# Chapter 3

## Grammars

Course “Compiler Construction”
Martin Steffen
Spring 2024

# Chapter 3

## Learning Targets of Chapter "Grammars".

1. (context-free) grammars + BNF
2. ambiguity and other properties
3. terminology: tokens, lexemes
4. different trees connected to grammars/parsing
5. derivations, sentential forms

The chapter corresponds to [1, Section 3.1–3.2] (or [3, Chapter 3]).

# Chapter 3

## Outline of Chapter "Grammars".

# Section

## Introduction

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# Bird's eye view of a parser


Syntax

- *check* that the token sequence correspond to a *syntactically correct* program
  - if yes: yield *tree* as intermediate representation for subsequent phases
  - if not: give *understandable* error message(s)

# Trees, trees, more trees

**syntax trees**

*parse* tree or *concrete syntax tree* vs. *abstract syntax trees*

- derivation trees (derivation in a (context-free) grammar)
- mentioned tree forms hang together, dividing line a bit fuzzy
- output of a parser: AST

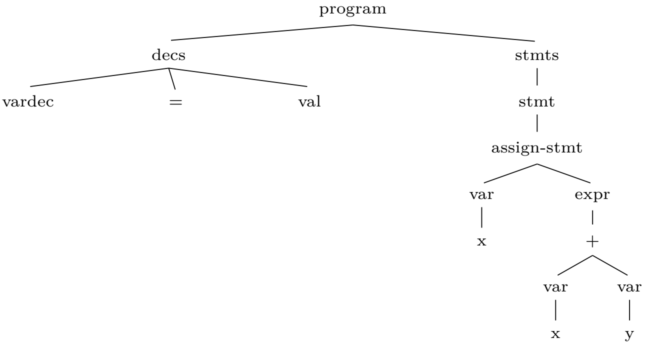
# (Context-free) grammars

- specifies the *syntactic structure* of a language
- here: grammar means CFG
- $G$ derives word $w$

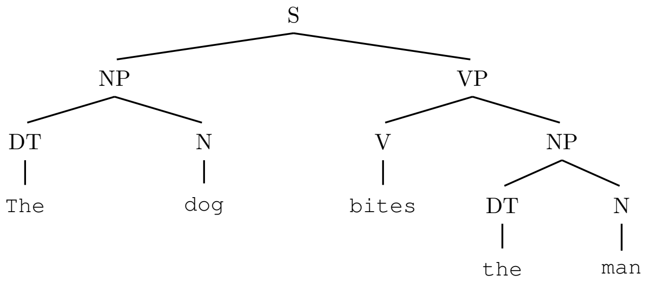
**Parsing**

Given a stream of “symbols” $w$ and a grammar $G$, find a *derivation* from $G$ that produces $w$.

# Schematic syntax tree



# Natural-language parse tree



# "Interface" between scanner and parser

- remember: task of scanner = "chopping up" the input char stream (throw away white space, etc.) and *classify* the pieces (1 piece = *lexeme*)
- classified lexeme = token
- sometimes we use $\langle \texttt{integer}, "42" \rangle$
  - `integer`: "class" or "type" of the token, also called *token name*
  - $"42"$ : *value of the token attribute* (or just value). Here: directly the *lexeme* (a string or sequence of chars)
- a note on (sloppyness/ease of) terminology: often: the token name is simply just called the token

the *token (symbol)* corrresponds there to terminal symbols (or terminals, for short)

# Section

## Context-free grammars and BNF notation

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# Grammars

- in this chapter(s): focus on context-free grammars
- thus here: grammar = CFG
- as in the context of regular expressions/languages: *language* = (typically infinite) set of words
- grammar = formalism to unambiguously specify a language
- intended language: all syntactically correct programs of a given progamming language

**Slogan**

A CFG describes the syntax of a programming language. [1]

[1] And some say, regular expressions describe its microsyntax.

# Context-free grammar

## Definition (CFG)

A *context-free grammar* $G$ is a 4-tuple $G = (\Sigma_T, \Sigma_N, S, P)$:

1. two disjoint finite alphabets of *terminals* $\Sigma_T$ and
2. *non-terminals* $\Sigma_N$,
3. one *start*-symbol $S \in \Sigma_N$ (a non-terminal), and
4. *productions* $P$ = finite subset of $\Sigma_N \times (\Sigma_N + \Sigma_T)^*$.

- terminal symbols: corresponds to tokens in parser = basic building blocks of syntax
- non-terminals: (e.g. "expression", "while-loop", "method-definition" ...)
- grammar: generating (via "derivations") languages
- parsing: the *inverse* problem

⇒ CFG = specification

# Further notions

- sentence and sentential form
- productions (or rules)
- derivation
- *language* of a grammar $\mathcal{L}(G)$
- parse tree

# BNF notation

- popular & common format to write CFGs, i.e., describe context-free languages
- named after *pioneering* (seriously) work on Algol 60
- notation to write productions/rules + some extra meta-symbols for convenience and grouping

**Slogan: Backus-Naur form**

What regular expressions are for regular languages is BNF for context-free languages.

# "Expressions" in BNF

$$exp \rightarrow exp \; op \; exp \mid (\, exp \,) \mid \mathbf{number} \qquad (1)$$
$$op \rightarrow + \mid - \mid *$$

- "$\rightarrow$" indicating productions and " $\mid$ " indicating alternatives
- convention: terminals written **boldface**, non-terminals *italic*
- also simple math symbols like "+" and "(" are meant above as terminals
- start symbol here: $exp$
- remember: terminals like **number** correspond to tokens, resp. token classes. The attributes/token values are not relevant here.

# Different notations

- BNF: notationally not 100% “standardized” across books/tools
- “classic” way (Algol 60):

```
<exp> ::=  <exp> <op> <exp>
        |  ( <exp> )
        |  NUMBER
<op>  ::=  + | − | *
```

- Extended BNF (EBNF) and yet another style

$$\begin{array}{rcl} exp & \rightarrow & exp \; (\; \text{"}+\text{"} \;|\; \text{"}-\text{"} \;|\; \text{"}*\text{"} \;) \; exp \\ & | & \text{"("} \; exp \; \text{")"} \;|\; \text{"}number\text{"} \end{array} \tag{2}$$

- note: parentheses as terminals vs. as *metasymbols*

# Different ways of writing the same grammar

- directly written as 6 pairs (6 rules, 6 productions) from $\Sigma_N \times (\Sigma_N \cup \Sigma_T)^*$, with "$\rightarrow$" as nice looking "separator":

$$
\begin{aligned}
exp &\rightarrow exp\ op\ exp \\
exp &\rightarrow (\ exp\ ) \\
exp &\rightarrow \mathbf{number} \\
op &\rightarrow + \\
op &\rightarrow - \\
op &\rightarrow *
\end{aligned}
\tag{3}
$$

- choice of non-terminals: irrelevant (except for human readability):

$$
\begin{aligned}
E &\rightarrow E\ O\ E \mid (\ E\ ) \mid \mathbf{number} \\
O &\rightarrow + \mid - \mid *
\end{aligned}
\tag{4}
$$

- still: we count 6 productions

# Grammars as language generators

## Deriving a word:

Start from start symbol. Pick a "matching" rule to rewrite the current word to a new one; repeat until *terminal* symbols, only.

- *non-deterministic* process
- rewrite relation for derivations:
  - one step rewriting: $w_1 \Rightarrow w_2$
  - one step using rule $n$: $w_1 \Rightarrow_n w_2$
  - many steps: $\Rightarrow^*$ , etc.

## Language of grammar $G$

$$\mathcal{L}(G) = \{s \mid \mathit{start} \Rightarrow^* s \text{ and } s \in \Sigma_T^*\}$$

# Example derivation for (number−number)∗number

$$
\begin{aligned}
\underline{exp} &\Rightarrow \underline{exp}\ op\ exp \\
&\Rightarrow (\underline{exp})\ op\ exp \\
&\Rightarrow (\underline{exp}\ op\ exp)\ op\ exp \\
&\Rightarrow (\mathbf{n}\ \underline{op}\ exp)\ op\ exp \\
&\Rightarrow (\mathbf{n}-\underline{exp})\ op\ exp \\
&\Rightarrow (\mathbf{n}-\mathbf{n})\underline{op}\ exp \\
&\Rightarrow (\mathbf{n}-\mathbf{n})*\underline{exp} \\
&\Rightarrow (\mathbf{n}-\mathbf{n})*\mathbf{n}
\end{aligned}
$$

- underline the "place" where a rule is used, i.e., an *occurrence* of the non-terminal symbol is being rewritten/expanded
- here: *leftmost* derivation[2]

[2] We'll come back to that later, it will be important.

# Right-most derivation

$$
\begin{aligned}
\underline{exp} &\Rightarrow exp\ op\ \underline{exp} \\
&\Rightarrow exp\ \underline{op}\ \mathbf{n} \\
&\Rightarrow \underline{exp}{*}\mathbf{n} \\
&\Rightarrow (exp\ op\ \underline{exp}){*}\mathbf{n} \\
&\Rightarrow (exp\ \underline{op}\ \mathbf{n}){*}\mathbf{n} \\
&\Rightarrow (\underline{exp}{-}\mathbf{n}){*}\mathbf{n} \\
&\Rightarrow (\mathbf{n}{-}\mathbf{n}){*}\mathbf{n}
\end{aligned}
$$

- other ("mixed") derivations for the same word possible

# Some easy requirements for reasonable grammars

- all symbols (terminals and non-terminals): should occur in a some word derivable from the start symbol
- words containing only non-terminals should be derivable
- an example of a silly grammar $G$ (start-symbol $A$)

$$\begin{array}{lcl} A & \rightarrow & B\mathbf{x} \\ B & \rightarrow & A\mathbf{y} \\ C & \rightarrow & \mathbf{z} \end{array}$$

- $\mathcal{L}(G) = \emptyset$
- those "sanitary conditions": minimal "common sense" requirements

# Parse tree

- derivation: if viewed as sequence of steps ⇒ linear "structure"
- order of individual steps: irrelevant
- ⇒ order not needed for subsequent phases
- parse tree: structure for the *essence* of derivation
- also called concrete syntax tree.

$$^{1}\ exp$$

- numbers in the tree
  - *not* part of the parse tree, indicate order of derivation, only
  - here: leftmost derivation

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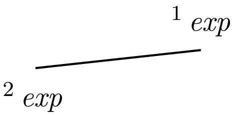


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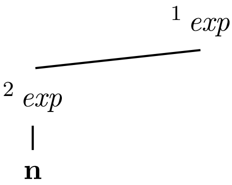


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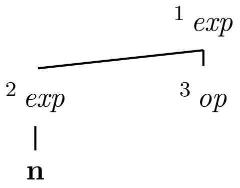


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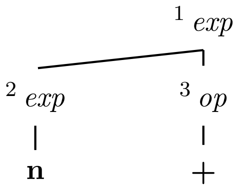


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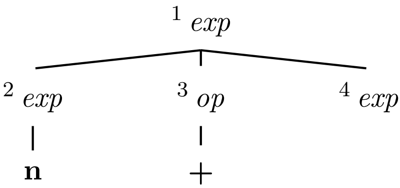


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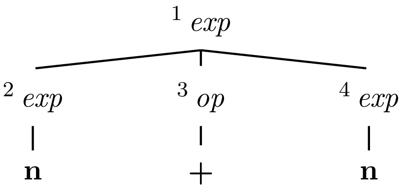


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# Another parse tree (numbers for right-most derivation)

$^{1}\ exp$

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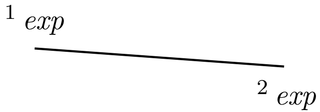

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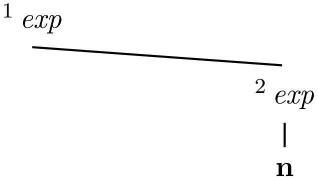

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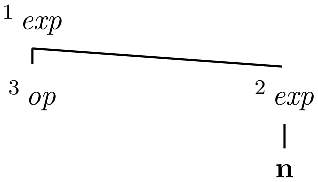

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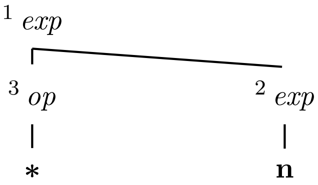

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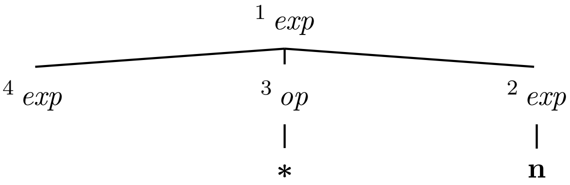

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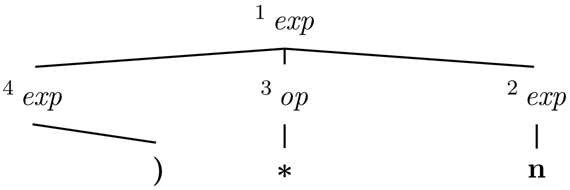

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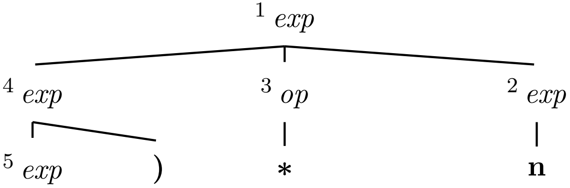

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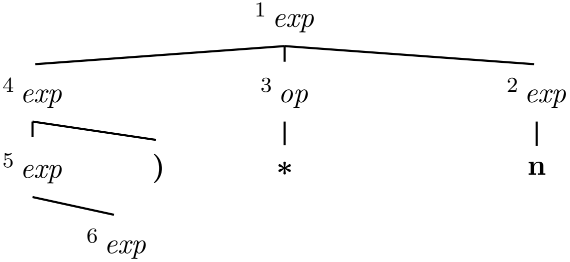



INF5110 – Compiler Construction

Targets & Outline

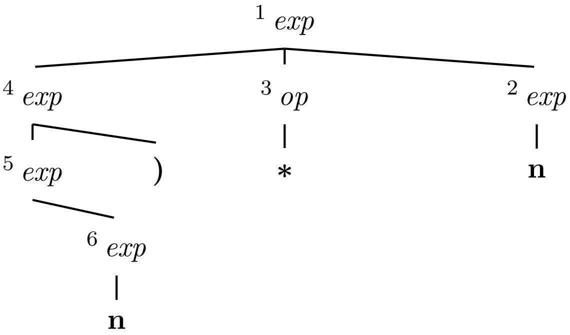
Introduction

Context-free grammars and BNF notation

Ambiguity

Chomsky hierarchy

# Another parse tree (numbers for right-most derivation)



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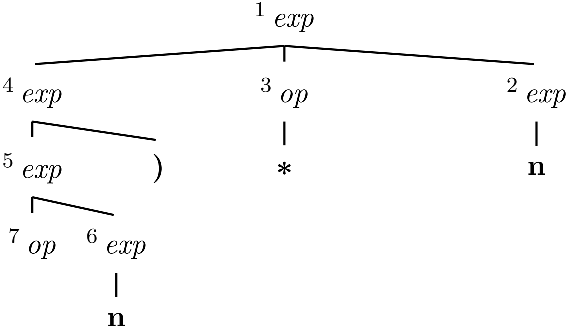

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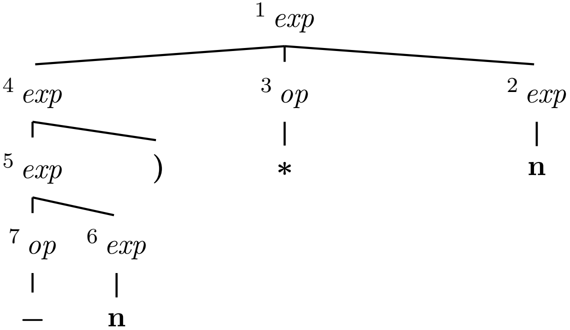

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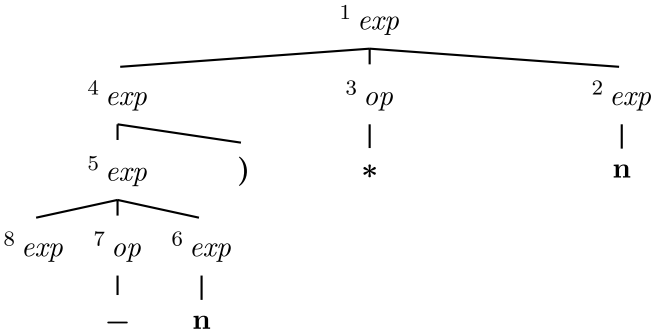

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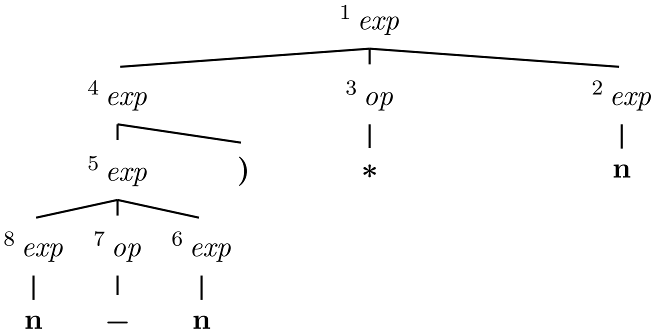

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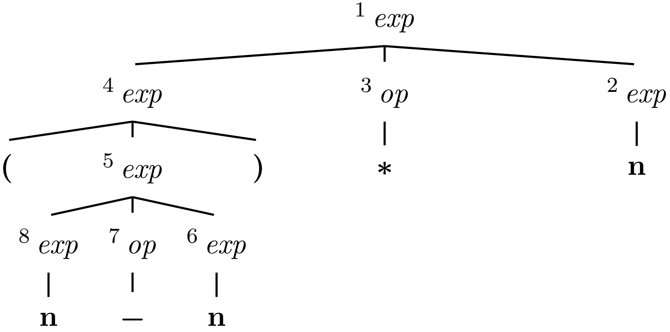
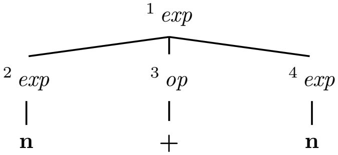
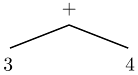


# Abstract syntax tree

- parse tree: contains still unnecessary details
- specifically: *parentheses* or similar, used for grouping
- tree-structure: can express the intended grouping already
- remember: tokens may contain also attribute values (e.g.: full token for token class **n** contains values like "42" ...)



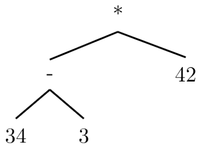



# AST vs. CST

- parse tree
  - important *conceptual* structure, to talk about grammars and derivations
  - most likely *not explicitly implemented* in a parser
- AST is a *concrete* data structure
  - important IR of the syntax (for the language being implemented)
  - written in the meta-language
  - therefore: nodes like $+$ and $3$ *are no longer tokens or lexemes*
  - concrete data stuctures in the meta-language (C-structs, instances of Java classes, or what suits best)
  - the figure is meant schematic, only
  - produced by the parser, used by later phases
  - note also: we use $3$ in the AST, where lexeme was `"3"`

  ⇒ at some point, the lexeme *string* (for numbers) is translated to a *number* in the meta-language (typically already by the lexer)

# Plausible schematic AST (for the other parse tree)




- this AST: rather "simplified" version of the CST
- an AST closer to the CST (just dropping the parentheses): in principle nothing "wrong" with it either

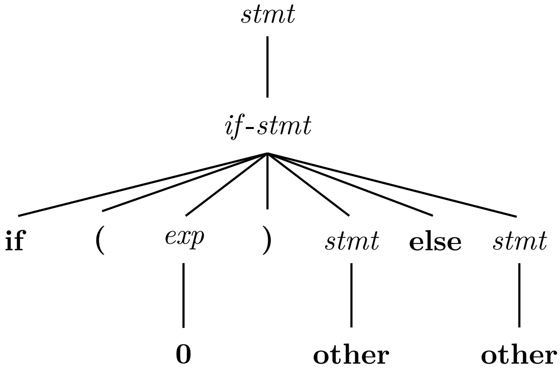
# Conditionals

## Conditionals $G_1$

$$
\begin{array}{rcl}
stmt & \rightarrow & \textit{if-stmt} \mid \textbf{other} \\
\textit{if-stmt} & \rightarrow & \textbf{if}\ (\ exp\ )\ stmt \\
& \mid & \textbf{if}\ (\ exp\ )\ stmt\ \textbf{else}\ stmt \\
exp & \rightarrow & \mathbf{0} \mid \mathbf{1}
\end{array}
\tag{5}
$$

# Parse tree

**if ( 0 ) other else other**



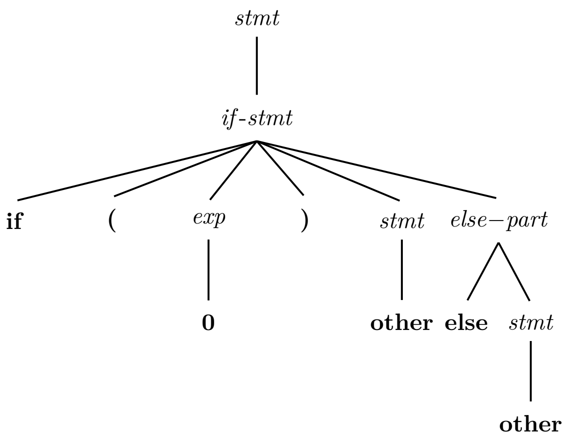
# Another grammar for conditionals

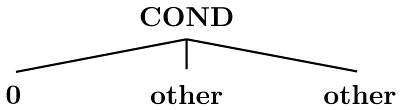
**Conditionals $G_2$**

$$
\begin{array}{rcl}
stmt & \rightarrow & if\text{-}stmt \mid \textbf{other} \\
if\text{-}stmt & \rightarrow & \textbf{if}\,(\,exp\,)\,stmt\;else{-}part \\
else{-}part & \rightarrow & \textbf{else}\,stmt \mid \epsilon \\
exp & \rightarrow & \textbf{0} \mid \textbf{1}
\end{array} \tag{6}
$$

$\epsilon$ = empty word

# A further parse tree + an AST



# Section

## Ambiguity

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# Tempus fugit . . .

picture source: wikipedia

# Ambiguous grammar

**Definition (Ambiguous grammar)**

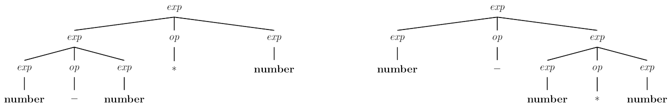
A grammar is *ambiguous* if there exists a word with *two different* parse trees.

Remember grammar from equation (1):

$$\begin{array}{rcl} exp & \rightarrow & exp \;\; op \;\; exp \mid (\, exp \,) \mid \textbf{number} \\ op & \rightarrow & + \mid - \mid * \end{array}$$

Consider:

$$\mathbf{n} - \mathbf{n} * \mathbf{n}$$


exp
exp
op
exp
exp
op
exp
*
number
number
−
number
exp
exp
op
exp
number
−
exp
op
exp
number
*
number

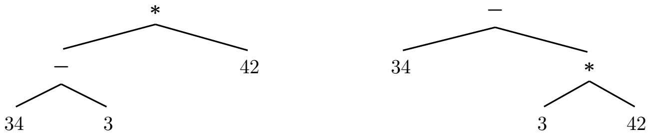
# 2 resulting ASTs


different parse trees ⇒ different ASTs ⇒ different meaning

**Side remark: different meaning**

The issue of "different meaning" may in practice be subtle: is $(x + y) - z$ the same as $x + (y - z)$?

# 2 resulting ASTs




different parse trees ⇒ different ASTs ⇒ different meaning

**Side remark: different meaning**

The issue of "different meaning" may in practice be subtle: is $(x + y) - z$ the same as $x + (y - z)$? In principle yes, but what about MAXINT ?

# Precendence & associativity

- one way to make a grammar unambiguous (or less ambiguous)
- for instance:

| binary op's | precedence | associativity |
|---|---|---|
| $+, -$ | low | left |
| $\times, /$ | higher | left |
| $\uparrow$ | highest | right |

- $a \uparrow b$ written in standard math as $a^b$:

$$
\begin{aligned}
5 + 3/5 \times 2 + 4 \uparrow 2 \uparrow 3 &= \\
5 + 3/5 \times 2 + 4^{2^3} &= \\
(5 + ((3/5 \times 2)) + (4^{(2^3)}))\ .
\end{aligned}
$$

- mostly fine for *binary* ops, but usually also for unary ones (postfix or prefix)

# Unambiguity without imposing explicit associativity and precedence

- removing ambiguity by reformulating the grammar
- precedence for op's: *precedence cascade*
  - some bind stronger than others ($*$ more than $+$)
  - introduce separate *non-terminal* for each precedence level (here: terms and factors)

# Expressions, revisited

- *associativity*
  - *left*-assoc: write the corresponding rules in *left-recursive* manner, e.g.:

$$exp \to exp\ addop\ term \mid term$$

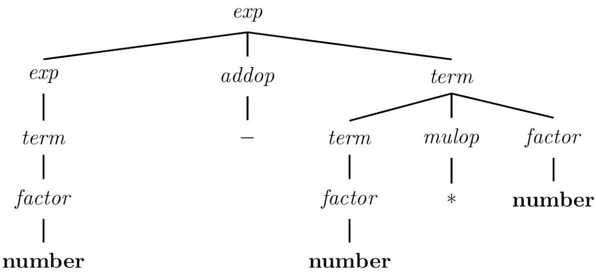
  - *right*-assoc: analogous, but right-recursive
  - *non*-assoc:

$$exp \to term\ addop\ term \mid term$$

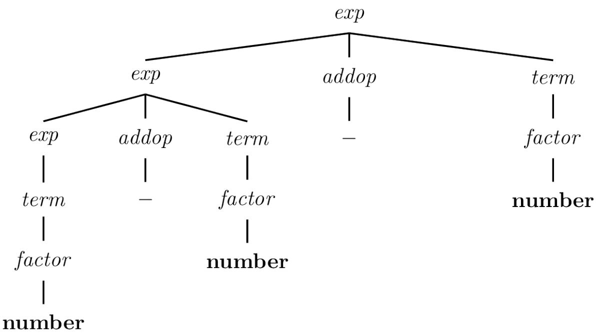
## factors and terms

$$
\begin{aligned}
exp &\to exp \;\; addop \;\; term \mid term \qquad (7)\\
addop &\to + \mid - \\
term &\to term \;\; mulop \;\; factor \mid factor \\
mulop &\to * \\
factor &\to (\, exp\,) \mid \textbf{number}
\end{aligned}
$$

# $34 - 3 * 42$


exp
exp
addop
term
term
−
term
mulop
factor
factor
factor
*
number
number
number

# $34 - 3 - 42$


exp
exp
addop
term
exp
addop
term
–
factor
term
–
factor
number
factor
number
number

# Real life example

left associative

**Operator Precedence**

Java performs operations assuming the following ordering (or ***precedence***) rules if parentheses are not used to determine the order of evaluation (operators on the same line are evaluated in left-to-right order subject to the conditional evaluation rule for && and ||). The operations are listed below from highest to lowest precedence (we use ⟨exp⟩ to denote an atomic or parenthesized expression):

| | |
|---|---|
| postfix ops | [] . (⟨exp⟩) ⟨exp⟩ ++ ⟨exp⟩ −− |
| prefix ops | ++⟨exp⟩ −−⟨exp⟩ −⟨exp⟩ ˜⟨exp⟩ !⟨exp⟩ |
| creation/cast | **new** (⟨type⟩)⟨exp⟩ |
| mult./div. | * / % |
| add./subt. | + − |
| shift | << >> >>> |
| comparison | < <= > >= **instanceof** |
| equality | == != |
| bitwise-and | & |
| bitwise-xor | ˆ |
| bitwise-or | \| |
| and | && |
| or | \|\| |
| conditional | ⟨bool_exp⟩? ⟨true_val⟩: ⟨false_val⟩ |
| assignment | = |
| op assignment | += −= *= /= %= |
| bitwise assign. | >>= <<= >>>= |
| boolean assign. | &= ˆ= \|= |

# Another example

cppreference.com    Create account    Search

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C++ / C++ language / Expressions

## C++ Operator Precedence

The following table lists the precedence and associativity of C++ operators. Operators are listed top to bottom, in descending precedence.

<table>
<tr><th>Precedence</th><th>Operator</th><th>Description</th><th>Associativity</th></tr>
<tr><td>1</td><td>::</td><td>Scope resolution</td><td rowspan="2">Left-to-right</td></tr>
<tr><td>2</td><td>a++ a--<br>type() type{}<br>a()<br>a[]<br>. -></td><td>Suffix/postfix increment and decrement<br>Functional cast<br>Function call<br>Subscript<br>Member access</td></tr>
<tr><td>3</td><td>++a --a<br>+a -a<br>! ~<br>(type)<br>*a<br>&a<br>sizeof<br>new new[]<br>delete delete[]</td><td>Prefix increment and decrement<br>Unary plus and minus<br>Logical NOT and bitwise NOT<br>C-style cast<br>Indirection (dereference)<br>Address-of<br>Size-of[note 1]<br>Dynamic memory allocation<br>Dynamic memory deallocation</td><td>Right-to-left</td></tr>
<tr><td>4</td><td>.* ->*</td><td>Pointer-to-member</td><td rowspan="11">Left-to-right</td></tr>
<tr><td>5</td><td>a*b a/b a%b</td><td>Multiplication, division, and remainder</td></tr>
<tr><td>6</td><td>a+b a-b</td><td>Addition and subtraction</td></tr>
<tr><td>7</td><td><< >></td><td>Bitwise left shift and right shift</td></tr>
<tr><td>8</td><td>< <=<br>> >=</td><td>For relational operators < and ≤ respectively<br>For relational operators > and ≥ respectively</td></tr>
<tr><td>9</td><td>== !=</td><td>For relational operators = and ≠ respectively</td></tr>
<tr><td>10</td><td>a&b</td><td>Bitwise AND</td></tr>
<tr><td>11</td><td>^</td><td>Bitwise XOR (exclusive or)</td></tr>
<tr><td>12</td><td>|</td><td>Bitwise OR (inclusive or)</td></tr>
<tr><td>13</td><td>&&</td><td>Logical AND</td></tr>
<tr><td>14</td><td>||</td><td>Logical OR</td></tr>
<tr><td>15</td><td>a?b:c<br>throw<br>=<br>+= -=<br>*= /= %=<br><<= >>=<br>&= ^= |=</td><td>Ternary conditional[note 2]<br>throw operator<br>Direct assignment (provided by default for C++ classes)<br>Compound assignment by sum and difference<br>Compound assignment by product, quotient, and remainder<br>Compound assignment by bitwise left shift and right shift<br>Compound assignment by bitwise AND, XOR, and OR</td><td>Right-to-left</td></tr>
<tr><td>16</td><td>,</td><td>Comma</td><td>Left-to-right</td></tr>
</table>

1. ↑ The operand of `sizeof` can't be a C-style type cast: the expression `sizeof (int) * p` is unambiguously interpreted as `(sizeof(int)) * p`, but not `sizeof((int)*p)`.
2. ↑ The expression in the middle of the conditional operator (between `?` and `:`) is parsed as if parenthesized: its precedence relative to `?:` is ignored.

When parsing an expression, an operator which is listed on some row of the table above with a precedence will be bound tighter (as if by parentheses) to its arguments than any operator that is listed on a row further below it with a lower precedence. For example, the expressions `std::cout << a & b` and `*p++` are parsed as `(std::cout << a) & b` and `*(p++)`, and not as `std::cout << (a & b)` or `(*p)++`.

Operators that have the same precedence are bound to their arguments in the direction of their associativity. For example, the expression `a = b = c` is parsed as `a = (b = c)`, and not as `(a = b) = c` because of right-to-left associativity of assignment, but `a + b - c` is parsed `(a + b) - c` and not `a + (b - c)` because of left-to-right associativity of addition and subtraction.

Associativity specification is redundant for unary operators and is only shown for completeness: unary prefix operators always associate right-to-left (`delete ++*p` is `delete(++(*p))`) and unary postfix operators always associate left-to-right (`a[1][2]++` is `((a[1])[2])++`). Note that the associativity is meaningful for member access operators, even though they are grouped with unary postfix operators: `a.b++` is parsed `(a.b)++` and not `a.(b++)`.

Operator precedence is unaffected by operator overloading. For example, `std::cout << a ? b : c;` parses as

INF5110 – Compiler Construction

Targets & Outline

Introduction

Context-free grammars and BNF notation

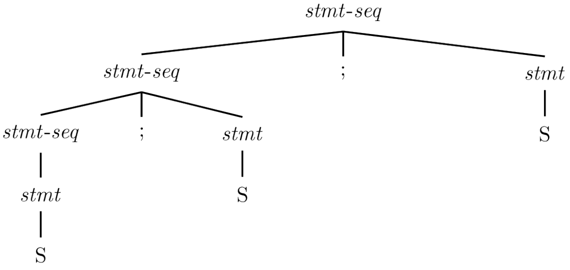
Ambiguity

Chomsky hierarchy

# Non-essential ambiguity

## left-assoc

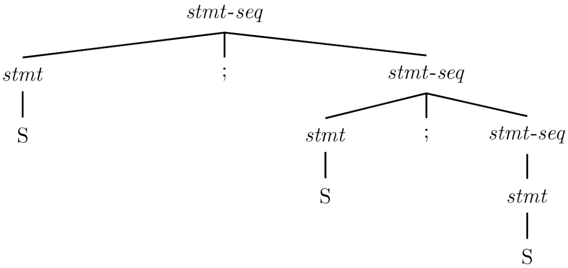
$$
\begin{array}{rcl}
stmts & \rightarrow & stmts\,;stmt \\
 & | & stmt \\
stmt & \rightarrow & S
\end{array}
$$



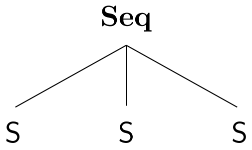
# Non-essential ambiguity (2)

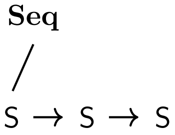
## right-assoc representation instead

$$\begin{array}{rcl} stmts & \rightarrow & stmt\,;\,stmts \\ & | & stmt \\ stmt & \rightarrow & S \end{array}$$



# Possible AST representations



# Dangling else

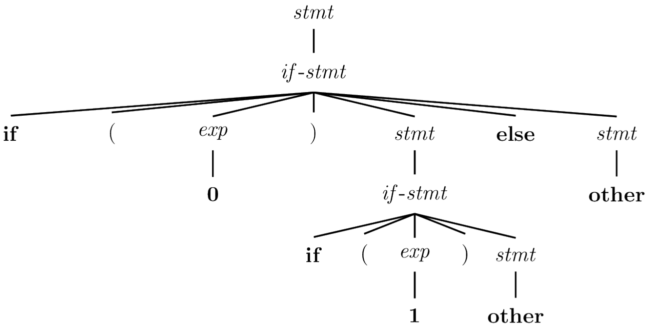
### Nested if's

$$\textbf{if}\,(\,0\,)\,\textbf{if}\,(\,1\,)\,\textbf{other else other}$$

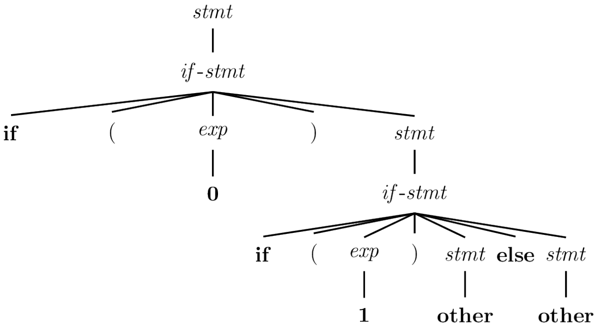
Remember grammar from equation (5):

$$
\begin{array}{rcl}
\mathit{stmt} & \rightarrow & \mathit{if\text{-}stmt} \;\mid\; \textbf{other} \\
\mathit{if\text{-}stmt} & \rightarrow & \textbf{if}\;(\,\mathit{exp}\,)\;\mathit{stmt} \\
 & \mid & \textbf{if}\;(\,\mathit{exp}\,)\;\mathit{stmt}\;\textbf{else}\;\mathit{stmt} \\
\mathit{exp} & \rightarrow & \mathbf{0} \;\mid\; \mathbf{1}
\end{array}
$$

# Should it be like this . . .



# . . . or like this




- common convention: connect **else** to closest "free" (= dangling) occurrence

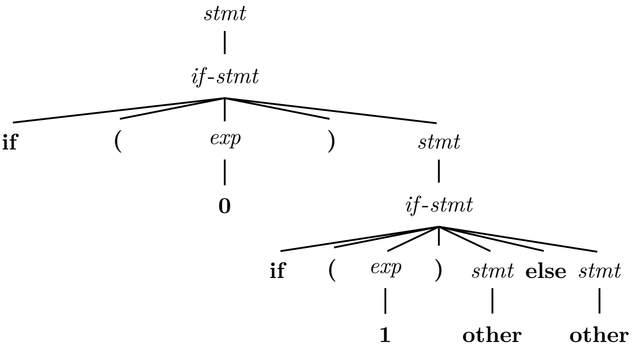
# Unambiguous grammar

## Grammar

$$
\begin{array}{rcl}
\textit{stmt} & \rightarrow & \textit{matched\_stmt} \mid \textit{unmatch\_stmt} \\
\textit{matched\_stmt} & \rightarrow & \textbf{if} \; ( \, \textit{exp} \, ) \; \textit{matched\_stmt} \; \textbf{else} \; \textit{matched\_stmt} \\
 & \mid & \textbf{other} \\
\textit{unmatch\_stmt} & \rightarrow & \textbf{if} \; ( \, \textit{exp} \, ) \; \textit{stmt} \\
 & \mid & \textbf{if} \; ( \, \textit{exp} \, ) \; \textit{matched\_stmt} \; \textbf{else} \; \textit{unmatch\_stmt} \\
\textit{exp} & \rightarrow & \mathbf{0} \mid \mathbf{1}
\end{array}
$$

- never an unmatched statement inside a matched one
- complex grammar, seldomly used
- instead: ambiguous one, with extra “rule”: connect each **else** to closest free **if**
- alternative: *different* syntax, e.g.,
  - *mandatory* **else**,
  - or require **endif**

# CST


stmt
if-stmt
if
(
exp
)
stmt
0
if-stmt
if
(
exp
)
stmt
else
stmt
1
other
other

# Adding sugar: extended BNF

- make CFG-notation more "convenient" (but without more theoretical expressiveness)
- syntactic sugar

**EBNF**

Main additional notational freedom: use *regular expressions* on the rhs of productions. They can contain terminals and non-terminals.

- EBNF: officially standardized, but often: all "sugared" BNFs are called EBNF
- in the standard:
  - $\alpha^*$ written as $\{\alpha\}$
  - $\alpha?$ written as $[\alpha]$
- supported (in the standardized form or other) by some parser tools, but not in all
- remember equation (2)

# EBNF examples

$$A \rightarrow \beta\{\alpha\} \qquad \text{for} \quad A \rightarrow A\alpha \mid \beta$$

$$A \rightarrow \{\alpha\}\beta \qquad \text{for} \quad A \rightarrow \alpha A \mid \beta$$

$$\begin{array}{rcl} \mathit{stmts} & \rightarrow & \mathit{stmt}\ \{\mathbf{;}\,\mathit{stmt}\} \\ \mathit{stmts} & \rightarrow & \{\mathit{stmt}\,\mathbf{;}\}\ \ \mathit{stmt} \\ \mathit{if}\text{-}\mathit{stmt} & \rightarrow & \mathbf{if}\ \mathbf{(}\ \mathit{exp}\ \mathbf{)}\ \mathit{stmt}[\mathbf{else}\ \mathit{stmt}] \end{array}$$

greek letters: for non-terminals or terminals.

# Some yacc style grammar

```
/* Infix notation calculator--calc */
%{
#define YYSTYPE double
#include <math.h>
%}

/* BISON Declarations */
%token NUM
%left '-' '+'
%left '*' '/'
%left NEG     /* negation--unary minus */
%right '^'    /* exponentiation        */

/* Grammar follows */
%%
input:    /* empty string */
        | input line
;

line:     '\n'
        | exp '\n'  { printf ("\t%.10g\n", $1); }
;

exp:      NUM                { $$ = $1;         }
        | exp '+' exp        { $$ = $1 + $3;    }
        | exp '-' exp        { $$ = $1 - $3;    }
        | exp '*' exp        { $$ = $1 * $3;    }
        | exp '/' exp        { $$ = $1 / $3;    }
        | '-' exp  %prec NEG { $$ = -$2;        }
        | exp '^' exp        { $$ = pow ($1, $3); }
        | '(' exp ')'        { $$ = $2;         }
;
%%
```

# Section

## Chomsky hierarchy

Chapter 3 “Grammars”
Course “Compiler Construction”
Martin Steffen
Spring 2024

# The Chomsky hierarchy

- linguist Noam Chomsky [? ]
- important classification of (formal) languages (sometimes Chomsky-Schützenberger)
- 4 levels: type 0 languages – type 3 languages
- levels related to machine models that generate/recognize them
- so far: regular languages and CF languages

# Overview

| | rule format | languages | machines | closed |
|---|---|---|---|---|
| 3 | $A \to aA$ , $A \to a$ | regular | NFA, DFA | all |
| 2 | $A \to \alpha_1 \beta \alpha_2$ | CF | pushdown automata | $\cup$, $*$, $\circ$ |
| 1 | $\alpha_1 A \alpha_2 \to \alpha_1 \beta \alpha_2$ | context-sensitive | (linearly restricted automata) | all |
| 0 | $\alpha \to \beta$, $\alpha \neq \epsilon$ | recursively enumerable | Turing machines | all, except complement |

## Conventions

- terminals $a, b, \ldots \in \Sigma_T$,
- non-terminals $A, B, \ldots \in \Sigma_N$
- general words $\alpha, \beta \ldots \in (\Sigma_T \cup \Sigma_N)^*$

# Phases of a compiler & hierarchy

## "Simplified" design?

1 big grammar for the whole compiler? Or at least a CSG for the front-end, or a CFG combining parsing and scanning?

possible, but a bad idea:

- efficiency
- bad design
- especially combining scanner + parser in one BNF:
  - grammar would be needlessly large
  - separation of concerns: much clearer/ more efficient design
- for scanner/parsers: regular expressions + (E)BNF: simply the formalisms of choice!
  - front-end needs to do more than checking syntax, CFGs not expressive enough
  - for level-2 and higher: situation gets less clear-cut, plain CSG not too useful for compilers

# References I

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