Chapter 7

Types and type checking

Course “Compiler Construction”
Martin Steffen
Spring 2024

‘

Chapter 7

Learning Targets of Chapter “Types and type checking”.

1. the concept of types
2. specific common types
3. type safety
4. type checking
5. polymorphism, subtyping and other complications

Chapter 7

Outline of Chapter “Types and type checking”.

Section

Introduction

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General remarks and overview

- Goal here:
  - what are *types*?
  - static vs. dynamic typing
  - how to describe types *syntactically?*
  - how to *represent* and use types in a compiler?
- coverage of various types
  - basic types (often predefined/built-in)
  - type constructors
  - values of a type
  - type operators
  - representation at run-time
  - run-time tests and special problems (array, union, record, pointers)
- specification and implementation of type systems/type checkers
- advanced concepts

Why types?

- crucial, user-visible abstraction describing program behavior
- one view: type describes a set of (mostly related) *values*
- static typing: checking/enforcing a type discipline at compile time
- dynamic typing: same at run-time, mixtures possible
- completely untyped languages: very rare to non-existant, types were part of PLs from the start.

Milner's dictum ("type safety")

Well-typed programs cannot go wrong!

- *strong* typing:[1] rigorously prevent "misuse" of data
- types useful for later phases and optimizations
- documentation and partial specification

[1] The terminology is rather fuzzy, and perhaps changed a bit over time.

Types: in first approximation

Conceptually

- semantic view: set of values *plus* a set of corresponding operations
- syntactic view: notation to *construct* basic elements of the type (its values) *plus* "procedures" operating on them
- compiler implementor's view: data of the same type have same underlying memory representation

further classification:

- built-in/predefined vs. *user-defined* types
- basic/base/elementary/primitive types vs. compound types
- type constructors: building more compex types from simpler ones
- reference vs. value types

Section

Various types and their representation

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Some typical basic types

base types			
`int`	`0, 1, ...`	$+, -, *, /$	integers
`real`	`5.05E4 ...`	`+,-,*`	real numbers
`bool`	`true`, `false`	`and or (|) ...`	booleans
`char`	`'a'`		characters

⋮

- often HW support for some of those (including some of the op's)
- elements of `int` are not exactly mathematical *integers*, same for `real`
- often variations offered: `int32`, `int64`
- often implicit conversions and relations between basic types
  - which the type system has to specify/check for legality
  - which the compiler has to implement

Some compound types

compound types		
`array[0..9] of real`		`a[i+1]`
`list`	`[], [1;2;3]`	concat
`string`	`"text"`	concat ...
`struct / record`		`r.x`

...

- mostly reference types
- when built in, special "easy syntax" (same for basic built-in types)
  - `4 + 5` as opposed to `plus(4,5)`
  - `a[6]` as opposed to `array_access(a, 6)` ...
- parser/lexer aware of built-in types/operators (special precedences, associativity, etc.)
- cf. functionality "built-in/predefined" via libraries

Abstract data types

- unit of *data* together with *functions/procedures/operations* ... operating on them
- encapsulation + interface
- often: separation between exported and internal operations
  - for instance `public`, `private` ...
  - or via separate interfaces
- (static) classes in Java: may be used/seen as ADTs, methods are then the "operations"

```
ADT  begin
    integer i;
    real x;
    int proc total(int a) {
       return i * x + a  // or: ``total = i * x + a''
    }
end
```

Type constructors: building new types

- array type
- record type (also known as struct-types)
- union type
- pair/tuple type
- pointer type
  - explict as in C
  - implict distinction between reference and value types, hidden from programmers (e.g. Java)
- *signatures* (specifying methods / procedures / subroutines / functions) as type
- function type constructor, incl. higher-order types (in functional languages)
- (names of) classes and subclasses
- ...

Arrays

Array type

```
array [<indextype>] of <component type>
```

- elements (arrays) = (finite) functions from index-type to component type
- allowed index-types:
  - non-negative (unsigned) integers?, `from ... to ...`?
  - other types?: enumerated types, characters
- things to keep in mind:
  - indexing outside the array bounds?
  - are the array bounds (statically) known to the compiler?
  - dynamic arrays (extensible at run-time)?

One and more-dimensional arrays

- one-dimensional: efficiently implementable in standard hardware (relative memory addressing, known offset)
- two or more dimensions

```
array [1..4] of array [1..3] of real
array [1..4, 1..3] of real
```

- one can see it as "array of arrays" (Java), an array is typically a reference type
- conceptually "two-dimensional"- *linear layout* in memory (language dependent)

Records (“structs”)

```
struct {
  real r;
  int  i;
}
```

- values: “labelled tuples” (`real` × `int`)
- constructing elements, e.g.

`struct point`

- access (read or update): *dot-notation* `x.i`
- implementation: linear memory layout given by the (types of the) attributes
- attributes accessible by statically fixed offsets
- fast access
- cf. objects as in Java

Tuple/product types

- $T_1 \times T_2$ (or in ascii `T_1 * T_2`)
- elements are *tuples*: for instance: `(1, "text")` is element of `int * string`
- generalization to n-tuples:

value	type
`(1, "text", true)`	`int * string * bool`
`(1, ("text", true))`	`int * (string * bool)`

- structs can be seen as "labeled tuples", resp. tuples as "anonymous structs"
- tuple types: common in functional languages,
- in C/Java-like languages: n-ary tuple types often only implicit as input types for procedures/methods (part of the "signature")

Union types (C-style again)

```
union {
  real r;
  int  i
}
```

- related to sum types (outside C)
- (more or less) represents disjoint union of values of "participating" types
- access in C (confusingly enough): dot-notation `u.i`

Union types in C and type safety

- union types is C: bad example for (safe) type disciplines, as it's simply type-unsafe, basically an *unsafe* hack ...

Union type (in C):

- nothing much more than a directive to allocate enough memory to hold largest member of the union.
- in the example: `real` takes more space than `int`

- implementor's (= low level) focus and memory allocation, not "proper usage focus" or assuring strong typing

⇒ bad example of modern use of types

- better (type-safe) implementations known since

⇒ variant record ("tagged"/"discriminated" union) or even inductive data types

Variant records from Pascal

```
record case isReal: boolean of
  true:  (r:real);
  false: (i:integer);
```

- "variant record"
- non-overlapping memory layout[2]
- programmer responsible to set and check the "discriminator" self
- enforcing type-safety-wise: not really an improvement :-(

[2] Again, that's an implementor-centric view, not a user-centric one.

Inductive types in ML and similar

- *type-safe* and powerful
- allows *pattern matching*

```
IsReal of real | IsInteger of int
```

- allows *recursive* definitions ⇒ inductive data types:

```
 type int_bintree =
    Node of int * int_bintree * int_bintree
  |  Nil
```

- `Node`, `Leaf`, `IsReal`: *constructors* (cf. languages like Java)
- constructors used as discriminators in “union” types

```
type exp =
    Plus of exp * exp
  | Minus of exp * exp
  | Number of int
  | Var of string
```

Recursive data types in C

does not work

```
struct int_bintree {
  int val;
  struct int_bintree left, right;
}
```

- note: *implementation* in ML: also uses "pointers" (but hidden from the user)
- no nil-pointers in ML (and `NIL` is not a nil-pointer, it's a constructor)

Recursive data types in C

“indirect” recursion

```
struct int_bintree {
  int val;
  struct int_bintree *left, *right;
};
```

- note: *implementation* in ML: also uses “pointers” (but hidden from the user)
- no nil-pointers in ML (and `NIL` is not a nil-pointer, it’s a constructor)

Recursive data types in C

"indirect" recursion

```
struct int_bintree {
  int val;
  struct int_bintree *left, *right;
};
```

In Java: references implicit

```
class IntBinTreeNode {
 int val;
 IntBinTreeNode left, right;
}
```

- note: *implementation* in ML: also uses "pointers" (but hidden from the user)
- no nil-pointers in ML (and `NIL` is not a nil-pointer, it's a constructor)

Pointer types

- *pointer* type: notation in C: `int*`
- " * ": can be seen as type constructor

```
int* p;
```

- random other languages: `^integer` in Pascal, `int ref` in ML
- value: *address* of (or reference/pointer to) values of the underlying type
- operations: *dereferencing* and determining the address of an data item (and C allows " *pointer arithmetic* ")

```
var a:  ^integer  (* pointer to an integer    *)
var b:   integer
...
a := &i            (* i an int var             *)
                   (* a :=  new integer ok too *)
b:= ^a + b
```

Implicit dereferencing

- many languages: more or less hide existence of pointers
- cf. reference vs. value types often: automatic/implicit dereferencing

```
C r;
C r = new C();
```

- "sloppy" speaking: " `r` is an object (which is an instance of class `C` /which is of type `C`)",
- slightly more precise: variable " `r` contains an object... "
- precise: "variable `r` will contain a reference to an object"
- `r.field` corresponds to something like " `(*r).field`, similar in Simula ...

Programming with pointers

- "popular" source of errors
- test for non-null-ness often required
- explicit pointers: can lead to problems in block-structured language (when handled non-expertly)
- watch out for parameter passing
- aliasing
- null-pointers: "the billion-dollar-mistake"
- take care of concurrency

Function variables

```
program Funcvar;
var pv : Procedure (x: integer);  (* procedur var    *)

   Procedure Q();
   var
      a : integer;
      Procedure P(i : integer);
      begin
         a:= a+i;     (* a def'ed outside              *)
      end;
   begin
      pv := @P;       (* ``return'' P (as side effect) *)
   end;               (* "@" dependent on dialect      *)
begin                 (* here: free Pascal             *)
   Q();
   pv(1);
end.
```

Function variables and nested scopes

- tricky part here: nested scope + function definition *escaping* surrounding function/scope.
- here: inner procedure "returned" via assignment to function variable
- stack discipline of dynamic memory management?
- related also: functions allowed as return value?
  - Pascal: not directly possible (unless one "returns" them via function-typed reference variables like here)
  - C: possible, but *nested* function definitions not allowed
- combination of nested function definitions and functions as official return values (and arguments): higher-order functions
- Note: functions as arguments less problematic than as return values.

Function signatures

- define the "header" (also "signature") of a function[3]
- in the discussion: we don't distinguish mostly: functions, procedures, methods, subroutines.
- functional type (independent of the name f): `int`$\rightarrow$`int`

Modula-2

```
var f: procedure (integer): integer;
```

C

```
int (*f) (int)
```

- values: all functions (procedures ...) with the given signature
- problems with block structure and free use of procedure variables.

[3] Actually, an identfier of the function is mentioned as well.

Escaping

```
program Funcvar;
var pv : Procedure (x: integer);  (* procedur var     *)

   Procedure Q();
   var
      a : integer;
      Procedure P(i : integer);
      begin
         a:= a+i;     (* a def'ed outside              *)
      end;
   begin
      pv := @P;       (* ``return'' P (as side effect) *)
   end;               (* "@" dependent on dialect      *)
begin                 (* here: free Pascal             *)
  Q();
  pv(1);
end.
```

- at the end of line 15: variable `a` no longer exists
- possible safe usage: only assign to such variables (here `pv`) a new value (= function) at the same blocklevel the variable is declared

Classes and subclasses

Parent class

```
class A {
  int i;
  void f() {...}
}
```

Subclass B

```
class B extends A {
  int i
  void f() {...}
}
```

Subclass C

```
class C extends A {
  int i
  void f() {...}
}
```

- classes resemble records, and subclasses variant types, but additionally
  - visibility: local methods possible (besides fields)
  - subclasses
  - objects mostly created dynamically, *no* references into the stack
  - subtyping and polymorphism (subtype polymorphism): a reference typed by `A` can also point to `B` or `C` objects
- special problems: not really many, nil-pointer still possible

Access to object members: late binding

- notation `rA.i` or `rA.f()`
- dynamic binding, late-binding, virtual access, dynamic dispatch ...: all mean roughly the same
- central mechanism in many OO language, in connection with inheritance

Virtual access `rA.f()` (methods)

"deepest" `f` in the run-time class of the *object*, `rA` points to

- remember: "most-closely nested" access of variables in nested lexical block
- Java:
  - methods "in" objects are only dynamically bound (but there are class methods too)
  - instance variables not, neither static methods "in" classes.

Example: fields and methods

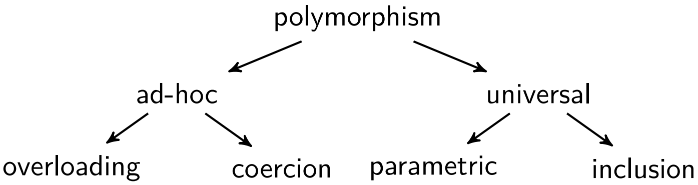
```
public class Shadow {
    public static void main(String[] args){
        C2 c2 = new C2();
        c2.n();
    }
}

class C1 {
    String s = "C1";
    void m () {System.out.print(this.s);}
}


class C2 extends C1 {
    String s = "C2";
    void n () {this.m();}
}
```

Diverse notions

- *Overloading*
  - common for (at least) standard, built-in operations
  - also possible for user defined functions/methods ...
  - disambiguation via (static) types of arguments
  - "ad-hoc" polymorphism
  - implementation:
    - put types of parameters as "part" of the name
    - look-up gives back a set of alternatives
- (generic) polymporphism

```
swap(var x,y:  anytype)
```

Polymorphism



Section

Equality of types

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Classes as types

- classes = types? Not so fast
- more precise view:
    - design decision in Java and similar languages (but not all/even not all class-based OOLs): that class *names* are used in the role of (names of) types.
- other roles of classes (in class-based OOLs)
    - generator of objects (via constructor, again with the same name)[4]
    - containing *code* that implements the instances

[4] Not for Java's *static* classes etc, obviously.

When are 2 types "equal"?

- type equivalence
- surprisingly many different answers possible
- implementor's focus (deprecated): type int and short are equal, because they "are" both 2 bytes
- type checker must often decide such equivalences
- related to a more fundamental question: what's a type?

Example: pairs of integers

```
type pair_of_ints = int * int;;
let x : pair_of_int = (1,4);;
```

Is "the" type of (values of) x

- `pair_of_ints`, or
- the product type `int * int`, or
- both, as they are equal, i.e., `pair_of_int` is an abbreviation of the product type (type synonym)?

Example with interfaces

```java
interface I1 { int m (int x); }
interface I2 { int m (int x); }
class C1 implements I1 {
    public int m(int y) {return y++;  }
}
class C2 implements I2 {
    public int m(int y) {return y++;  }
}

public class Noduck1 {
    public static void main(String[] arg) {
        I1 x1 = new C1();        // I2 not possible
        I2 x2 = new C2();
        x1 = x2;                 // ???
    }
}
```

Analogous when using classes in their roles as types

Structural vs. nominal equality

a, b

```
var a, b: record
    int i;
    double d
  end
```

c

```
var c: record
    int i;
    double d
  end
```

typedef

```
typedef idRecord: record
    int i;
    double d
  end
```

```
var d: idRecord;
var e: idRecord;;
```

what's possible?

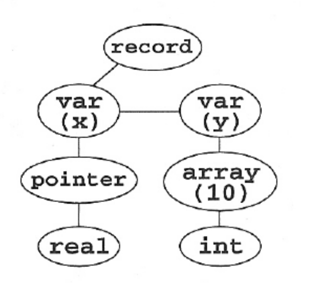
```
a := c;
a := d;

a := b;
d := e;
```

Types in the AST

- types are part of the syntax, as well
- represent: either in a separate symbol table, or part of the AST

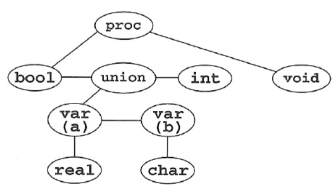
Record type

```
record
  x: pointer to real;
  y: array [10] of int
 end
```



Procedure header

```
proc(bool,
     union a: real; b:char end,
     int):void
 end
```



Structured types without names

$$
\begin{array}{rcl}
\textit{var-decls} & \rightarrow & \textit{var-decls}\,\mathbf{;}\,\textit{var-decl} \mid \textit{var-decl} \\
\textit{var-decl} & \rightarrow & \mathbf{id}\,\mathbf{:}\,\textit{type-exp} \\
\textit{type-exp} & \rightarrow & \textit{simple-type} \mid \textit{structured-type} \\
\textit{simple-type} & \rightarrow & \mathbf{int} \mid \mathbf{bool} \mid \mathbf{real} \mid \mathbf{char} \mid \mathbf{void} \\
\textit{structured-type} & \rightarrow & \mathbf{array}\,[\,\textit{num}\,]\,:\,\textit{type-exp} \\
 & \mid & \mathbf{record}\,\textit{var-decls}\,\mathbf{end} \\
 & \mid & \mathbf{union}\,\textit{var-decls}\,\mathbf{end} \\
 & \mid & \mathbf{pointerto}\,\textit{type-exp} \\
 & \mid & \mathbf{proc}\,\mathbf{(}\,\textit{type-exps}\,\mathbf{)}\,\textit{type-exp} \\
\textit{type-exps} & \rightarrow & \textit{type-exps}\,\mathbf{,}\,\textit{type-exp} \mid \textit{type-exp}
\end{array}
$$

Structural equality (1)

```
function typeEqual(t1, t2: TypeExp) : Boolean;
var temp: Boolean;
    p1, p2: TypeExp;
begin
  if   t1 and t2 are of simple type
  then return t1 = t2
  else if t1.kind = array and t2.kind = array
  then return t1.size = t2.size and typeEqual(t1.child, t2.child)
  else if      t1.kind = record and t2.kind = record
          or   t1.kind = union and t2.kind = union
      then begin
               p1 := t1.child;
               p2 := t2.child;
               temp := true;
               while temp and p1 ≠ nil and p2 ≠ nil
               do
                  if   p1.name ≠ p2.name
                  then temp := false
                  else
                  begin
                     p1 := p1.sibling;
                     p2 := p2.sibling;
                  end;
                return temp and p1 = nil and p2 = nil;
```

Structural equality (2)

```
  else if t1.kind = pointer and t2.kind = pointer
  then    return typeEqual(t1.child, t2.child)
  else if t1.kind = proc and t2.kind = proc
  then    begin
              p1 := t1.child;
              p2 := t2.child;
              temp := true;
              while temp and p1 ≠ nil and p2 ≠ nil
              do
                 if   not typeEqual(p1.child, p2.child)
                 then temp := false
                 else
                   begin
                     p1 := p1.sibling;
                     p2 := p2.sibling;
                   end;
                 return temp and p1 = nil and p2 = nil
                             and typeEqual(t1.child,t2.child=
                 end
  else if t1 and t2 are type names  (* if also names are checked *)
  then    return typeEqual(getTypeExp(t1), getTypeExp(t2)
  else    return false
end; (* typeEqual)
```

Structural equality

```
type texp = (* abstract syntax for types *)
    TBool
  | TInt
  | TFloat
  | TArray of int * texp
  | TRecord of string * (string * texp) list
  | TUnion  of string * (string * texp) list
  | TPointer of texp
  | TFunc    of texp * texp
```

Structural equality

```
let rec t_structequal ((t1, t2): texp * texp) =
  match (t1, t2) with
    (TBool, TBool) | (TInt,TInt) | (TFloat,TFloat) -> true
  | (TArray(size1, t1'), TArray (size2, t2')) ->
     ( size1 = size2 && t_structequal (t1', t2'))
  | TRecord(name_1,flist1), TRecord(name_2,flist2)
    | TUnion(name_1,flist1), TUnion(name_2,flist2) ->
     (name_1 = name_2 && t_structequal_fields (flist1 ,flist2))
  | (TPointer t1', TPointer t2') ->
     t_structequal(t1',t2')
  | (TFunc(s1',t1'), TFunc(s2',t2')) ->
     t_structequal (s1', s2') && t_structequal (t1',t2')
  | _ -> false
```

Types with names

$$
\begin{array}{rcl}
\textit{var-decls} & \to & \textit{var-decls}\,\mathbf{;}\,\textit{var-decl} \;\mid\; \textit{var-decl} \\
\textit{var-decl} & \to & \mathbf{id}\,\mathbf{:}\,\textit{simple-type-exp} \\
\textit{type-decls} & \to & \textit{type-decls}\,\mathbf{;}\,\textit{type-decl} \;\mid\; \textit{type-decl} \\
\textit{type-decl} & \to & \mathbf{id} = \textit{type-exp} \\
\textit{type-exp} & \to & \textit{simple-type-exp} \;\mid\; \textit{structured-type} \\
\textit{simple-type-exp} & \to & \textit{simple-type} \;\mid\; \mathbf{id} \\
\textit{simple-type} & \to & \mathbf{int} \mid \mathbf{bool} \mid \mathbf{real} \mid \mathbf{char} \mid \mathbf{void} \\
\textit{structured-type} & \to & \mathbf{array}\,[\,\textit{num}\,] : \textit{simple-type-exp} \\
 & \mid & \mathbf{record}\;\textit{var-decls}\;\mathbf{end} \\
 & \mid & \mathbf{union}\;\textit{var-decls}\;\mathbf{end} \\
 & \mid & \mathbf{pointerto}\;\textit{simple-type-exp} \\
 & \mid & \mathbf{proc}\,(\,\textit{type-exps}\,)\;\textit{simple-type-exp} \\
\textit{type-exps} & \to & \textit{type-exps}\,,\,\textit{simple-type-exp} \\
 & \mid & \textit{simple-type-exp}
\end{array}
$$

Name equality

- all types have "names", and two types are equal iff their names are equal
- type equality checking: obviously simpler
- of course: type names may have *scopes*. . . .

```
function typeEqual(t1, t2: TypeExp): Boolean;
var temp: boolean
    p1, p2: TypeExp;
begin
  if      t1 and t2 are of simple type
  then    return t1 = t2
  else if t1 and t2 are type names
  then    return t1 = t2
  else    return false;
end
```

Type aliases

- languages with type aliases (type synonyms): C, Pascal, ML ....
- often very convenient (`type Coordinate = float * float`)
- light-weight mechanism

type alias; make `t1` known also under name `t2`

```
type t2 = t1   // t2 is the ``same type''.
```

- also here: different choices wrt. *type equality*

Type aliases: different choices

Alias, for simple types

```
type t1 = int;
type t2 = int;
```

- often: `t1` and `t2` are the "same" type

Alias of structured types

```
type t1 = array [10] of int;
type t2 = array [10] of int;
type t3 = t2;
```

- mostly `t3` $\neq$ `t1` $\neq$ `t2`

Section

Type checking

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Type checking of expressions (and statements)

- types of subexpressions must "fit" to the expected types the contructs can operate on
- type checking: top-down and *bottom-up* task

⇒ *synthesized* attributes, when using AGs

- Here: using an attribute grammar specification of the type checker
  - type checking conceptually done *while parsing* (as actions of the parser)
  - more common: type checker operates on the AST *after* the parser has done its job
- type system vs. type checker
  - type system: specification of the rules governing the use of types in a language, type discipline
  - type checker: algorithmic formulation of the type system (resp. implementation thereof)

Grammar for statements and expressions

$$
\begin{aligned}
\textit{program} &\rightarrow \textit{var-decls}\,\mathbf{;}\,\textit{stmts} \\
\textit{var-decls} &\rightarrow \textit{var-decls}\,\mathbf{;}\,\textit{var-decl} \mid \textit{var-decl} \\
\textit{var-decl} &\rightarrow \mathbf{id}\,\mathbf{:}\,\textit{type-exp} \\
\textit{type-exp} &\rightarrow \mathbf{int} \mid \mathbf{bool} \mid \mathbf{array}\,[\,\textit{num}\,]\,:\,\textit{type-exp} \\
\textit{stmts} &\rightarrow \textit{stmts}\,\mathbf{;}\,\textit{stmt} \mid \textit{stmt} \\
\textit{stmt} &\rightarrow \mathbf{if}\ \textit{exp}\ \mathbf{then}\ \textit{stmt} \mid \mathbf{id} \mathbin{\mathbf{:=}} \textit{exp} \\
\textit{exp} &\rightarrow \textit{exp} + \textit{exp} \mid \textit{exp}\,\mathbf{or}\,\textit{exp} \mid \textit{exp}\,[\,\textit{exp}\,]
\end{aligned}
$$

Type checking as semantic rules

Grammar Rule	Semantic Rules
var-decl → **id** : *type-exp*	*insert*(**id** .*name*, *type-exp.type*)
type-exp → **int**	*type-exp.type* := *integer*
type-exp → **bool**	*type-exp.type* := *boolean*
$type\text{-}exp_1$ → **array** [**num**] **of** $type\text{-}exp_2$	$type\text{-}exp_1$.*type* := *makeTypeNode*(*array*, **num** .*size*, $type\text{-}exp_2$.*type*)
stmt → **if** *exp* **then** *stmt*	**if not** *typeEqual*(*exp.type*, *boolean*) **then** *type-error*(*stmt*)
stmt → **id** := *exp*	**if not** *typeEqual*(*lookup*(**id** .*name*), *exp.type*) **then** *type-error*(*stmt*)
exp_1 → exp_2 + exp_3	**if not** (*typeEqual*(exp_2 .*type*, *integer*) **and** *typeEqual*(exp_3 .*type*, *integer*)) **then** *type-error*(exp_1) ; exp_1 .*type* := *integer*
exp_1 → exp_2 **or** exp_3	**if not** (*typeEqual*(exp_2 .*type*, *boolean*) **and** *typeEqual*(exp_3 .*type*, *boolean*)) **then** *type-error*(exp_1) ; exp_1 .*type* := *boolean*
exp_1 → exp_2 [exp_3]	**if** *isArrayType*(exp_2 .*type*) **and** *typeEqual*(exp_3 .*type*, *integer*) **then** exp_1 .*type* := exp_2 .*type.child1* **else** *type-error*(exp_1)
exp → **num**	*exp.type* := *integer*
exp → **true**	*exp.type* := *boolean*
exp → **false**	*exp.type* := *boolean*
exp → **id**	*exp.type* := *lookup*(**id** .*name*)

More “modern” presentation

- representation as derivation rules
- Γ: notation for symbol table
  - $\Gamma(x)$: look-up
  - $\Gamma, x : T$: insert
- more compact representation
- one reason: “errors” left implicit.

Type checking (expressions)

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T}\ \textsc{TE-Id} \qquad \frac{}{\Gamma \vdash \textbf{true} : \texttt{bool}}\ \textsc{TE-True} \qquad \frac{}{\Gamma \vdash \textbf{false} : \texttt{bool}}\ \textsc{T-Fals}$$

$$\frac{}{\Gamma \vdash n : \texttt{int}}\ \textsc{TE-Num}$$

$$\frac{\Gamma \vdash exp_2 : \texttt{array\_of}\ T \qquad \Gamma \vdash exp_3 : \texttt{int}}{\Gamma \vdash exp_2\,[\,exp_3\,] : T}\ \textsc{TE-Array}$$

$$\frac{\Gamma \vdash exp_1 : \texttt{bool} \qquad \Gamma \vdash exp_2 : \texttt{bool}}{\Gamma \vdash exp_1\ \textbf{or}\ exp_2 : \texttt{bool}}\ \textsc{Te-Or}$$

$$\frac{\Gamma \vdash exp_1 : \texttt{int} \qquad \Gamma \vdash exp_2 : \texttt{int}}{\Gamma \vdash exp_1 + exp_2 : \texttt{int}}\ \textsc{TE-Plus}$$

Declarations and statements

$$\frac{\Gamma, x : \texttt{int} \vdash \textit{rest} : \texttt{ok}}{\Gamma \vdash x \,\mathbf{:}\, \mathbf{int;}\ \textit{rest} : \texttt{ok}}\ \textsc{TD-Int} \qquad \frac{\Gamma, x : \texttt{bool} \vdash \textit{rest} : \texttt{ok}}{\Gamma \vdash x \,\mathbf{:}\, \mathbf{bool;}\ \textit{rest} : \texttt{ok}}\ \textsc{TD-Bool}$$

$$\frac{\Gamma \vdash \textit{num} : \texttt{int} \qquad \Gamma(\textit{type-exp}) = T \qquad \Gamma, x : \texttt{array}\ \textit{num}\ \texttt{of}\ T \vdash \textit{rest} : \texttt{ok}}{\Gamma \vdash x \,\mathbf{:}\, \mathbf{array}\,[\,\textit{num}\,] : \textit{type-exp}\,\mathbf{;}\, \textit{rest} : \texttt{ok}}\ \textsc{TD-Array}$$

$$\frac{\Gamma \vdash x : T \qquad \Gamma \vdash \textit{exp} : T}{\Gamma \vdash x \mathbin{\mathbf{:=}} \textit{exp} : \texttt{ok}}\ \textsc{TS-Assign} \qquad \frac{\Gamma \vdash \textit{exp} : \texttt{bool} \qquad \Gamma \vdash \textit{stmt} : \texttt{ok}}{\Gamma \vdash \mathbf{if}\ \textit{exp}\ \mathbf{then}\ \textit{stmt} : \texttt{ok}}$$

$$\frac{\Gamma \vdash \textit{stmt}_1 : \texttt{ok} \qquad \Gamma \vdash \textit{stmt}_2 : \texttt{ok}}{\Gamma \vdash \textit{stmt}_1\,\mathbf{;}\,\textit{stmt}_2 : \texttt{ok}}\ \textsc{TS-Seq}$$

References I

Bibliography

[1] Louden, K. (1997). *Compiler Construction, Principles and Practice*. PWS Publishing.