{n}$$

$$\underline{\mathbf{n}}+\mathbf{n} \hookrightarrow \underline{E+\mathbf{n}} \hookrightarrow \underline{E} \hookrightarrow E'$$

Viable prefixes of right-sentential forms and handles

- right-sentential form: $E + \mathbf{n}$
- viable prefixes of RSF
  - prefixes of that RSF *on the stack*
  - here: 3 viable prefixes of that RSF: E, $E+$, $E + \mathbf{n}$
- *handle*: remember the definition earlier
- here: for instance in the sentential form $\mathbf{n} + \mathbf{n}$
  - handle is production $E \to \mathbf{n}$ on the *left* occurrence of $\mathbf{n}$ in $\mathbf{n} + \mathbf{n}$ (let's write $\mathbf{n}_1 + \mathbf{n}_2$ for now)
  - note: in the stack machine:
    - the left $\mathbf{n}_1$ on the stack
    - rest $+ \mathbf{n}_2$ on the input (unread, because of LR(0))
- if the parser engine detects handle $\mathbf{n}_1$ on the stack, it does a *reduce*-step
- However (later): reaction depends on current *state* of the parser engine

A typical situation during LR-parsing



After a shift, the next reduction to be made is a reduction with the production:

C -> t1

Then, after two shifts, we will make a reduction with the production:

D -> t2 t3

Then, what's next?

General design for an LR-engine

- some ingredients clarified up-to now:
    - bottom-up tree building as reverse right-most derivation,
    - stack vs. input,
    - shift and reduce steps
- however: 1 ingredient missing: next step of the engine may depend on
    - top of the stack (“handle”)
    - look ahead on the input (but not for LL(0))
    - and: current state of the machine

But what are the states of an LR-parser?

General idea:

Construct an NFA (and ultimately DFA) which works on the stack (not the input). The alphabet consists of terminals and non-terminals $\Sigma_T \cup \Sigma_N$. The language

$$Stacks(G) = \{\alpha \ \mid \text{α may occur on the stack during LR-parsing of a sentence in $\mathcal{L}(G)$}\}$$

is regular!

LR(0) parsing as easy pre-stage

- LR(0): in practice *too simple*, but easy conceptual step towards LR(1), SLR(1) etc.
- LR(1): in practice good enough, LR(k) not used for $k > 1$
- to build the automaton: LR(0)-items

LR(0) items

LR(0) item

production with specific “parser position” . in its right-hand side

- . : “meta-symbol” (not part of the production)

LR(0) item for a production $A \to \beta\gamma$

$$A \to \beta.\gamma$$

- item with dot at the beginning: *initial* item
- item with dot at the end: *complete* item

Grammar for parentheses: 3 productions

$$\begin{array}{rcl} S' & \to & S \\ S & \to & (\, S\,)\, S \;\mid\; \epsilon \end{array}$$

8 items

$$\begin{array}{rcl} S' & \to & .S \\ S' & \to & S. \\ S & \to & .\,(\, S\,)\, S \\ S & \to & (\, .S\,)\, S \\ S & \to & (\, S.\,)\, S \\ S & \to & (\, S\,)\, .S \\ S & \to & (\, S\,)\, S. \\ S & \to & . \end{array}$$

- $S \to \epsilon$ gives $S \to .$ as item (not $S \to \epsilon.$ and $S \to .\epsilon$)

Grammar for addition: 3 productions

$$\begin{array}{rcl} E' & \to & E \\ E & \to & E + \textbf{number} \mid \textbf{number} \end{array}$$

(coincidentally also:) 8 items

$$\begin{array}{rcl} E' & \to & .E \\ E' & \to & E. \\ E & \to & .E + \textbf{number} \\ E & \to & E. + \textbf{number} \\ E & \to & E + .\textbf{number} \\ E & \to & E + \textbf{number}. \\ E & \to & .\textbf{number} \\ E & \to & \textbf{number}. \end{array}$$

Finite automata of items

- general set-up: *items* as states in an automaton
- automaton: "operates" *not* on the input, but the stack
- automaton either
    - first NFA, afterwards made deterministic (subset construction), or
    - directly DFA

States formed of sets of items

In a state marked by/containing item

$$A \to \beta.\gamma$$

- β on the *stack*
- γ: to be treated next (terminals on the input, but can contain also non-terminals(!))

State transitions of the NFA

- $X \in \Sigma$
- two kinds of transitions

Terminal or non-terminal

$$\boxed{A \to \alpha.X\eta} \xrightarrow{X} \boxed{A \to \alpha X.\eta}$$

ϵ $(X \to \beta)$

$$\boxed{A \to \alpha.X\eta} \xrightarrow{\epsilon} \boxed{X \to .\beta}$$

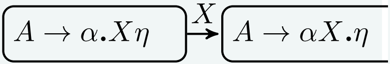
- In case $X =$ *terminal* (i.e. token) $=$
  - the left step corresponds to a shift step
- for non-terminals (see next slide):
  - interpretation more complex: non-terminals are officially never on the input
  - note: in that case, item $A \to \alpha.X\eta$ has two (kinds of) outgoing transitions

Transitions for non-terminals and ϵ

- so far: we never pushed a non-terminal from the input to the stack, we replace in a reduce-step the right-hand side by a left-hand side
- but: replacement in a reduce steps can be seen as
  1. pop right-hand side off the stack,
  2. instead, “assume” corresponding non-terminal on input,
  3. eat the non-terminal an push it on the stack.
- two kinds of transitions
- assume production $X \rightarrow \beta$ and *initial* item $X \rightarrow .\beta$

Transitions (repeated)

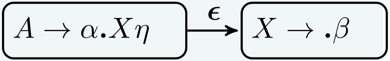
Terminal or non-terminal



$$A \to \alpha.X\eta \xrightarrow{X} A \to \alpha X.\eta$$

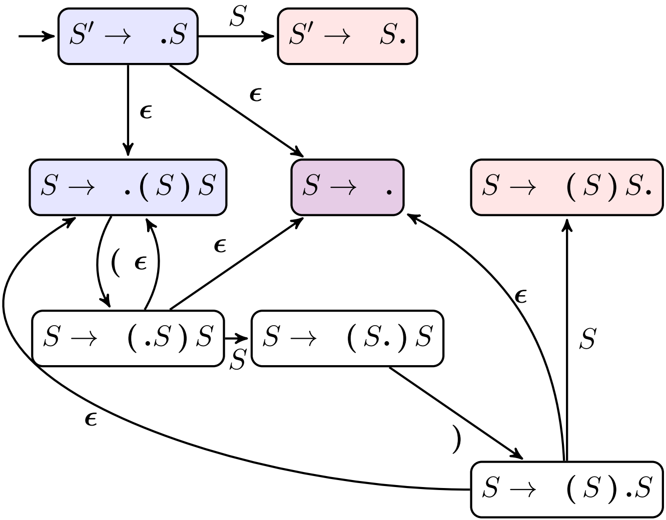
Epsilon (X: non-terminal here)

Given production $X \to \beta$:



$$A \to \alpha.X\eta \xrightarrow{\epsilon} X \to .\beta$$

NFA: parentheses



Initial and final states

initial states:

- we made our lives *easier*: assume one *extra* start symbol say S' (augmented grammar)

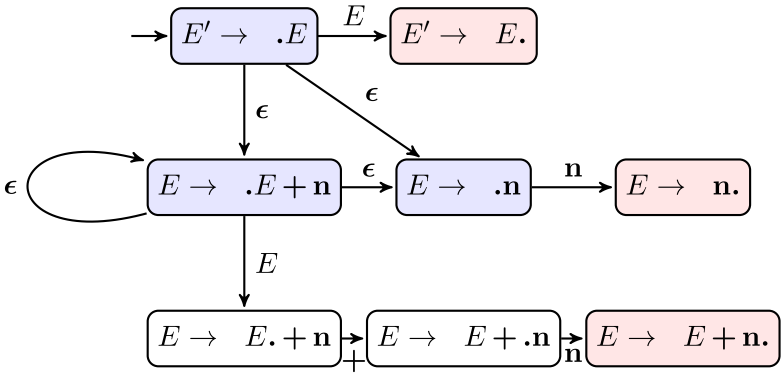
⇒ initial item $S' \rightarrow .S$ as (only) initial state

final states:

acceptance condition of the *overall* machine: a bit more complex

- input must be empty
- stack must be empty except the (new) start symbol
- NFA has a word to say about acceptence
  - but *not* in form of being in an accepting state
  - so: no accepting *states*
  - but: accepting *action* (see later)

NFA: addition



Determinizing: from NFA to DFA

- standard subset-construction[7]
- states then contain *sets* of items
- important: ϵ-closure
- also: *direct* construction of the DFA possible

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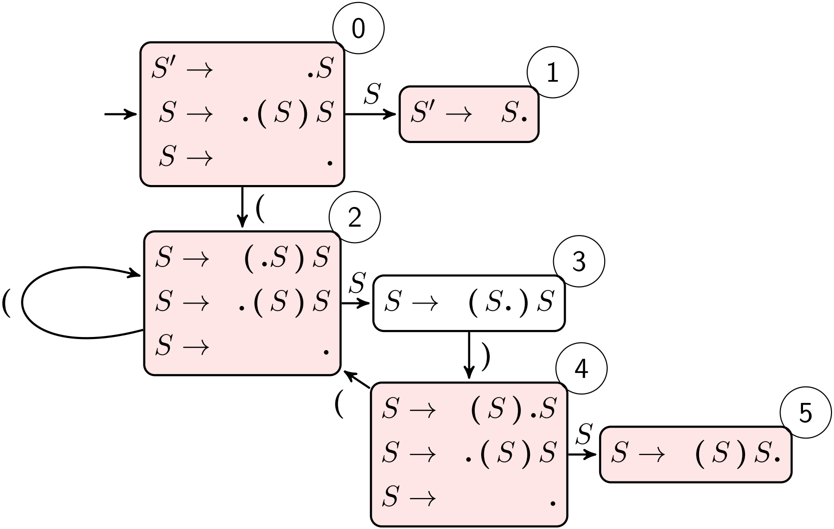
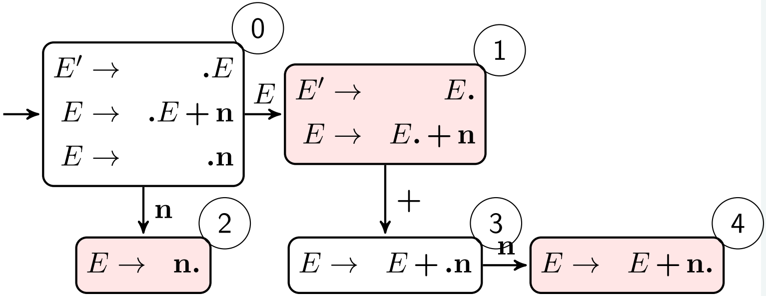
[7] Technically, we don't require here a *total* transition function, we leave out any error state.

DFA: parentheses



DFA: addition



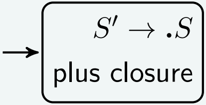
Direct construction of an LR(0)-DFA

- quite easy: just build in the closure directly…

ϵ-closure

- if $A \to \alpha.B\gamma$ is an item in a state where
- there are productions $B \to \beta_1 \mid \beta_2 \ldots$ then
- add items $B \to .\beta_1$, $B \to .\beta_2$ … to the state
- continue that process, until saturation

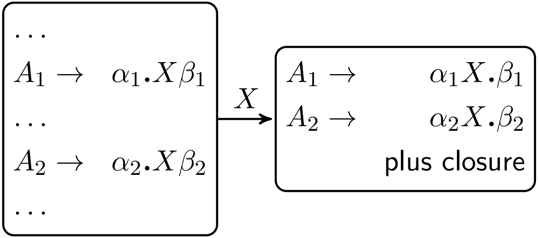
initial state



$S' \to .S$

plus closure

Direct DFA construction: transitions



$\dots$

$A_1 \to \quad \alpha_1.X\beta_1$

$\dots$

$A_2 \to \quad \alpha_2.X\beta_2$

$\dots$

X

$A_1 \to \quad \alpha_1 X.\beta_1$

$A_2 \to \quad \alpha_2 X.\beta_2$

plus closure

- X: terminal or non-terminal, both treated uniformely
- *All* items of the form $A \to \alpha.X\beta$ must be included in the post-state
- and all others (indicated by "$\dots$") in the pre-state: not included

How does the DFA do the shift/reduce and the rest?

- we have seen: bottom-up parse tree generation
- we have seen: shift-reduce and the stack vs. input
- we have seen: the construction of the DFA

But: how does it hang together?

We need to interpret the "set-of-item-states" in the light of the stack content and figure out the reaction in terms of

- transitions in the automaton
- stack manipulations (shift/reduce)
- acceptance
- input (apart from shifting) not relevant when doing LR(0)

and the reaction better be uniquely determined . . . .

Stack contents and state of the automaton

- remember: at any config. of stack/input in a run
  1. stack contains words from Σ^*
  2. DFA operates deterministically on such words
- the stack contains "abstraction of the past":
- when feeding that "past" on the stack into the automaton
  - starting with the oldest symbol (not in a LIFO manner)
  - starting with the DFA's initial state

  ⇒ stack content determines state of the DFA
- actually: each prefix also determines uniquely a state
- top state:
  - state after the complete stack content
  - corresponds to the current state of the stack-machine

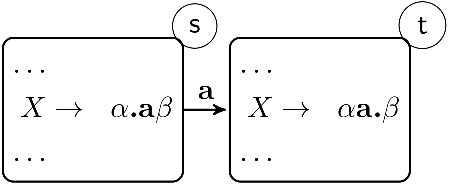
  ⇒ crucial when determining *reaction*

State transition allowing a shift

- assume: top-state (= current state) contains item

$$X \to \alpha.\mathbf{a}\beta$$

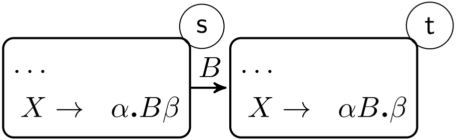
- construction thus has transition as follows



- shift is possible
- if shift is *the* correct operation and $\mathbf{a}$ is terminal symbol corresponding to the current token: state afterwards $= t$

State transition: analogous for non-term's

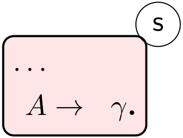
$X \to \alpha.B\beta$



- "goto = shift for non-terms"
- intuition: "second half of a reduce step"

State (not transition) where a reduce is possible

- remember: *complete items*
- assume top state s containing complete item $A \to \gamma.$



- a complete right-hand side ("handle") γ on the stack and thus done
- may be replaced by right-hand side A

$\Rightarrow$ reduce step

- builds up (implicitly) new parent node A in the bottom-up procedure
- Note: A on top of the stack instead of γ:
  - new top state!
  - remember the "goto-transition" (shift of a non-terminal)

Remarks: states, transitions, and reduce steps

- ignoring the ϵ-transitions (for the NFA)
- there are 2 “kinds” of transitions in the DFA
  1. terminals: reals shifts
  2. non-terminals: “following a reduce step”

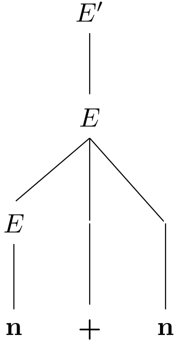
No edges to represent (all of) a reduce step!

- if a reduce happens, parser engine *changes state*!
- however: this state change is not represented by a transition in the DFA (or NFA for that matter)
- especially *not* by outgoing errors of completed items

- if the (rhs of the) handle is *removed* from top stack ⇒
  - “go back to the (top) state before that handle had been added”: *no edge for that*
- later: stack notation simply remembers the state as part of its configuration

Example: LR parsing for addition (given the tree)

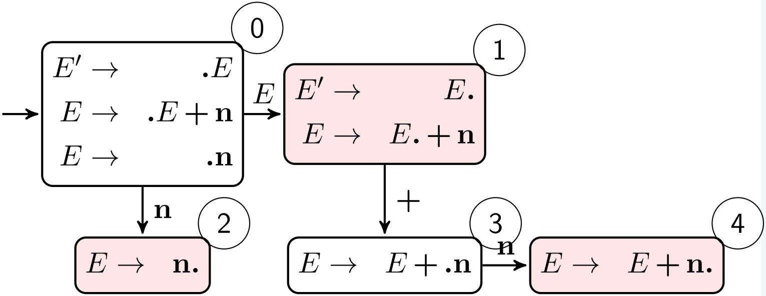
$$\begin{array}{rcl} E' & \rightarrow & E \\ E & \rightarrow & E + \mathbf{n} \mid \mathbf{n} \end{array}$$



	parse stack	input	action
1	$\$$	$\mathbf{n} + \mathbf{n}\,\$$	shift
2	$\$\,\mathbf{n}$	$+\,\mathbf{n}\,\$$	red:. $E \rightarrow \mathbf{n}$
3	$\$\,E$	$+\,\mathbf{n}\,\$$	shift
4	$\$\,E +$	$\mathbf{n}\,\$$	shift
5	$\$\,E + \mathbf{n}$	$\$$	reduce $E \rightarrow E + \mathbf{n}$
6	$\$\,E$	$\$$	red.: $E' \rightarrow E$
7	$\$\,E'$	$\$$	accept

note: line 3 vs line 6!; both contain E on top of stack

DFA of addition example



- note line 3 vs. line 6
- both stacks $= E \Rightarrow$ same (top) state in the DFA (state 1)

LR(0) grammars

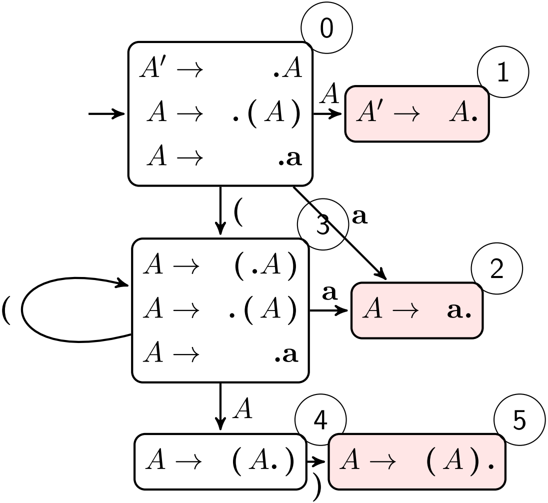
LR(0) grammar

The top-state alone determines the next step.

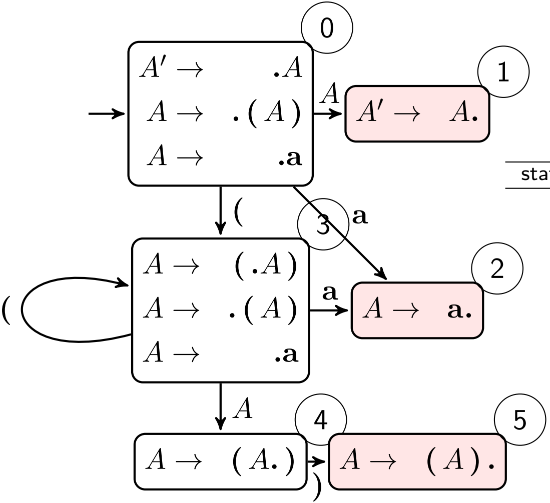
- especially: no shift/reduce conflicts in the form shown
- thus: previous addition-grammar is *not LR(0)*

Simple parentheses

$$A \rightarrow (A) \mid \mathbf{a}$$

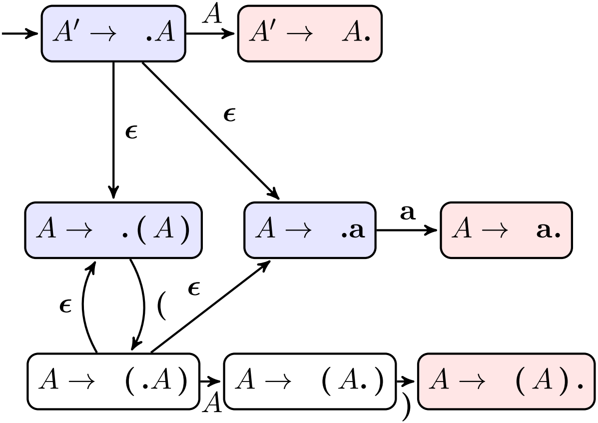


Simple parentheses is LR(0)



state	possible action
0	only shift
1	only red: $(A' \to A)$
2	only red: $(A \to \mathbf{a})$
3	only shift
4	only shift
5	only red $(A \to (\,A\,))$

NFA for simple parentheses (bonus slide)



Parsing table for an LR(0) grammar

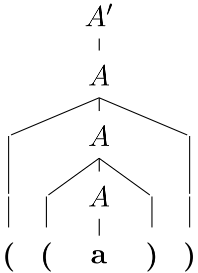
- table structure: slightly different for SLR(1), LALR(1), and LR(1) (see later)
- note: the “goto” part: “shift” on non-terminals (only 1 non-terminal A here)
- corresponding to the A-labelled transitions

state	action	rule	input			goto
			(	$\mathbf{a}$	)	A
0	shift		3	2		1
1	reduce	$A' \to A$				
2	reduce	$A \to \mathbf{a}$				
3	shift		3	2		4
4	shift				5	
5	reduce	$A \to (\, A\,)$				

Parsing of $((\mathbf{a}))$

stage	parsing stack	input	action
1	$\$_0$	$((\mathbf{a}))\$$	shift
2	$\$_0(_3$	$(\mathbf{a}))\$$	shift
3	$\$_0(_3(_3$	$\mathbf{a}))\$$	shift
4	$\$_0(_3(_3\mathbf{a}_2$	$))\$$	reduce $A \rightarrow \mathbf{a}$
5	$\$_0(_3(_3A_4$	$))\$$	shift
6	$\$_0(_3(_3A_4)_5$	$)\$$	reduce $A \rightarrow (A)$
7	$\$_0(_3A_4$	$)\$$	shift
8	$\$_0(_3A_4)_5$	$\$$	reduce $A \rightarrow (A)$
9	$\$_0A_1$	$\$$	accept

Parse tree of the parse



- As said:
  - the reduction "contains" the parse-tree
  - reduction: builds it bottom up
  - reduction in reverse: contains a *right-most* derivation (which is "top-down")
- accept action: corresponds to the parent-child edge $A' \rightarrow A$ of the tree

Parsing of erroneous input

- empty slots it the table: “errors”

stage	parsing stack	input	action
1	$\$_0$	$(\,(\,\mathbf{a}\,)\,\$$	shift
2	$\$_0(_3$	$(\,\mathbf{a}\,)\,\$$	shift
3	$\$_0(_3(_3$	$\mathbf{a}\,)\,\$$	shift
4	$\$_0(_3(_3\mathbf{a}_2$	$)\,\$$	reduce $A \to \mathbf{a}$
5	$\$_0(_3(_3A_4$	$)\,\$$	shift
6	$\$_0(_3(_3A_4)_5$	$\$$	reduce $A \to (\,A\,)$
7	$\$_0(_3A_4$	$\$$	????

stage	parsing stack	input	action
1	$\$_0$	$(\,)\,\$$	shift
2	$\$_0(_3$	$)\,\$$	?????

Invariant

important general invariant for LR-parsing: never shift something “illegal” onto the stack

LR(0) parsing algo, given DFA

let s be the current state, on top of the parse stack

1. s contains $A \rightarrow \alpha.X\beta$, where X is a *terminal*
   - shift X from input to top of stack. The new *state* pushed on the stack: state t where $s \xrightarrow{X} t$
   - else: if s does not have such a transition: *error*
2. s contains a complete item (say $A \rightarrow \gamma.$): reduce by rule $A \rightarrow \gamma$:
   - A reduction by $S' \rightarrow S$: accept, if input is empty; else error:
   - else:

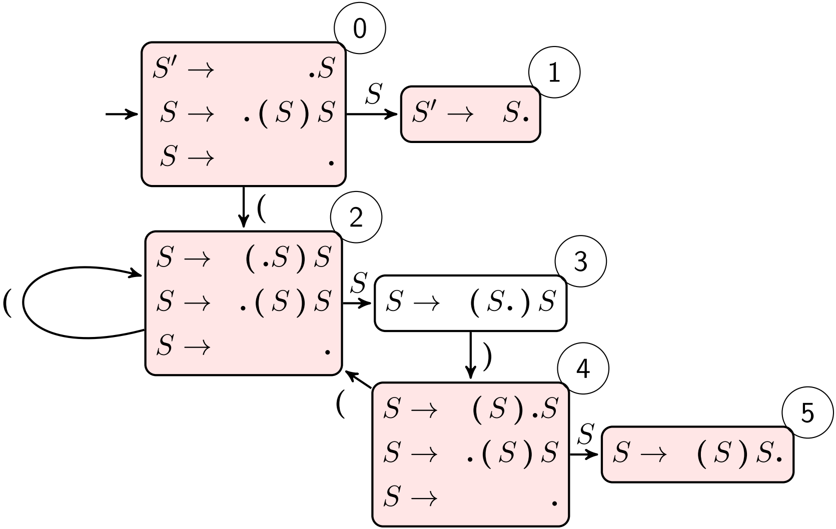
     pop: remove γ (including "its" states from the stack)

     back up: assume to be in state u which is *now* head state

     push: push A to the stack, new head state t where $u \xrightarrow{A} t$ (in the DFA)

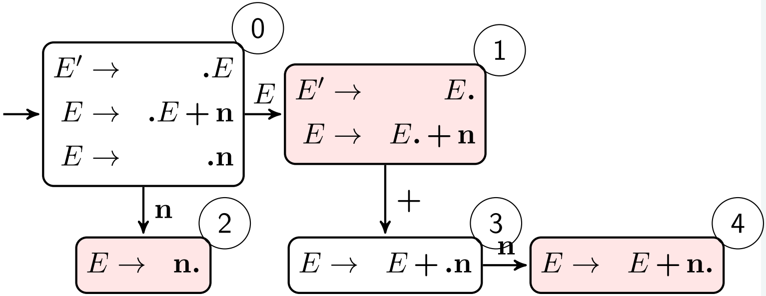
DFA parentheses again: LR(0)?

$$\begin{array}{rcl} S' & \to & S \\ S & \to & (\,S\,)\,S \;\mid\; \epsilon \end{array}$$

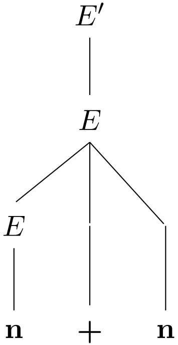


DFA addition again: LR(0)?

$$\begin{array}{rcl} E' & \to & E \\ E & \to & E + \textbf{number} \mid \textbf{number} \end{array}$$



Decision? If only we knew the ultimate tree already (expecially the parts still to come). . .



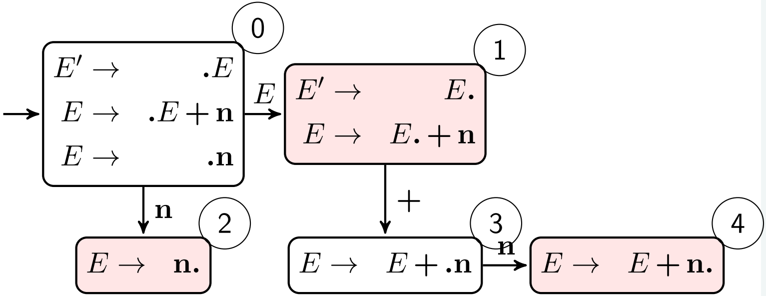
	parse stack	input	action
1	$\$$	$\mathbf{n}+\mathbf{n}\,\$$	shift
2	$\$\,\mathbf{n}$	$+\mathbf{n}\,\$$	red:. $E \to \mathbf{n}$
3	$\$\,E$	$+\mathbf{n}\,\$$	shift
4	$\$\,E+$	$\mathbf{n}\,\$$	shift
5	$\$\,E+\mathbf{n}$	$\$$	reduce $E \to E+\mathbf{n}$
6	$\$\,E$	$\$$	red.: $E' \to E$
7	$\$\,E'$	$\$$	accept

- current stack: represents already known part of the parse tree
- since we don't have the future parts of the tree yet:

⇒ look-ahead on the input (without building the tree yet)

- LR(1) and its variants: *look-ahead of 1*

Addition grammar (again)



- *How to make a decision in state 1?* (here: shift vs. reduce)

⇒ look at the next input symbol (in the token)

One look-ahead

- LR(0), not useful, too weak
- add look-ahead, here of *1 input symbol* (= token)
- different variations of that idea (with slight difference in expresiveness)
- tables slightly changed (compared to LR(0))
- but: *still* can use the LR(0)-DFAs

Resolving LR(0) reduce/reduce conflicts

LR(0) reduce/reduce conflict:

$$\boxed{\begin{array}{r} \ldots \\ A \to \alpha. \\ \ldots \\ B \to \beta. \end{array}}$$

Resolving LR(0) reduce/reduce conflicts

LR(0) reduce/reduce conflict:

$$\boxed{\begin{array}{r} \dots \\ A \to \alpha. \\ \dots \\ B \to \beta. \end{array}}$$

SLR(1) solution: use follow sets of non-terms

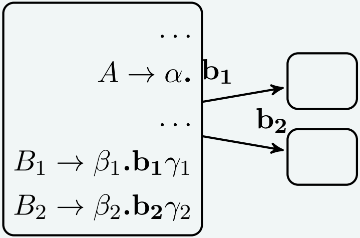
- If $Follow(A) \cap Follow(B) = \emptyset$

⇒ next symbol (in `token`) decides!

  - if `token` $\in Follow(\alpha)$ then reduce using $A \to \alpha$
  - if `token` $\in Follow(\beta)$ then reduce using $B \to \beta$
  - ...

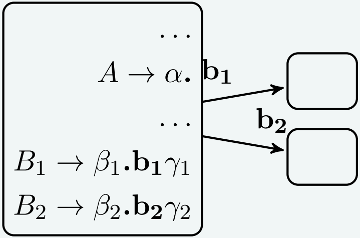
Resolving LR(0) shift/reduce conflicts

LR(0) shift/reduce conflict:



Resolving LR(0) shift/reduce conflicts

LR(0) shift/reduce conflict:



$\dots$

$A \to \alpha.$

$\dots$

$B_1 \to \beta_1.\mathbf{b_1}\gamma_1$

$B_2 \to \beta_2.\mathbf{b_2}\gamma_2$

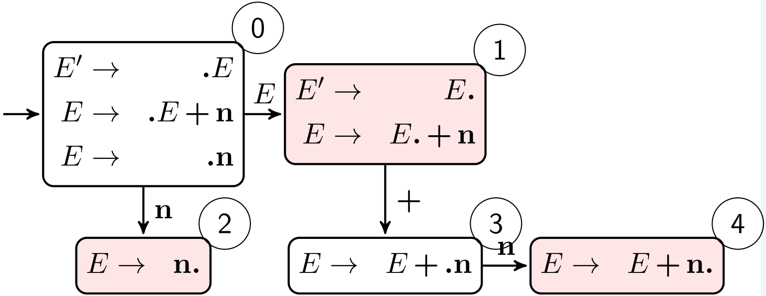
SLR(1) solution: again: use follow sets of non-terms

- If $\mathit{Follow}(A) \cap \{\mathbf{b_1}, \mathbf{b_2}, \dots\} = \emptyset$

⇒ next symbol (in `token`) decides!

  - if `token` $\in \mathit{Follow}(A)$ then *reduce* using $A \to \alpha$, non-terminal A determines new top state
  - if `token` $\in \{\mathbf{b_1}, \mathbf{b_2}, \dots\}$ then *shift*. Input symbol $\mathbf{b_i}$ determines new top state
  -

Revisit addition one more time



- $Follow(E') = \{\$\}$

$\Rightarrow$
  - shift for $+$
  - reduce with $E' \to E$ for $\$$ (which corresponds to accept, in case the input is empty)

SLR(1) algo

let s be the current state, on top of the parse stack

1. s contains $A \to \alpha.X\beta$, where X is a terminal and X is the next token on the input, then
   - shift X from input to top of stack. The new *state* pushed on the stack: state t where $s \xrightarrow{X} t$[8]
2. s contains a *complete* item (say $A \to \gamma.$) and the next token in the input is in $Follow(A)$: *reduce* by rule $A \to \gamma$:
   - A reduction by $S' \to S$: *accept*, if input is empty[9]
   - else:

     pop: remove γ (including "its" states from the stack)

     back up: assume to be in state u which is *now* head state

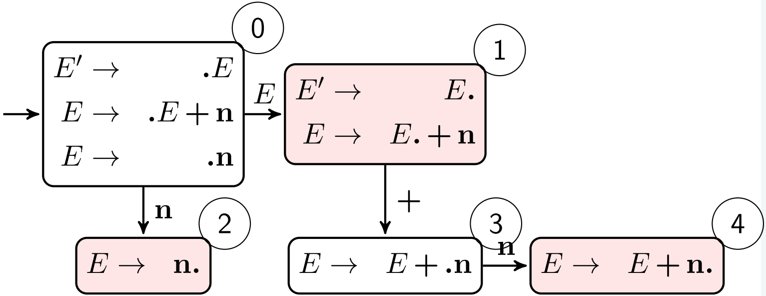
     push: push A to the stack, new head state t where $u \xrightarrow{A} t$
3. if next token is such that neither 1. or 2. applies: *error*

---

[8] Cf. to the LR(0) algo: since we checked the existence of the transition before, the else-part is missing now.

[9] Cf. to the LR(0) algo: This happens *now* only if next token is $.

Parsing table for SLR(1)



state	input			goto
	$\mathbf{n}$	$+$	$\$$	E
0	$s:2$			1
1		$s:3$	accept	
2		$r:(E \to \mathbf{n})$		
3	$s:4$			
4		$r:(E \to E + \mathbf{n})$	$r:(E \to E + \mathbf{n})$	

for state 2 and 4: $\mathbf{n} \notin Follow(E)$

Parsing table: remarks

- SLR(1) parsing table: rather similar-looking to the LR(0) one
- differences: reflect the differences in: LR(0)-algo vs. SLR(1)-algo
- same number of rows in the table (= same number of states in the DFA)
- only: colums "arranged" differently
  - LR(0): each state uniformely: either shift or else reduce (with given rule)
  - now: non-uniform, dependent on the input
- it should be obvious:
  - SLR(1) may resolve LR(0) conflicts
  - but: if the follow-set conditions are not met: SLR(1) *shift-shift* and/or SLR(1) *shift-reduce* conflicts
  - would result in non-unique entries in SLR(1)-table[10]

---

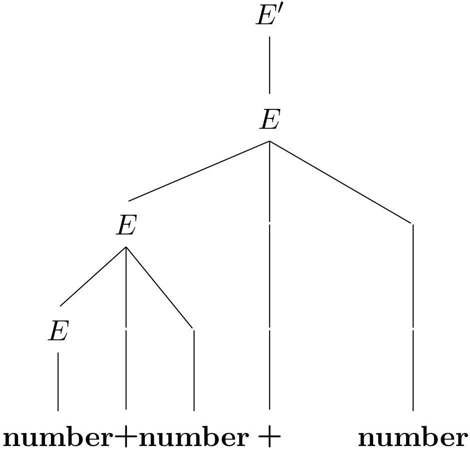
[10] by which it, strictly speaking, would no longer be an SLR(1)-table :-)

SLR(1) parser run (= “reduction”)

state	input			goto
	$\mathbf{n}$	$+$	$\mathbf{\$}$	E
0	$s:2$			1
1		$s:3$	accept	
2		$r:(E \to \mathbf{n})$		
3	$s:4$			
4		$r:(E \to E + \mathbf{n})$	$r:(E \to E + \mathbf{n})$	

stage	parsing stack	input	action
1	$\mathbf{\$}_0$	$\mathbf{n} + \mathbf{n} + \mathbf{n}\,\mathbf{\$}$	shift: 2
2	$\mathbf{\$}_0\mathbf{n}_2$	$+ \mathbf{n} + \mathbf{n}\,\mathbf{\$}$	reduce: $E \to \mathbf{n}$
3	$\mathbf{\$}_0E_1$	$+ \mathbf{n} + \mathbf{n}\,\mathbf{\$}$	shift: 3
4	$\mathbf{\$}_0E_1+_3$	$\mathbf{n} + \mathbf{n}\,\mathbf{\$}$	shift: 4
5	$\mathbf{\$}_0E_1+_3\mathbf{n}_4$	$+ \mathbf{n}\,\mathbf{\$}$	reduce: $E \to E + \mathbf{n}$
6	$\mathbf{\$}_0E_1$	$\mathbf{n}\,\mathbf{\$}$	shift 3
7	$\mathbf{\$}_0E_1+_3$	$\mathbf{n}\,\mathbf{\$}$	shift 4
8	$\mathbf{\$}_0E_1+_3\mathbf{n}_4$	$\mathbf{\$}$	reduce: $E \to E + \mathbf{n}$
9	$\mathbf{\$}_0E_1$	$\mathbf{\$}$	accept

Corresponding parse tree



Revisit the parentheses again: SLR(1)?

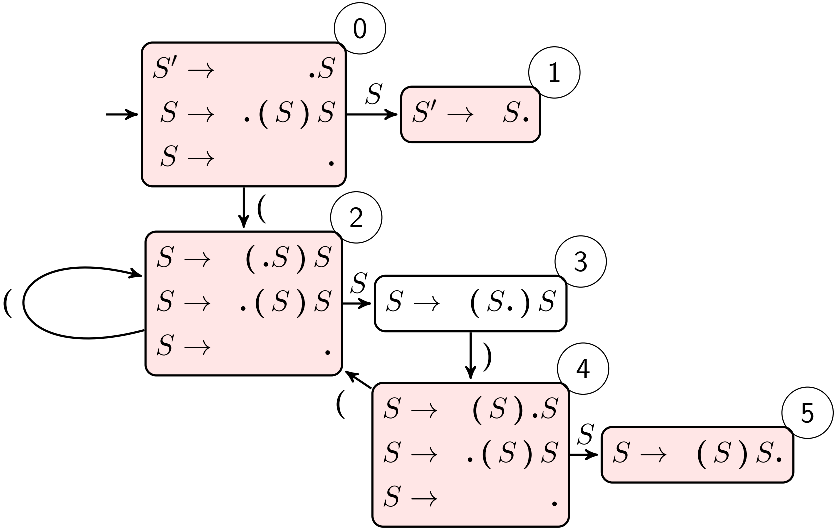
Grammar: parentheses

$$\begin{array}{rcl} S' & \rightarrow & S \\ S & \rightarrow & (\,S\,)\,S \;\mid\; \epsilon \end{array}$$

Follow set

$$Follow(S) = \{), \$\}$$

DFA for parentheses



SLR(1) parse table

state	input			goto
	(	)	$	S
0	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	1
1			accept	
2	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	3
3		$s:4$		
4	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	5
5		$r: S \to (\,S\,)\,S$	$r: S \to (\,S\,)\,S$	

Parentheses: SLR(1) parser run (= "reduction")

state	input			goto
	(	)	\$	S
0	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	1
1			accept	
2	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	3
3		$s:4$		
4	$s:2$	$r: S \to \epsilon$	$r: S \to \epsilon$	5
5		$r: S \to (\,S\,)\,S$	$r: S \to (\,S\,)\,S$	

stage	parsing stack	input	action
1	$\$_0$	$(\,)\,(\,)\,\$$	shift: 2
2	$\$_0(_2$	$)\,(\,)\,\$$	reduce: $S \to \epsilon$
3	$\$_0(_2S_3$	$)\,(\,)\,\$$	shift: 4
4	$\$_0(_2S_3)_4$	$(\,)\,\$$	shift: 2
5	$\$_0(_2S_3)_4(_2$	$)\,\$$	reduce: $S \to \epsilon$
6	$\$_0(_2S_3)_4(_2S_3$	$)\,\$$	shift: 4
7	$\$_0(_2S_3)_4(_2S_3)_4$	$\$$	reduce: $S \to \epsilon$
8	$\$_0(_2S_3)_4(_2S_3)_4S_5$	$\$$	reduce: $S \to (\,S\,)\,S$
9	$\$_0(_2S_3)_4S_5$	$\$$	reduce: $S \to (\,S\,)\,S$
10	$\$_0S_1$	$\$$	accept

Ambiguity & LR-parsing

- LR(k) (and LL(k)) grammars: *unambiguous*
- definition/construction: free of shift/reduce and reduce/reduce conflict (given the chosen level of look-ahead)
- However: ambiguous grammar tolerable, if (remaining) conflicts can be solved “meaningfully” otherwise:

Additional means of disambiguation:

1. by specifying associativity / precedence “externally”
2. by “living with the fact” that LR parser (commonly) *prioritizes shifts over reduces*

- for the second point (“let the parser decide according to its preferences”):
  - use sparingly and cautiously
  - typical example: *dangling-else*
  - even if parsers makes a decision, programmar may or may not “understand intuitively” the resulting parse tree (and thus AST)

Example of an ambiguous grammar

$$
\begin{array}{rcl}
\mathit{stmt} & \rightarrow & \mathit{if\text{-}stmt} \mid \textbf{other} \\
\mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt} \\
 & \mid & \textbf{if}\ (\ \mathit{exp}\ )\ \mathit{stmt}\ \textbf{else}\ \mathit{stmt} \\
\mathit{exp} & \rightarrow & \mathbf{0} \mid \mathbf{1}
\end{array}
$$

In the following, E for exp, etc.

Simplified conditionals

Simplified "schematic" if-then-else

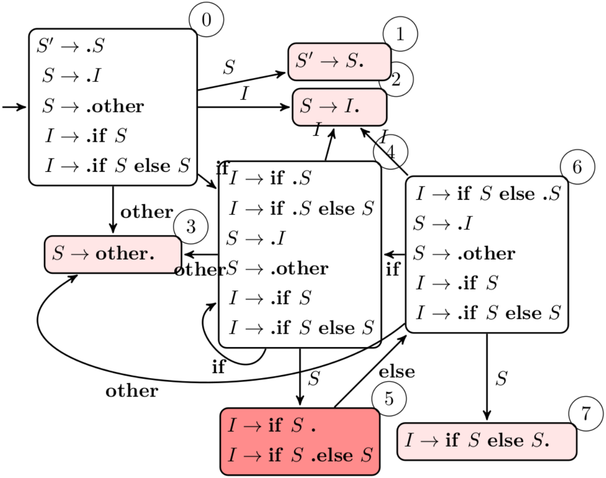
$$
\begin{array}{lcl}
S & \rightarrow & I \mid \textbf{other} \\
I & \rightarrow & \textbf{if}\ S \mid \textbf{if}\ S\ \textbf{else}\ S
\end{array}
$$

Follow-sets

	$Follow$
S'	$\{\$\}$
S	$\{\$, \textbf{else}\}$
I	$\{\$, \textbf{else}\}$

- since ambiguous: at least one conflict must be somewhere

DFA of LR(0) items



Simple conditionals: parse table

Grammar

$$
\begin{array}{lcll}
S & \to & I & (1) \\
  & \mid & \textbf{other} & (2) \\
I & \to & \textbf{if}\ S & (3) \\
  & \mid & \textbf{if}\, S\ \textbf{else}\ S & (4)
\end{array}
$$

SLR(1)-parse-table, conflict “resolved”

state	input				goto	
	if	**else**	**other**	**$**	S	I
0	$s:4$		$s:3$		1	2
1				accept		
2		$r:1$		$r:1$		
3		$r:2$		$r:2$		
4	$s:4$		$s:3$		5	2
5		$s:6$		$r:3$		
6	$s:4$		$s:3$		7	2
7		$r:4$		$r:4$		

- *shift-reduce conflict* in state 5: reduce with *rule 3* vs. shift (to state 6)
- conflict there: resolved in favor of *shift* to 6
- note: extra start state left out from the table

Parser run (= reduction)

state	input				goto	
	if	**else**	**other**	**$**	S	I
0	$s:4$		$s:3$		1	2
1				accept		
2		$r:1$		$r:1$		
3		$r:2$		$r:2$		
4	$s:4$		$s:3$		5	2
5		$s:6$		$r:3$		
6	$s:4$		$s:3$		7	2
7		$r:4$		$r:4$		

stage	parsing stack	input	action
1	$\$_0$	**if if other else other $**	shift: 4
2	$\$_0\mathbf{if}_4$	**if other else other $**	shift: 4
3	$\$_0\mathbf{if}_4\mathbf{if}_4$	**other else other $**	shift: 3
4	$\$_0\mathbf{if}_4\mathbf{if}_4\mathbf{other}_3$	**else other $**	reduce: 2
5	$\$_0\mathbf{if}_4\mathbf{if}_4S_5$	**else other $**	shift 6
6	$\$_0\mathbf{if}_4\mathbf{if}_4S_5\mathbf{else}_6$	**other $**	shift: 3
7	$\$_0\mathbf{if}_4\mathbf{if}_4S_5\mathbf{else}_6\mathbf{other}_3$	**$**	reduce: 2
8	$\$_0\mathbf{if}_4\mathbf{if}_4S_5\mathbf{else}_6S_7$	**$**	reduce: 4
9	$\$_0\mathbf{if}_4I_2$	**$**	reduce: 1
10	$\$_0S_1$	**$**	accept

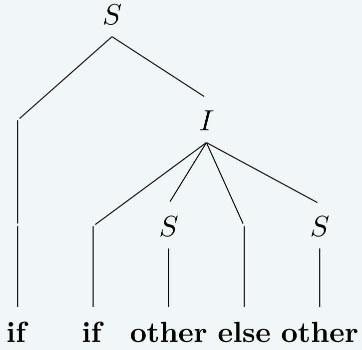
Parser run, different choice

state	input				goto	
	if	**else**	**other**	**$**	S	I
0	$s:4$		$s:3$		1	2
1				accept		
2		$r:1$		$r:1$		
3		$r:2$		$r:2$		
4	$s:4$		$s:3$		5	2
5		$s:6$		$r:3$		
6	$s:4$		$s:3$		7	2
7		$r:4$		$r:4$		

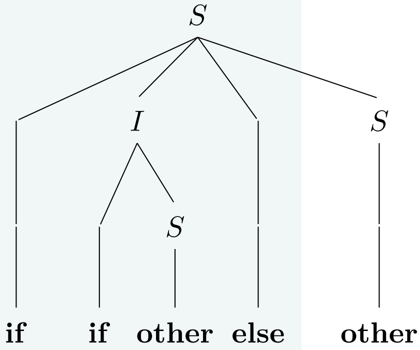
stage	parsing stack	input	action
1	$\$_0$	**if if other else other $**	shift: 4
2	$\$_0\textbf{if}_4$	**if other else other $**	shift: 4
3	$\$_0\textbf{if}_4\textbf{if}_4$	**other else other $**	shift: 3
4	$\$_0\textbf{if}_4\textbf{if}_4\textbf{other}_3$	**else other $**	reduce: 2
5	$\$_0\textbf{if}_4\textbf{if}_4S_5$	**else other $**	reduce 3
6	$\$_0\textbf{if}_4I_2$	**else other $**	reduce 1
7	$\$_0\textbf{if}_4S_5$	**else other $**	shift 6
8	$\$_0\textbf{if}_4S_5\textbf{else}_6$	**other $**	shift 3
9	$\$_0\textbf{if}_4S_5\textbf{else}_6\textbf{other}_3$	**$**	reduce 2
10	$\$_0\textbf{if}_4S_5\textbf{else}_6S_7$	**$**	reduce 4
11	$\$_0S_1$	**$**	accept

Parse trees for the “simple conditions”

shift-precedence: conventional



“wrong” tree



standard “dangling else” convention

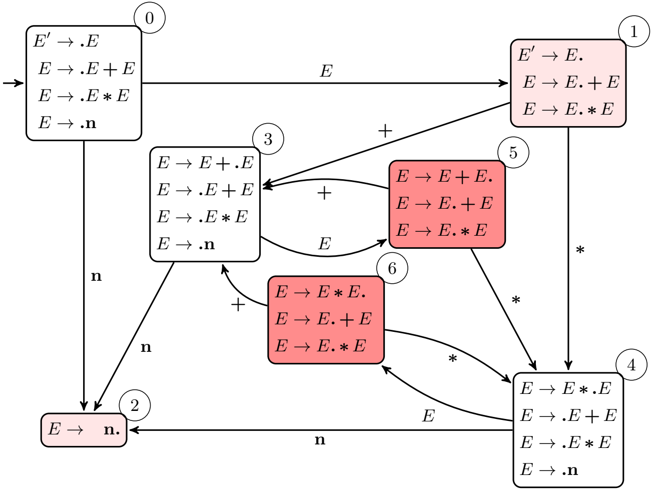
“an **else** belongs to the last previous, still open (= dangling) if-clause”

Use of ambiguous grammars

- advantage of ambiguous grammars: often simpler
- if ambiguous: grammar guaranteed to have conflicts
- can be (often) resolved by specifying *precedence* and *associativity*
- supported by tools like `yacc` and `CUP` …

$$
\begin{array}{rcl}
E' & \to & E \\
E & \to & E+E \mid E*E \mid \textbf{number}
\end{array}
$$

DFA for + and ×


0
$E' \to .E$
$E \to .E + E$
$E \to .E * E$
$E \to .\mathbf{n}$
1
$E' \to E.$
$E \to E. + E$
$E \to E. * E$
2
$E \to \mathbf{n}.$
3
$E \to E + .E$
$E \to .E + E$
$E \to .E * E$
$E \to .\mathbf{n}$
4
$E \to E * .E$
$E \to .E + E$
$E \to .E * E$
$E \to .\mathbf{n}$
5
$E \to E + E.$
$E \to E. + E$
$E \to E. * E$
6
$E \to E * E.$
$E \to E. + E$
$E \to E. * E$
E
$+$
$+$
E
$*$
$\mathbf{n}$
$*$
$\mathbf{n}$
$+$
$*$
E
$\mathbf{n}$

States with conflicts

- state 5
  - stack contains ...$E + E$
  - for input $\$$: reduce, since shift not allowed form $\$$
  - for input $+$; reduce, as $+$ is *left-associative*
  - for input $*$: shift, as $*$ has *precedence* over $+$
- state 6:
  - stack contains ...$E * E$
  - for input $\$$: reduce, since shift not allowed form $\$$
  - for input $+$; reduce, a $*$ has *precedence* over $+$
  - for input $*$: reduce, as $*$ is *left-associative*
- see also the table on the next slide

Parse table $+$ and $\times$

state	input				goto
	$\mathbf{n}$	$+$	$*$	$\mathbf{\$}$	E
0	$s:2$				1
1		$s:3$	$s:4$	accept	
2		$r:E\to\mathbf{n}$	$r:E\to\mathbf{n}$	$r:E\to\mathbf{n}$	
3	$s:2$				5
4	$s:2$				6
5		$r:E\to E+E$	$s:4$	$r:E\to E+E$	
6		$r:E\to E*E$	$r:E\to E*E$	$r:E\to E*E$	

How about exponentiation (written $\uparrow$ or $**$)?

Defined as *right-associative*. See exercise

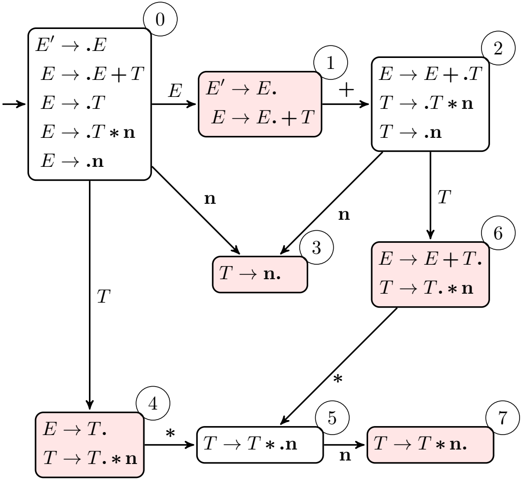
Compare: unambiguous grammar for + and *

Unambiguous grammar: precedence and left-assoc built in

$$
\begin{array}{rcl}
E' & \rightarrow & E \\
E & \rightarrow & E + T \mid T \\
T & \rightarrow & T * \mathbf{n} \mid \mathbf{n}
\end{array}
$$

	Follow	
E'	$\{\$\}$	(as always for start symbol)
E	$\{\$, +\}$	
T	$\{\$, +, *\}$	

DFA for unambiguous + and ×



DFA remarks

- the DFA now is SLR(1)
  - check states with *complete* items

    state 1: $Follow(E') = \{\$\}$
    state 4: $Follow(E) = \{\$, +\}$
    state 6: $Follow(E) = \{\$, +\}$
    state 3/7: $Follow(T) = \{\$, +, *\}$
  - in no case there's a shift/reduce conflict (check the outgoing edges vs. the follow set)
  - there's not reduce/reduce conflict either

LR(1) parsing

- most general from of LR(1) parsing
- aka: *canonical* LR(1) parsing
- usually: considered as unecessarily “complex” (i.e. LALR(1) or similar is good enough)
- “stepping stone” towards LALR(1)

Basic restriction of SLR(1)

Uses *look-ahead*, yes, but only *after* it has built a non-look-ahead DFA (based on LR(0)-items)

A help to remember

SLR(1) “improved” LR(0) parsing LALR(1) is “crippled” LR(1) parsing.

Limits of SLR(1) grammars

Assignment grammar fragment[11]

$$\begin{aligned} stmt &\rightarrow call\text{-}stmt \mid assign\text{-}stmt \\ call\text{-}stmt &\rightarrow \textbf{identifier} \\ assign\text{-}stmt &\rightarrow var \mathrel{\textbf{:=}} exp \\ var &\rightarrow [\, exp \,] \mid \textbf{identifier} \\ exp &\rightarrow var \mid \textbf{number} \end{aligned}$$

Assignment grammar fragment, simplified

$$\begin{aligned} S &\rightarrow \textbf{id} \mid V \mathrel{\textbf{:=}} E \\ V &\rightarrow \textbf{id} \\ E &\rightarrow V \mid \textbf{n} \end{aligned}$$

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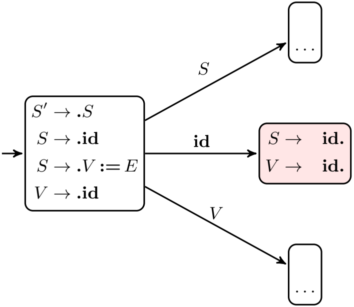
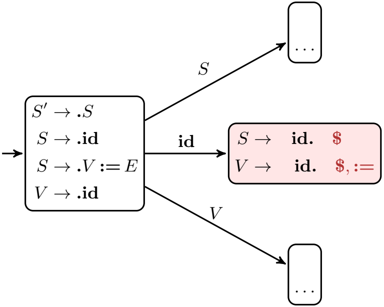
[11] Inspired by Pascal, analogous problems in C . . .

non-SLR(1): Reduce/reduce conflict



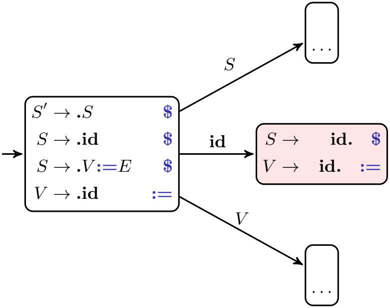
	First	*Follow*
S	**id**	**\$**
V	**id**	**\$**, **:=**
E	**id**, **number**	**\$**

non-SLR(1): Reduce/reduce conflict



	First	*Follow*
S	**id**	**$**
V	**id**	**$**, **:=**
E	**id**, **number**	**$**

Situation can be saved: more look-ahead



LALR(1) (and LR(1)): Being more precise with the follow-sets

- LR(0)-items: too "indiscriminate" wrt. the follow sets
- remember the definition of SLR(1) conflicts
- LR(0)/SLR(1)-states:
  - sets of items[12] due to subset construction
  - the items are LR(0)-items
  - follow-sets as an *after-thought*

Add precision in the states of the automaton already

Instead of using LR(0)-items and, when the LR(0) DFA is done, try to add a little disambiguation with the help of the follow sets for states containing complete items, better make more fine-grained items from the very start:

- LR(1) items
- each *item* with "specific follow information": look-ahead

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[12] That won't change in principle (but the items get more complex)

LR(1) items

- main idea: simply make the look-ahead part of the item
- obviously: proliferation of states[13]

LR(1) items

$$[A \to \alpha.\beta, \mathbf{a}] \tag{10}$$

- $\mathbf{a}$: terminal/token, including $\mathbf{\$}$

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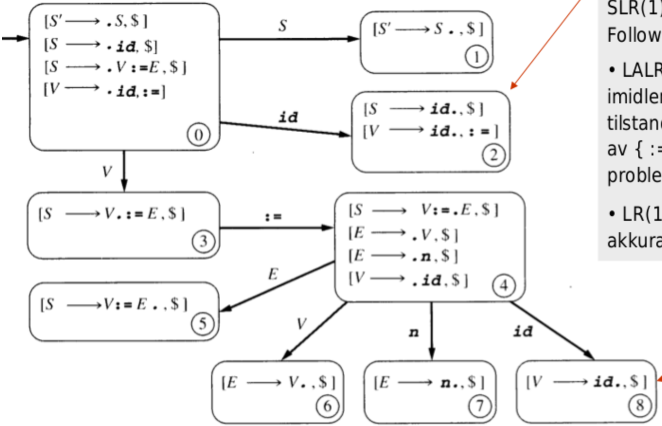
[13] Not to mention if we wanted look-ahead of $k > 1$, which in practice is not done, though.

LALR(1)-DFA (or LR(1)-DFA)



SLR(1),
Follow(

- LALR(
imidlert
tilstand
av { :=
problem

- LR(1)
akkurat

Remarks on the DFA

- Cf. state *2* (seen before)
    - in SLR(1): problematic (reduce/reduce), as $Follow(V) = \{\textbf{:=}, \textbf{\$}\}$
    - now: diambiguation, by the added information
- LR(1) would give the same DFA

Full LR(1) parsing

- AKA: canonical LR(1) parsing
- the *best* you can do with 1 look-ahead
- unfortunately: big tables
- pre-stage to LALR(1)-parsing

SLR(1)	**LALR(1)**
LR(0)-item-based parsing, with *afterwards* adding some extra "pre-compiled" info (about follow-sets) to increase expressivity	LR(1)-item-based parsing, but *afterwards* throwing away precision by collapsing states, to save space

LR(1) transitions: arbitrary symbol

- transitions of the NFA (not DFA)

X-transition

$$\boxed{[A \to \ \alpha.X\beta, \mathbf{a}]} \xrightarrow{X} \boxed{[A \to \ \alpha X.\beta, \mathbf{a}]}$$

LR(1) transitions: ϵ

ϵ-transition

for all

$$B \to \beta_1 \mid \beta_2 \ldots \quad \text{and all} \quad \mathbf{b} \in \mathit{First}(\gamma\mathbf{a})$$

$$\boxed{[A \to \alpha.B\gamma \quad ,\mathbf{a}]} \xrightarrow{\epsilon} \boxed{[B \to .\beta \quad ,\mathbf{b}]}$$

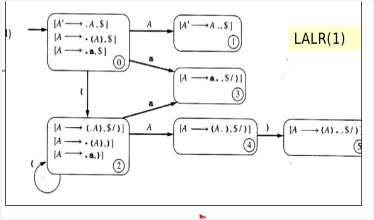
including special case ($\gamma = \epsilon$)

for all $B \to \beta_1 \mid \beta_2 \ldots$

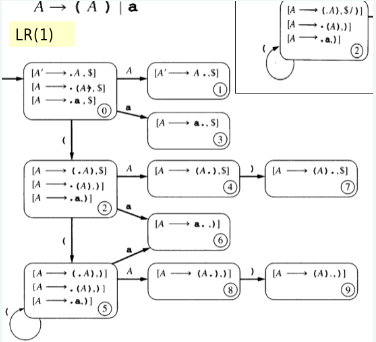
$$\boxed{[A \to \alpha.B \quad ,\mathbf{a}]} \xrightarrow{\epsilon} \boxed{[B \to .\beta \quad ,\mathbf{a}]}$$

LALR(1) vs LR(1)

LALR(1)



LR(1)



Core of LR(1)-states

- actually: not done that way in practice
- main idea: *collapse* states with the same *core*

Core of an LR(1) state

= set of *LR(0)*-items (i.e., ignoring the look-ahead)

- observation: core of the LR(1) item = LR(0) item
- 2 LR(1) states with the same core have same outgoing edges, and those lead to states with the same core

LALR(1)-DFA by as collapse

- collapse all states with the same core
- based on above observations: edges are also consistent
- Result: almost like a LR(0)-DFA but additionally
  - still each individual item has still look ahead attached: the union of the "collapsed" items
  - especially for states with *complete* items $[A \to \alpha, \mathbf{a}, \mathbf{b}, \ldots]$ is smaller than the follow set of A
  - $\Rightarrow$ less unresolved conflicts compared to SLR(1)

Concluding remarks of LR / bottom up parsing

- all constructions (here) based on BNF (not EBNF)
- *conflicts* (for instance due to ambiguity) can be solved by
    - reformulate the grammar, but generarate the same language[14]
    - use *directives* in parser generator tools like `yacc`, `CUP`, `bison` (precedence, assoc.)
    - or (not yet discussed): solve them later via *semantical analysis*
    - NB: *not all* conflics are solvable, also not in LR(1) (remember ambiguous languages)

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[14] If designing a new language, there's also the option to massage the language itself. Note also: there are *inherently* ambiguous *languages* for which there is no *unambiguous* grammar.

LR/bottom-up parsing overview

	advantages	remarks
LR(0)	defines states *also* used by SLR and LALR	not really used, many conflicts, very weak
SLR(1)	clear improvement over LR(0) in expressiveness, even if using the same number of states. Table typically with 50K entries	weaker than LALR(1). but often good enough. Ok for hand-made parsers for *small* grammars
LALR(1)	almost as expressive as LR(1), but number of states as LR(0)!	method of choice for most generated LR-parsers
LR(1)	*the* method covering *all* bottom-up, one-look-ahead parseable grammars	large number of states (typically 11M of entries), mostly LALR(1) preferred

Remember: once the *table* specific for LR(0), . . . is set-up, the parsing algorithms all work *the same*

Error handling

Minimal requirement

Upon “stumbling over” an error (= deviation from the grammar): give a *reasonable* & *understandable* error message, indicating also error *location*. Potentially stop parsing

- for parse error *recovery*
    - one cannot really recover from the fact that the program has an error (an syntax error is a syntax error), but
    - after giving decent error message:
        - move on, potentially jump over some subsequent code,
        - until parser can *pick up* normal parsing again
        - so: meaningfull checking code even following a first error
    - avoid: reporting an avalanche of subsequent *spurious* errors (those just “caused” by the first error)
    - “pick up” again after semantic errors: easier than for syntactic errors

Error messages

- important:
  - avoid error messages that only occur because of an already reported error!
  - report error as early as possible, if possible at the *first point* where the program cannot be extended to a correct program.
  - make sure that, after an error, one doesn't end up in an *infinite loop* without reading any input symbols.
- What's a good error message?
  - assume: that the method `factor()` chooses the alternative (exp) but that it , when control returns from method `exp()`, does not find a)
  - one could report : `right parenthesis missing`
  - But this may often be confusing, e.g. if what the program text is: `( a + b c )`
  - here the `exp()` method will terminate after `( a + b`, as `c` cannot extend the expression). You should therefore rather give the message `error in expression or right parenthesis missing`.

Error recovery in bottom-up parsing

- *panic recovery* in LR-parsing
  - simple form
  - the only one we shortly look at
- upon error: recovery ⇒
  - pops parts of the stack
  - ignore parts of the input
- until "on track again"
- but: how to do that
- additional problem: *non-determinism*
  - table: constructed *conflict-free* under normal operation
  - upon error (and clearing parts of the stack + input): no guarantee it's clear how to continue

⇒ heuristic needed (like panic mode recovery)

Panic mode idea

- try a fresh start,
- promising "fresh start" is: a possible goto action
- thus: back off and take the *next* such goto-opportunity

Possible error situation

	parse stack	input	action
1	$\mathbf{\$}_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3\mathbf{(}_4\mathbf{d}_5\mathbf{e}_6$	$\mathbf{f}$ $\mathbf{)}\,\mathbf{gh}\ldots\mathbf{\$}$	no entry for $\mathbf{f}$

state	input					goto			
	$\ldots$	$)$	$\mathbf{f}$	$\mathbf{g}$	$\ldots$	$\ldots$	A	B	$\ldots$
$\ldots$									
3							u	v	
4			—				—	—	
5			—				—	—	
6		—	—				—	—	
$\ldots$									
u		—	—	reduce . . .					
v		—	—	shift : 7					
$\ldots$									

Possible error situation

	parse stack	input	action
1	$\$_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3(_4\mathbf{d}_5\mathbf{e}_6$	$\mathbf{f}\ )\,\mathbf{gh}\ldots\$$	no entry for **f**
2	$\$_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3 B_v$	$\mathbf{gh}\ \ldots\$$	back to normal
3	$\$_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3 B_v \mathbf{g}_7$	$\mathbf{h}\ \ldots\$$	...

state	input					goto			
	...	)	**f**	**g**	...	...	A	B	...
...									
3							u	v	
4			–				–	–	
5			–				–	–	
6		–	–				–	–	
...									
u		–	–	reduce ...					
v		–	–	shift : 7					
...									

Panic mode recovery

Algo

1. *Pop* states for the stack *until* a state is found with non-empty goto entries
2.
   - If there's legal action on the current input token from one of the goto-states, push token on the stack, *restart* the parse.
   - If there's several such states: *prefer shift* to a reduce
   - Among possible reduce actions: prefer one whose associated non-terminal is least general
3. if no legal action on the current input token from one of the goto-states: *advance input* until there is a legal action (or until end of input is reached)

Example again

	parse stack	input	action
1	$\mathbf{\$}_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3\mathbf{(}_4\mathbf{d}_5\mathbf{e}_6$	$\mathbf{f}\ )\,\mathbf{gh}\ldots\mathbf{\$}$	no entry for $\mathbf{f}$

- first pop, until in state 3
- then jump over input
  - until next input $\mathbf{g}$
  - since $\mathbf{f}$ and $)$ cannot be treated
- choose to goto v (shift in that state)

Example again

	parse stack	input	action
1	$\mathbf{\$}_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3\mathbf{(}_4\mathbf{d}_5\mathbf{e}_6$	$\mathbf{f}\ \mathbf{)}\,\mathbf{gh}\ldots\mathbf{\$}$	no entry for $\mathbf{f}$
2	$\mathbf{\$}_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3B_v$	$\mathbf{gh}\ \ldots\mathbf{\$}$	back to normal
3	$\mathbf{\$}_0\mathbf{a}_1\mathbf{b}_2\mathbf{c}_3B_v\mathbf{g}_7$	$\mathbf{h}\ \ldots\mathbf{\$}$	...

- first pop, until in state 3
- then jump over input
  - until next input $\mathbf{g}$
  - since $\mathbf{f}$ and $)$ cannot be treated
- choose to goto v (shift in that state)

Panic mode may loop forever

	parse stack	input	action
1	$\$_0$	$(\, \mathbf{n}\ \mathbf{n}\,)\, \$$	
2	$\$_0 (_6$	$\mathbf{n}\ \mathbf{n}\,)\, \$$	
3	$\$_0 (_6 \mathbf{n}_5$	$\mathbf{n}\,)\, \$$	
4	$\$_0 (_6 factor_4$	$\mathbf{n}\,)\, \$$	
6	$\$_0 (_6 term_3$	$\mathbf{n}\,)\, \$$	
7	$\$_0 (_6 exp_{10}$	$\mathbf{n}\,)\, \$$	panic!
8	$\$_0 (_6 factor_4$	$\mathbf{n}\,)\, \$$	been there before: stage 4!

Panicking and looping

	parse stack	input	action
1	$\$_0$	$(\,\mathbf{n}\ \mathbf{n}\,)\,\$$	
2	$\$_0(_6$	$\mathbf{n}\ \mathbf{n}\,)\,\$$	
3	$\$_0(_6\mathbf{n}_5$	$\mathbf{n}\,)\,\$$	
4	$\$_0(_6 factor_4$	$\mathbf{n}\,)\,\$$	
6	$\$_0(_6 term_3$	$\mathbf{n}\,)\,\$$	
7	$\$_0(_6 exp_{10}$	$\mathbf{n}\,)\,\$$	panic!
8	$\$_0(_6 factor_4$	$\mathbf{n}\,)\,\$$	been there before: stage 4!

- error raised in stage 7, no action possible
- panic:
  1. pop-off exp_{10}
  2. state 6: 3 goto's

	exp	$term$	$factor$
goto to	10	3	4
with $\mathbf{n}$ next: action there	—	reduce r_4	reduce r_6

3. no shift, so we need to decide between the two reduces
4. $factor$: less general, we take that one

How to deal with looping panic?

- make sure to detec loop (i.e. previous “configurations”)
- if loop detected: doen’t repeat but do something special, for instance
    - pop-off more from the stack, and try again
    - pop-off and *insist* that a shift is part of the options

Left out (from the book and the pensum)

- more info on error recovery
- expecially: more on `yacc` error recovery
- it’s not pensum, and for the oblig: need to deal with CUP-specifics (not classic `yacc` specifics even if similar) anyhow, and error recovery is not part of the oblig (halfway decent error *handling* is).

References I

Bibliography

[1] Appel, A. W. (1998). *Modern Compiler Implementation in ML/Java/C*. Cambridge University Press.

[2] Louden, K. (1997). *Compiler Construction, Principles and Practice*. PWS Publishing.