



# Chapter 10

## Code generation

Course "Compiler Construction"

Martin Steffen

Spring 2024





# Chapter 10

## Learning Targets of Chapter “Code generation”.

1. 2AC
2. cost model
3. register allocation
4. control-flow graph
5. local liveness analysis (data flow analysis)
6. “global” liveness analysis



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dead or alive

Local liveness<sup>++</sup>:  
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**Global analysis**

**Code generation algo**



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## Intro

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# Code generation

- note: *code generation* so far: AST<sup>+</sup> to **intermediate code**
  - three address intermediate code (3AIC)
  - P-code
- $\Rightarrow$  *intermediate code generation*
- i.e., we are still not there ...
- material here: based on the (old) *dragon book* [2] (but principles still ok)
- there is also a new edition [1]



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# Intro: code generation

- goal: translate intermediate code (= 3AI-code) to machine language
- machine language/assembler:
  - even *more* restricted
  - here: 2 address code
- limited number of *registers*
- different *address modes* with different *costs* (registers vs. main memory)

## Goals

- **efficient** code
- small code size also desirable
- but first of all: **correct** code



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# Code “optimization”

- often conflicting goals
- code generation: *prime* arena for achieving *efficiency*
- **optimal code**: undecidable anyhow (and: don't forget there's trade-offs).
- even for many more clearly defined subproblems: *untractable*

## “optimization”

interpreted as: *heuristics* to achieve “good code” (without hope for *optimal* code)

- due to importance of optimization at code generation
  - time to bring out the “heavy artillery”
  - so far: all techniques (parsing, lexing, even sometimes type checking) are computationally “easy”
  - at code generation/optimization: perhaps *invest* in aggressive, computationally complex and rather advanced techniques
  - **many** different techniques used



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## 2AC and costs of instructions

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## 2-address machine code used here

- “typical” op-codes, but not a instruction set of a *concrete* machine
- two address instructions
- Note: cf. 3-address-code intermediate representation vs. 2-address machine code
  - machine code is **not** lower-level/closer to HW because it has one argument less than 3AC
  - it's just one illustrative choice
  - the new Dragon book: uses **3-address-machine code**
- translation task from IR to 3AC or 2AC: comparable challenge



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# 2-address instructions format



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## Format

OP source dest

- note: *order* of arguments here (esp. for minus)
- restrictions on *source* and *target*
  - register or memory cell
  - source: can additionally be a constant

```
ADD a b // b := b + a
SUB a b // b := b - a
MUL a b // b := b * a
GOTO i // unconditional jump
```

- further opcodes for conditional jumps, procedure calls  
.....

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# Side remarks: 3A machine code

## Possible format

```
OP source1 source2 dest
```

- but: what's the *difference* to 3A *intermediate* code?
- apart from a more restricted instruction set:
- **restriction** on the **operands**, for example:
  - only *one* of the arguments allowed to be a memory access
  - *no fancy addressing* modes (indirect, indexed ... see later) for memory cells, only for registers
- not “too much” memory-register traffic back and forth per machine instruction
- example:

$$\&x = \&y + *z$$

may be 3A-intermediate code, but not 3A-machine code



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# Cost model

- “optimization”: need some well-defined “measure” of the “quality” of the produced code
- interested here in *execution* time
- not all instructions take the same time
- estimation of execution
- factors outside our control/not part of the cost model: effect of *caching*

## cost factors:

- size of instruction
  - it's here not about code size, but
  - instructions need to be *loaded*
  - longer instructions  $\Rightarrow$  perhaps longer load
- address modes (as *additional costs*: see later)
  - registers vs. main memory vs. constants
  - direct vs. indirect, or indexed access



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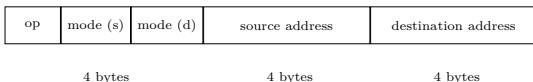
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# Instruction modes and additional costs



## Modes and cost model

mode	abbr.	address	added cost	
absolute	M	M	1	
register	R	R	0	
indexed	c(R)	$c + cont(R)$	1	
indirect register	*R	$cont(R)$	0	
indirect indexed	*c(R)	$cont(c + cont(R))$	1	
literal	#M	the value M	1	only for source

- indirect: useful for elements in “records” with known off-set
- indexed: useful for slots in arrays

# Examples $a := b + c$



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## Using registers (costs=?)

```
MOV b, R0 // R0 = b
ADD c, R0 // R0 = c + R0
MOV R0, a // a = R0
```

## Mem.-mem. ops (costs=?)

```
MOV b, a // a = b
ADD c, a // a = c + a
```

## Addresses in registers (costs=?)

```
MOV *R1, *R0 // *R0 = *R1
ADD *R2, *R0 // *R0 = *R2 + *R0
```

Assume R0, R1, and R2  
contain *addresses* for a, b,  
and c

## Storing back to mem. (costs=?)

```
ADD R2, R1 // R1 = R2 + R1
MOV R1, a // a = R1
```

Assume R1 and R2 contain  
*values* for b, and c

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# Examples $a := b + c$



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## Using registers (costs=6)

```
MOV b, R0 // R0 = b
ADD c, R0 // R0 = c + R0
MOV R0, a // a = R0
```

## Mem.-mem. ops (costs=6)

```
MOV b, a // a = b
ADD c, a // a = c + a
```

## Addresses in registers (costs=2)

```
MOV *R1, *R0 // *R0 = *R1
ADD *R2, *R0 // *R0 = *R2 + *R0
```

Assume R0, R1, and R2  
contain *addresses* for a, b,  
and c

## Storing back to mem. (costs=3)

```
ADD R2, R1 // R1 = R2 + R1
MOV R1, a // a = R1
```

Assume R1 and R2 contain  
*values* for b, and c

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## Basic blocks and control-flow graphs

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# Basic blocks

- machine code level equivalent of straight-line code
- (a largest possible) sequence of instructions without
  - jump out
  - jump in
- elementary unit of code analysis/optimization<sup>1</sup>
- amenable to analysis techniques like
  - static simulation/symbolic evaluation
  - abstract interpretation
- basic unit of code generation

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<sup>1</sup>Those techniques can also be used across basic blocks, but then they become more costly and challenging.



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# Control-flow graphs



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## CFG

basically: *graph* with

- nodes = basic blocks
  - edges = (potential) jumps (and “fall-throughs”)
- 
- here (as often): CFG on 3AIC (linear intermediate code)
  - also possible CFG on low-level code,
  - or also:
    - CFG extracted from AST<sup>2</sup>
    - here: the opposite: synthesizing a CFG from the linear code
  - explicit data structure (as another intermediate representation) or implicit only.

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<sup>2</sup>See also the exam 2016.

# From 3AC to CFG: “partitioning algo”

- remember: 3AIC contains *labels* and (conditional) jumps
- ⇒ algo rather straightforward
- the only complication: some labels can be ignored
  - we ignore procedure/method calls here
  - concept: “leader” representing the nodes/basic blocks

## Leader

- first line is a leader
- **GOTO** *i*: line labelled *i* is a leader
- instruction *after* a **GOTO** is a leader

## Basic block

instruction sequence from (and including) one leader to (but excluding) the next leader or to the end of code



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# Partitioning algo

```
.....  
.....  
.....  
if ... goto L5  
L1 .....  
L2 .....  
.....  
.....  
goto L3  
L5 .....  
.....  
L3 .....  
.....  
if ... goto L1  
.....  
goto L3
```



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# Partitioning algo

```
.....  
.....  
.....  
if ... goto L5  
-----  
L1 .....  
L2 .....  
.....  
.....  
goto L3  
-----  
L5 .....  
.....  
-----  
L3 .....  
.....  
if ... goto L1  
-----  
.....  
goto L3  
-----
```

- note: no line jumps to  $L_2$



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# 3AIC for factorial (from previous chapter)



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```
read x
t1 = x > 0
if_false t1 goto L1
fact = 1
label L2
t2 = fact * x
fact = t2
t3 = x - 1
x = t3
t4 = x == 0
if_false t4 goto L2
write fact
label L1
halt
```

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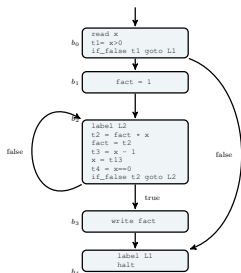
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# Factorial: CFG



- goto/conditional goto: never *inside* block
- not every block
  - ends in a goto
  - starts with a label
- ignored here: function/method calls, i.e., focus on
- *intra-procedural* cfg



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# Levels of analysis

- here: *three* levels where to apply code analysis / optimizations

## levels

1. **local**: per basic block (block-level)
  2. **global**: per function body/intra-procedural CFG
  3. (**inter-procedural**: really global, whole-program analysis)
- better terminology: block-local, procedure-local etc.
  - the “more global”, the more *costly* the analysis and, especially the optimization (if done at all)



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# Loops in CFGs

- *loop optimization*: “loops” are thankful places for optimizations
- important for analysis to *detect* loops (in the cfg)
- importance of *loop discovery*: not too important any longer in modern languages.

## Loops in a CFG vs. graph cycles

- concept of loops in CFGs **not** identical with **cycles** in a graph
- all **loops** are graph **cycles** but not vice versa
- intuitively: loops are cycles originating from source-level looping constructs (“while”)
- goto’s may lead to non-loop cycles in the CFG
- importance of loops: loops are “well-behaved” when considering certain optimizations/code transformations (goto’s can destroy that...)



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# Loops in CFGs



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## Loop $L$ with header $h$

Loop  $L$  in a CFG: set of nodes, including **header node**  
 $h \in L$ :

1. any node in  $L$ : a path in  $L$  to  $h$
2. a path in  $L$  from  $h$  to any node in  $L$
3. every edge that goes from outside  $L$  into  $L$  passes through  $h$

often additional assumption/condition: “root” node of a CFG (there’s only one) is *not* itself an entry of a loop

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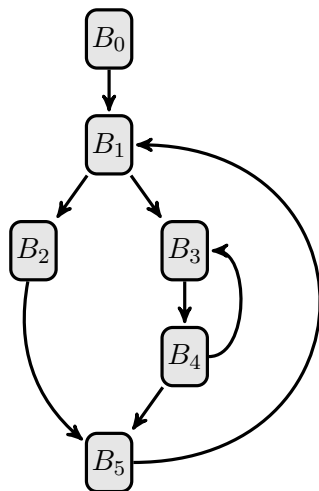
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# Loop example



- Loops:
  - $\{B_3, B_4\}$  (nested)
  - $\{B_4, B_3, B_1, B_5, B_2\}$
- Non-loop:
  - $\{B_1, B_2, B_5\}$
- unique entry marked red



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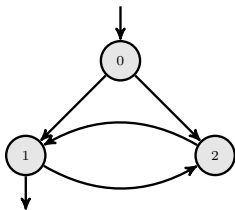
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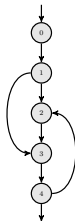
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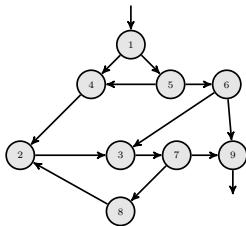
# Loop non-examples



(a)



(b)



(c)



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# Loops as fertile ground for optimizations



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```
while ( i < n ) { i++; A[ i ] = 3*k }
```

- possible optimizations
    - move  $3*k$  “out” of the loop
    - put frequently used variables into *registers* while in the loop (like  $i$ )
  - when moving out computation from the loop:
  - put it “right in front of the loop”
- ⇒ add extra node/basic block in front of the *entry* of the loop<sup>3</sup>

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<sup>3</sup>That's one of the motivations for unique entry.

# Data flow analysis in general

- general *analysis technique* working on CFGs
- **many** concrete forms of analyses
- such analyses: basis for (many) *optimizations*
- *data*: info stored in memory/temporaries/registers etc.
- *control*:
  - movement of the instruction pointer
  - abstractly represented by the CFG
    - inside elementary blocks: increment of the instruction pointer
    - edges of the CFG: (conditional) jumps
    - jumps together with RTE and calling convention

## Data flowing from (a) to (b)

Given the control flow (normally as CFG): is it *possible* or is it *guaranteed* (“may” vs. “must” analysis) that some “data” originating at one control-flow point (a) reaches control flow point (b).



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# Data flow as abstraction



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- data flow analysis **DFA**: fundamental and important *static* analysis technique
- it's impossible to decide statically if data from (a) *actually* "flows to" (b)
- ⇒ approximative (= abstraction)
- therefore: work on the CFG: if there are two options/outgoing edges: *consider both*
- Data-flow answers therefore **approximatively**
  - if it's *possible* that the data flows from (a) to (b)
  - it's *necessary* or unavoidable that data flows from (a) to (b)
- for *basic blocks*: **exact** answers possible

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## Liveness analysis (general)

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# Data flow analysis: Liveness

- prototypical / important data flow analysis
- especially important for register allocation

## Basic question

When (at which control-flow point) can I be *sure* that I don't need the current content of a variable (temporary, register) any more?

- optimization: if not needed for sure in the future: register can be used otherwise

## Definition (Live)

A “variable” is **live** at a given control-flow point if there *exists* an execution starting from there (given the level of abstraction), where the current content of the variable is *used* in the future.



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# Definitions and uses of variables

- “variables”: also temporary variables are meant.
- basic notions underlying most data-flow analyses (including liveness analysis)
- here: def’s and uses of *variables* (or temporaries etc.)
- all data, including intermediate results, has to be stored somewhere, in variables, temporaries, etc.

## Def’s and uses

- a “**definition**” of  $x$  = assignment to  $x$  (store to  $x$ )
- a “**use**” of  $x$ : read content of  $x$  (load  $x$ )
- variables can occur more than once, so
- a definition/use refers to *instances* or *occurrences* of variables (“use of  $x$  in line  $l$ ” or “use of  $x$  in block  $b$ ”)
- same for liveness: “ $x$  is live here, but not there”



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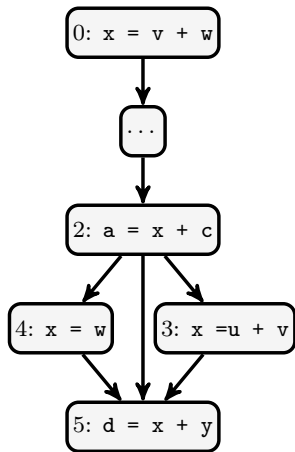
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# Defs, uses, and liveness



- $x$  is “defined” (= assigned to) in 0, 3, and 4
- $u$  is **live** “in” (= at the end of) block 2, as it *may* be used in 3
- a *non-live* variable at some point: “dead”, which means: the corresponding memory can be reclaimed
- *note*: here, liveness across block-boundaries = “global” (but blocks contain only one instruction here)



# Def-use or use-def analysis

- use-def: given a “use”: determine all possible “definitions”
- def-use: given a “def”: determine all possible “uses”
- for straight-line-code/inside one basic block
  - deterministic: each line has exactly one place where a given variable has been assigned to last (or else not assigned to in the block). Equivalently for uses.
- for whole CFG:
  - approximative (“may be used in the future”)
  - more advanced techniques (caused by presence of loops/cycles)
- def-use analysis:
  - closely connected to liveness analysis (basically the same)
  - *prototypical* data-flow question (same for use-def analysis), related to many data-flow analyses (but not all)



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# Calculation of def/uses (or liveness ...)

- three levels of complication
  1. inside basic block
  2. branching (but no loops)
  3. Loops
  4. [even more complex: inter-procedural analysis]

## For SLC/inside basic block

- deterministic result
- simple “one-pass” treatment enough
- similar to “static simulation”

## For whole CFG

- *iterative* algo needed
- dealing with non-determinism: over-approximation
- “closure” algorithms, similar to the way e.g., dealing with *first* and *follow* sets
- = fix-point algorithms



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## Local liveness: dead or alive

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# Inside one block: optimizing use of temporaries

- simple setting: *intra*-block analysis & optimization, only
- temporaries:
  - symbolic representations to hold intermediate results
  - generated on request, assuming unbounded numbers
  - intention: use **registers**
- limited about of register available (platform dependent)

## Assumption about temps (here)

- temp's *don't transfer* data across blocks ( $\neq$  program var's)
- ⇒ temp's *dead* at the beginning and at the end of a block
- but: variables have to be *assumed* live at the end of a block (block-local analysis, only)



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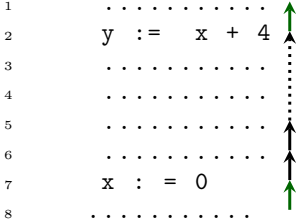
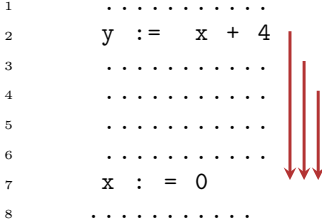
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# Forward vs. backward



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# Intra-block liveness

```
t1 := a - b
t2 := t1 * a
a  := t1 * t2
t1 := t1 - c
a  := t1 * a
```

- let's call **operand**: variables or temp's
- neither temp's nor vars in the example are "single assignment",
- but first occurrence of a temp in a block: a definition (but for temps it would often be the case, anyhow)
- uses of operands: on the rhs's, definitions on the lhs's
- not good enough to say " $t_1$  is live in line 4" (why?)



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# Single step per line: transfer function

- liveness-status of an operand: *different* from lhs vs. rhs in a given instruction
- informal definition: an operand is live at some occurrence, if its content is used some place in the future

## Definition (consider statement $x_1 := x_2 \text{ op } x_3$ )

- Variable  $x$  is live at the *beginning* of  $x_1 := x_2 \text{ op } x_3$ , if
  1. if  $x$  is  $x_2$  or  $x_3$ , or
  2. if  $x$  live at its *end*, if  $x$  and  $x_1$  are different variables
- A variable  $x$  is live at the *end* of an instruction,
  - if it's live at *beginning of the next* instruction
  - if no next instruction
    - temp's are dead
    - user-level variables are (assumed) live



# Algo: dead or alive (binary info)



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```
// _____ initialise T _____  
  for all entries: T[i,x] := D  
  except: for all variables a // but not temps  
          T[n,a] := L,  
//_____ backward pass _____  
for instruction i = n-1 down to 0  
  let current instruction at i+1: x := y op z;  
  T[i,o] := T[i+1,o] (for all other vars o)  
  T[i,x] := D // note order; x can ``equal`` y or z  
  T[i,y] := L  
  T[i,z] := L  
end
```

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# Algo: dead or alive (binary info) (2)

- Data structure  $T$ : table, mapping for each line/instruction  $i$  and variable: boolean status of “live”/“dead”
- represents liveness status per variable *at the end* (i.e. *rhs*) of that line
- basic block:  $n$  instructions, from 1 until  $n$ , where “line 0” represents the “sentry” imaginary line “before” the first line (no instruction in line 0)
- *backward scan* through instructions/lines from  $n$  to 0



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# Run of of the algo

line	<i>a</i>	<i>b</i>	<i>c</i>	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>
[0]	<i>L</i>	<i>L</i>	<i>L</i>	<i>D</i>	<i>D</i>
1	<i>L</i>	<i>L</i>	<i>L</i>	<i>L</i>	<i>D</i>
2	<i>D</i>	<i>L</i>	<i>L</i>	<i>L</i>	<i>L</i>
3	<i>L</i>	<i>L</i>	<i>L</i>	<i>L</i>	<i>D</i>
4	<i>L</i>	<i>L</i>	<i>L</i>	<i>L</i>	<i>D</i>
5	<i>L</i>	<i>L</i>	<i>L</i>	<i>D</i>	<i>D</i>

**Table:** Liveness analysis example: result of the analysis



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## Local liveness<sup>++</sup>: Dependence graph

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# Adding information: next-use

- more refined information
  - not just binary **dead-or-alive** but **next-use** info
- ⇒ three kinds of information
1. Dead:  $D$
  2. Live:
    - with *local* line number of *next use*:  $L(n)$
    - *potential* use of outside local basic block  $L(\perp)$
- otherwise: same algo



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# Algo: alive with next use



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```
// _____ initialise T _____  
  for all entries: T[i,x] := D  
  except: for all variables a // but not temps  
          T[n,a] := L( $\perp$ ),  
//_____ backward pass _____  
for instruction i = n-1 down to 0  
  let current instruction at i+1: x := y op z;  
  T[i,o] := T[i+1,o] (for all other vars o)  
  T[i,x] := D // note order; x can ``equal'' y or z  
  T[i,y] := L(i+1)  
  T[i,z] := L(i+1)  
end
```

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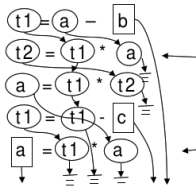
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# Run of the algo



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line	<i>a</i>	<i>b</i>	<i>c</i>	<i>t</i> <sub>1</sub>	<i>t</i> <sub>2</sub>
[0]	<i>L</i> (1)	<i>L</i> (1)	<i>L</i> (4)	<i>D</i>	<i>D</i>
1	<i>L</i> (2)	<i>L</i> (⊥)	<i>L</i> (4)	<i>L</i> (2)	<i>D</i>
2	<i>D</i>	<i>L</i> (⊥)	<i>L</i> (4)	<i>L</i> (3)	<i>L</i> (3)
3	<i>L</i> (5)	<i>L</i> (⊥)	<i>L</i> (4)	<i>L</i> (4)	<i>D</i>
4	<i>L</i> (5)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (5)	<i>D</i>
5	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>D</i>	<i>D</i>



```
1 t1 := a - b
2 t2 := t1 * a
3 a := t1 * t2
4 t1 := t1 - c
5 a := t1 * a
```

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# Dependency graph and def-use



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- small step from **next**-use of **all**-future-uses

## Def-use analysis

Connect definitions with all their uses  $\Rightarrow$  dependency graph

- straight-line code
- acyclic graph  $\Rightarrow$  DAG (or partial order)
- nodes: (lines of) instructions (or variable instance)

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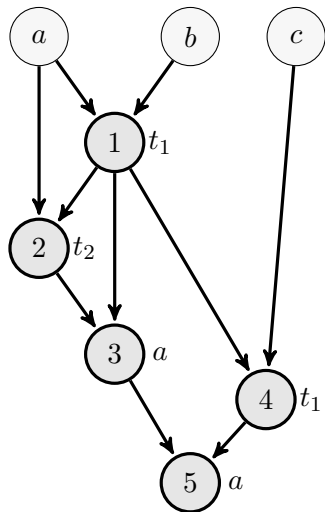
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# DAG of the block



```
1  t1 := a - b
2  t2 := t1 * a
3  a := t1 * t2
4  t1 := t1 - c
5  a := t1 * a
```



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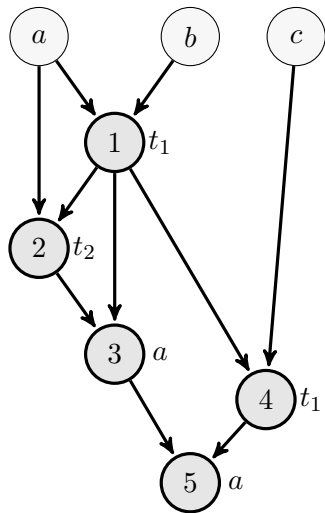
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# DAG of the block



- no linear order (as in code), only *partial order*
- *the next use*: meaningless
- but: *all "next" uses* visible
- node = occurrence of a variable
- e.g.: node 1 for "defining"  $t_1$  has *three* uses
- different "versions" (instances) of  $t_1$



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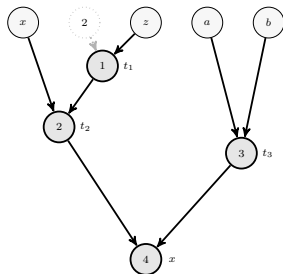
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# Dependence graphs for pure expressions



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```
t1 := 2 * z
t2 := x + t1
t3 := a + b
x := t2 - t3
```



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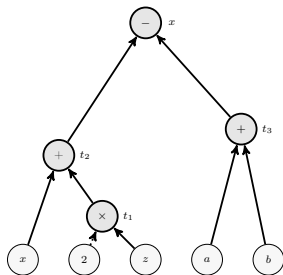
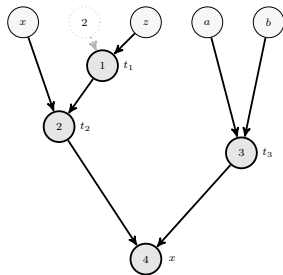
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# Dependence graphs for pure expressions: cf. AST!



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```
t1 := 2 * z
t2 := x + t1
t3 := a + b
x := t2 - t3
```



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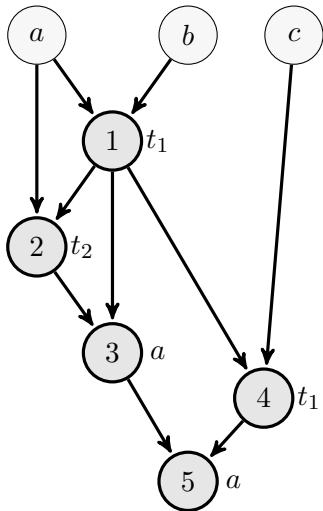
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# (S)SA format

```
t1 := a - b
t2 := t1 * a
a  := t1 * t2
t1 := t1 - c
a  := t1 * a
```



**Figure:** DAG for the 3AIC code block



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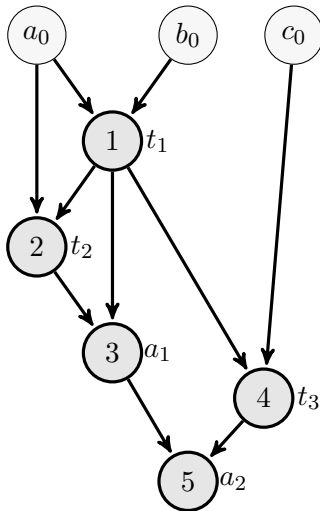
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# (S)SA format

```
t1 := a0 - b0
t2 := t1 * a0
a1 := t1 * t2
t3 := t1 - c0
a2 := t3 * a1
```



**Figure:** DAG for the 3AIC code block



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## Global analysis

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# Global data flow analysis

- block-local
  - block-local analysis (here liveness): *exact* information possible
  - block-local liveness: *1 backward scan*
  - important use of liveness: *register allocation*, temporaries typically don't survive blocks anyway
- **global**: working on complete CFG

## 2 complications

- **branching**: *non-determinism*, unclear which branch is taken
  - **loops** in the program (loops/cycles in the graph): simple *one pass* through the graph does not cut it any longer
  - *exact* answers no longer possible (undecidable)
- ⇒ work with safe **approximations**
- this is: general characteristic of DFA



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# Generalizing block-local liveness analysis



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- *assumptions* for block-local analysis
  - all program variables (assumed) *live* at the end of each basic block
  - all temps are assumed *dead* there.
- now: we do better, info across blocks

## at the end of each block:

which variables **may** be used in subsequent block(s).

- **now**: re-use of temporaries (and thus corresponding registers) across blocks possible
- remember local liveness algo: determined liveness status per var/temp *at the end* of each “line/instruction”

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# Connecting blocks in the CFG: *inLive* and *outLive*



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- CFG:
  - pretty conventional graph (nodes and edges, often designated start and end node)
  - *nodes* = basic blocks = contain straight-line code (here 3AIC)
  - being conventional graphs:
    - conventional representations possible
    - E.g. nodes with lists/sets/collections of immediate *successor nodes* plus immediate *predecessor nodes*
- remember: local liveness status
  - can be different *before* and *after* one single instruction
  - liveness status *before* expressed as dependent on status *after*

⇒ backward scan
- Now per block: *inLive* and *outLive*

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- tracing / approximating set of live variables<sup>4</sup> at the *beginning* and *end* per basic block
- *inLive* of a block: depends on
  - *outLive* of that block and
  - the SLC inside that block
- *outLive* of a block: depends on *inLive* of the *successor* blocks

## Approximation: To err on the safe side

Judging a variable (statically) live: always *safe*. Judging wrongly a variable *dead* (which actually will be used): **unsafe**

- goal: **smallest** (but **safe**) possible sets for *outLive* (and *inLive*)

---

<sup>4</sup>To stress “approximation”: *inLive* and *outLive* contain sets of *statically* live variables. If those are dynamically live or not is undecidable.



# Example: factorial CFG



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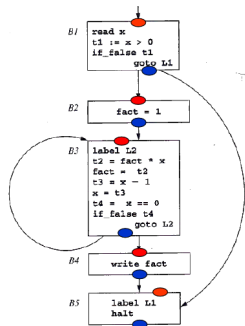
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- *inLive* and *outLive*
- picture shows arrows as *successor nodes*
- needed *predecessor nodes* (reverse arrows)

node/block	predecessors
$B_1$	$\emptyset$
$B_2$	$\{B_1\}$
$B_3$	$\{B_2, B_3\}$
$B_4$	$\{B_3\}$
$B_5$	$\{B_1, B_4\}$

# Block local info for global liveness/data flow analysis

- 1 CFG per procedure/function/method
- as for SLC: algo works **backwards**
- for each block: underlying block-local liveness analysis

## 3-valued block local status per variable

result of block-local live variable analysis

1. *locally live* on entry: variable used (before overwritten)
2. *locally dead* on entry: variable overwritten (before used)
3. status not locally determined: variable neither assigned to nor read locally

- for efficiency: **precompute** this info, before starting the global iteration  $\Rightarrow$  avoid *recomputation* for blocks in loops



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# Global DFA as iterative “completion algorithm”

- different names for the general approach
  - *closure* algorithm, *saturation* algo
  - *fixpoint* iteration
- basically: a big loop with
  - **iterating** a step approaching an intended solution by making current approximation of the solution *larger*
  - **until** the solution stabilizes
- similar (for example): calculation of first- and follow-sets
- often: realized as *worklist algo*
  - named after central data-structure containing the “work-still-to-be-done”
  - here possible: worklist containing nodes untreated wrt. liveness analysis (or DFA in general)



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# Example

```
a := 5
L1: x := 8
   y := a + x
   if_true x=0 goto L4
   z := a + x      // B3
   a := y + z
   if_false a=0 goto L1
   a := a + 1     // B2
   y := 3 + x
L5: a := x + y
   result := a + z
   return result // B6
L4: a := y + 8
   y := 3
   goto L5
```



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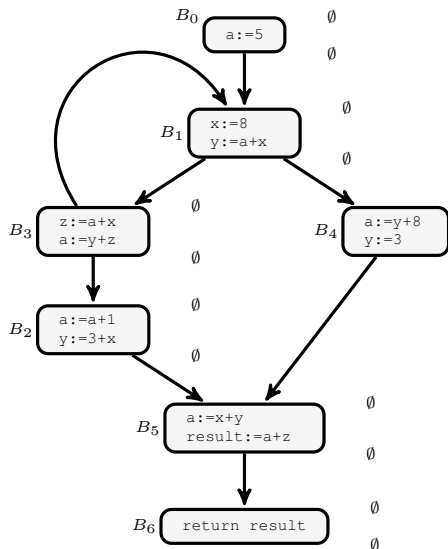
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# CFG: initialization



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- *inLive* and *outLive*: initialized to  $\emptyset$  everywhere
- note: start with (most) *unsafe* estimation
- extra (return) node
- but: analysis here *local per procedure*, only

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# Iterative algo



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## General schema

**Initialization** start with the “minimal” estimation ( $\emptyset$  everywhere)

**Loop** pick one node & update (= enlarge) liveness estimation in connection with that node

**Until** finish upon stabilization (= no further enlargement)

- order of treatment of nodes: in principle arbitrary<sup>5</sup>
- in tendency: following edges **backwards**
- comparison: for linear graphs (like inside a block):
  - no repeat-until-stabilize loop needed
  - 1 simple backward scan enough

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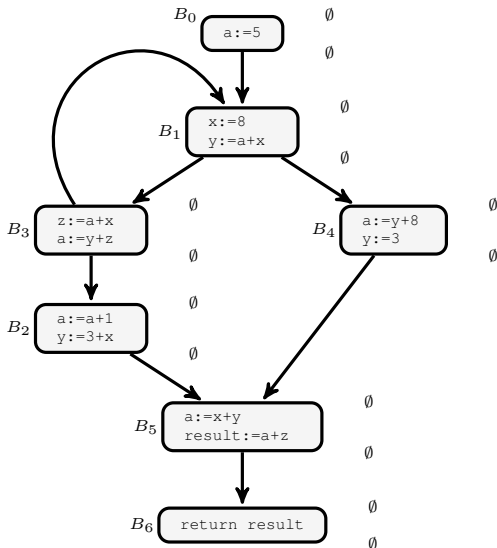
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<sup>5</sup>There may be more efficient and less efficient orders of treatment.

# Liveness: run



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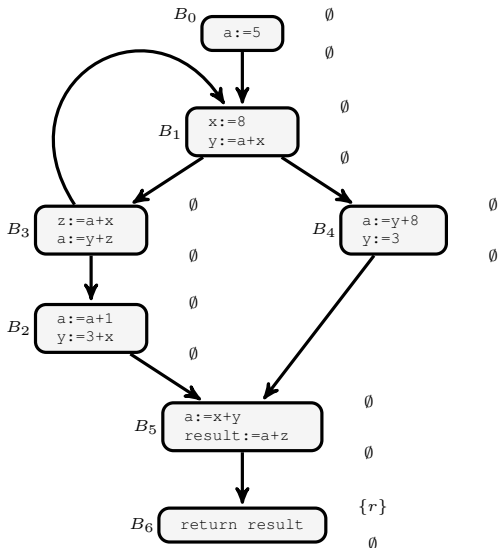
### Local liveness: dead or alive

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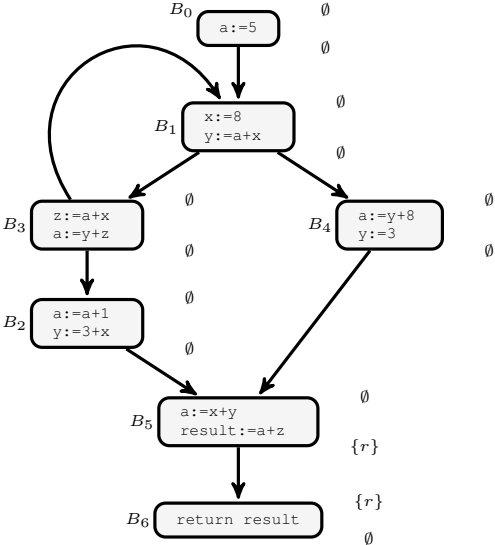
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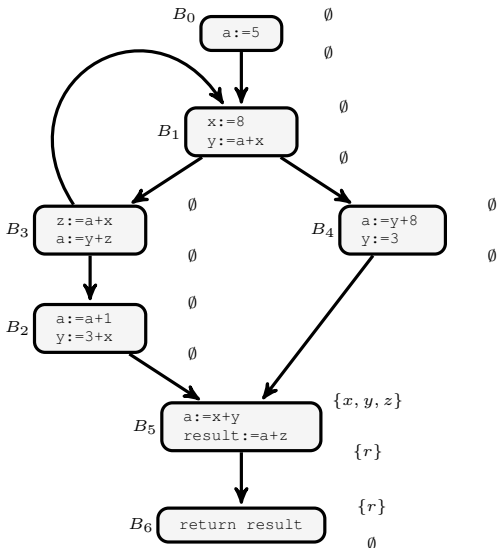
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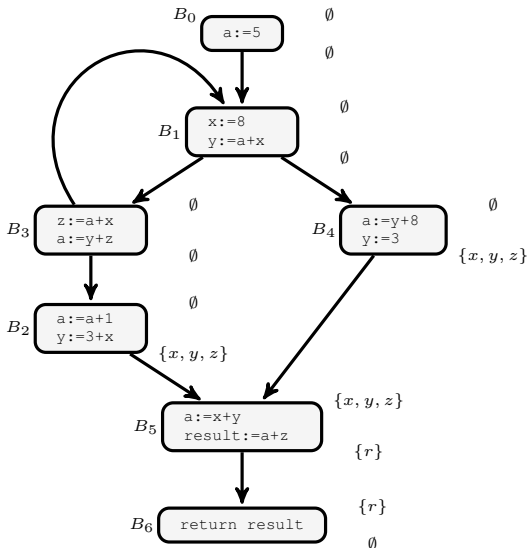
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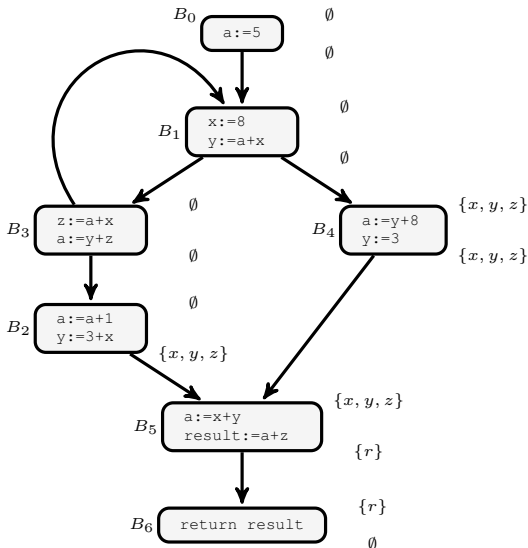
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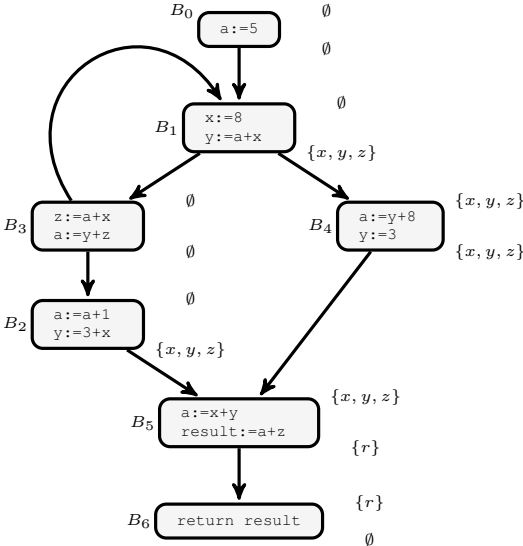
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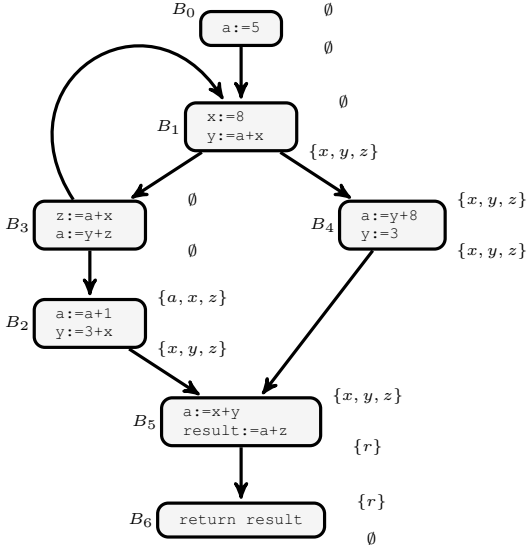
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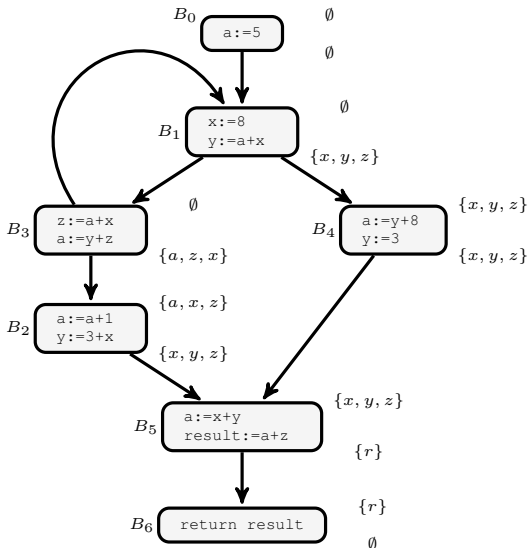
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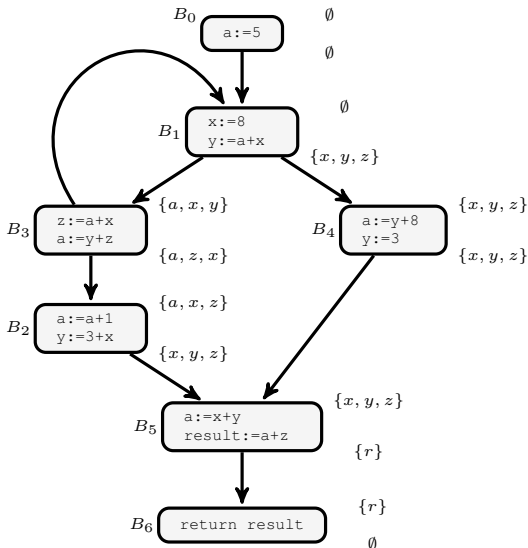
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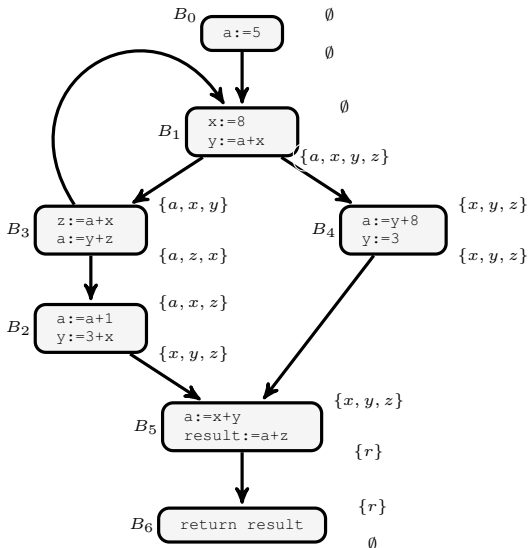
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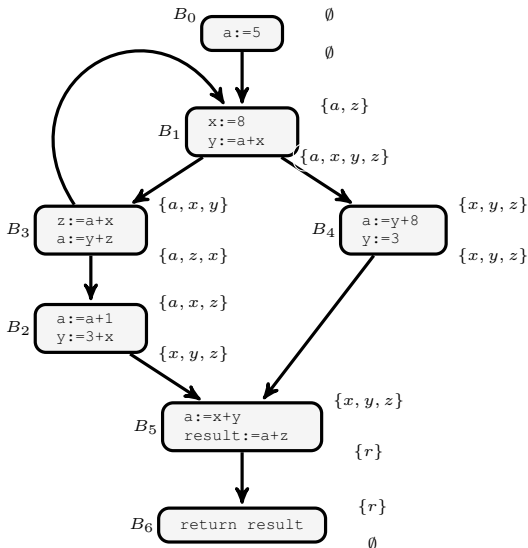
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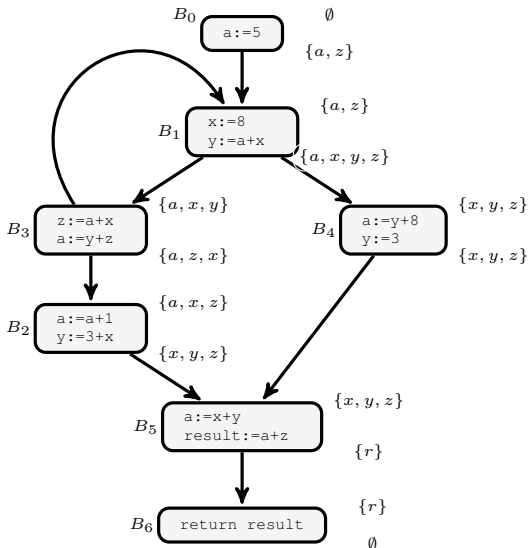
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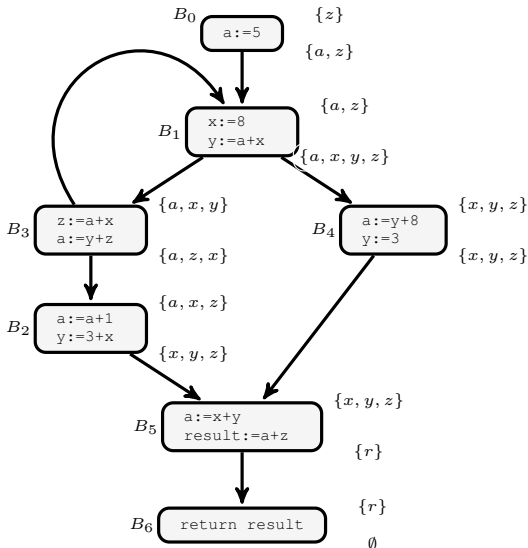
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# Liveness example: remarks

- the shown traversal strategy is (cleverly) backwards
- example resp. example run simplistic:
- the *loop* (and the choice of “evaluation” order):

## “harmless loop”

after having updated the *outLive* info for  $B_1$  following the edge from  $B_3$  to  $B_1$  *backwards* (propagating flow from  $B_1$  back to  $B_3$ ) **does not increase the current solution for  $B_3$**

- no need (in this particular order) for continuing the iterative search for stabilization
- in other examples: loop iteration cannot be avoided
- note also: end result (after stabilization) **independent from evaluation order!** (only some strategies may stabilize faster. . . )



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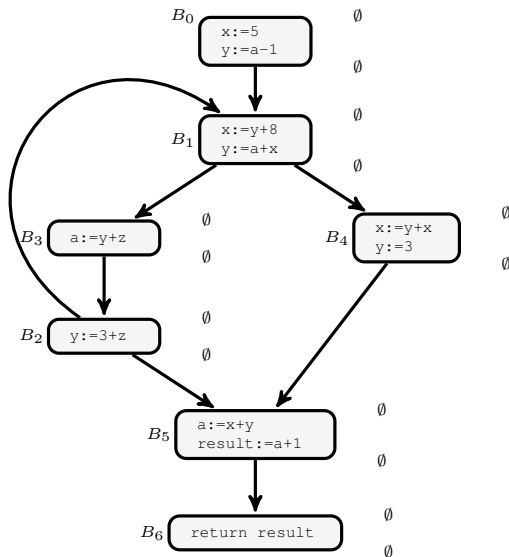
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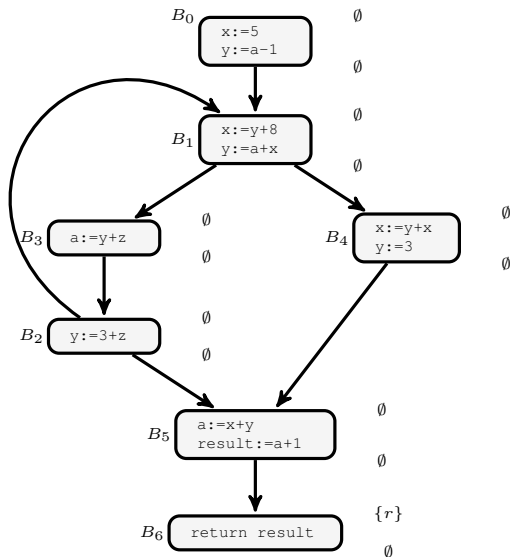
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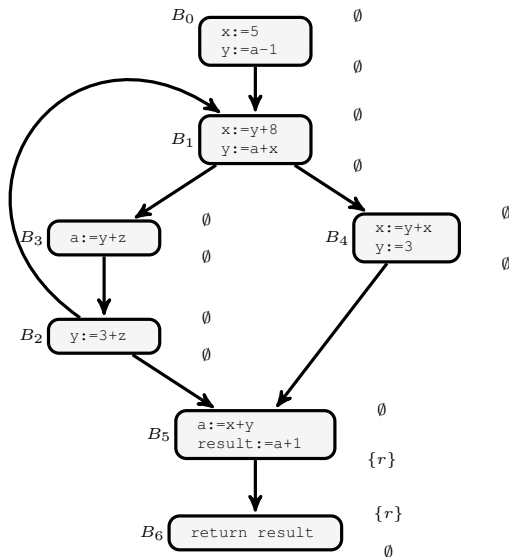
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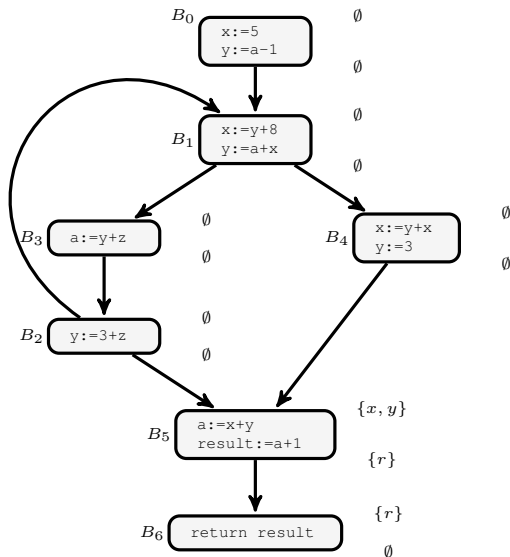
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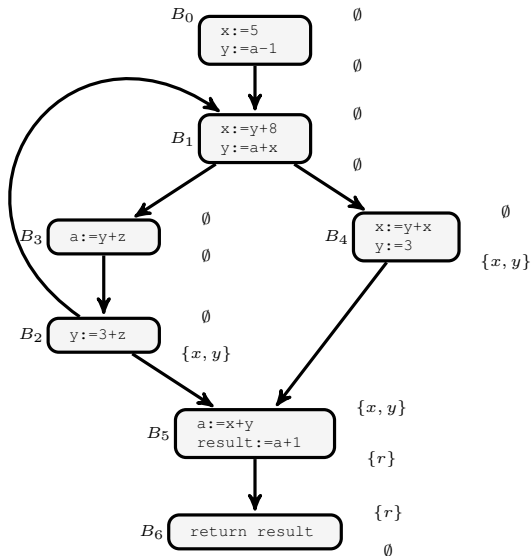
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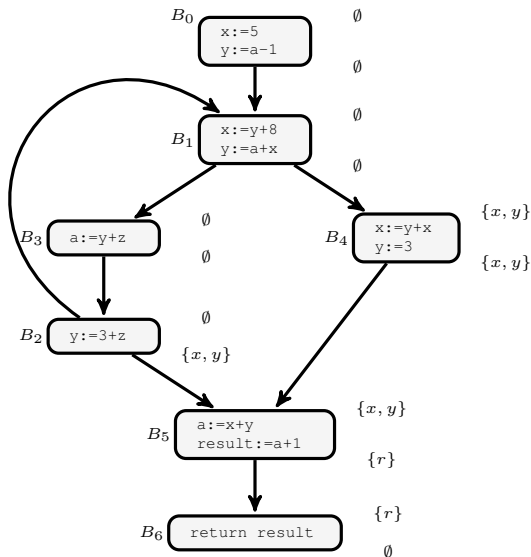
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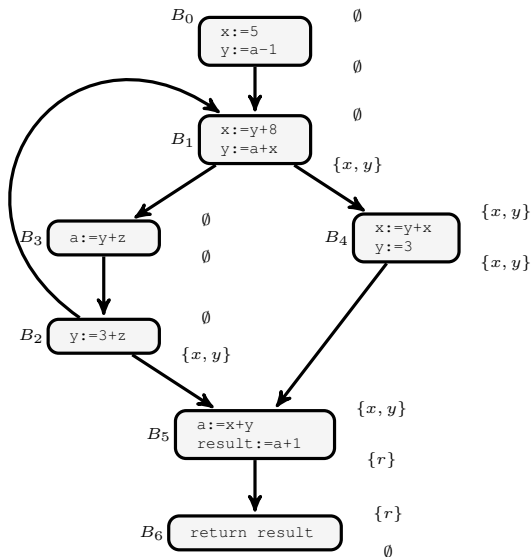
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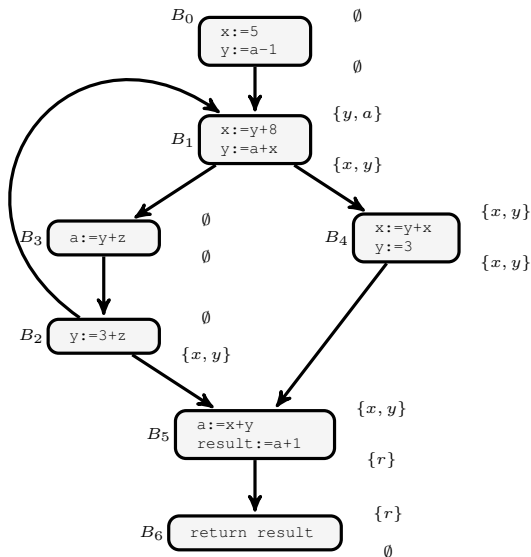
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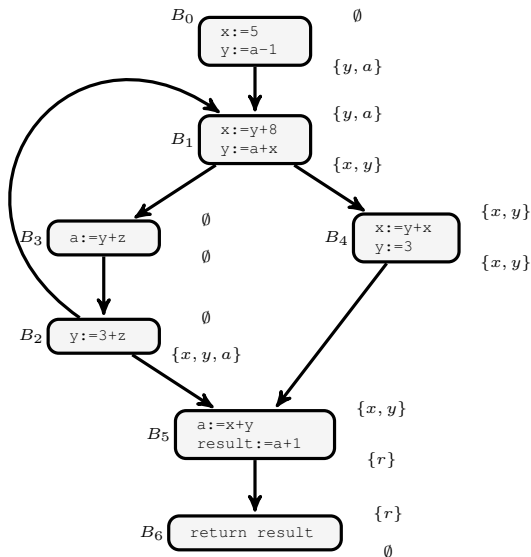
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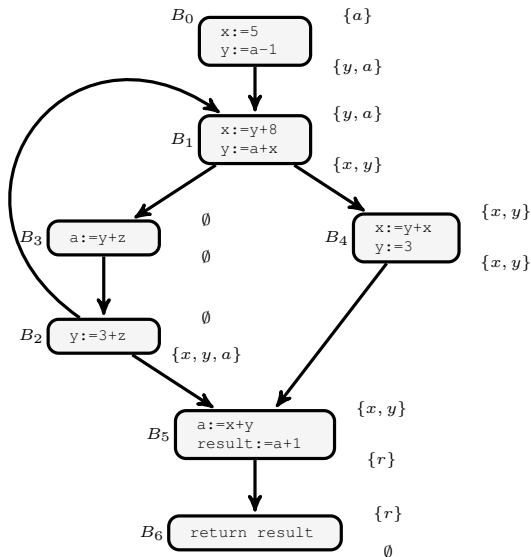
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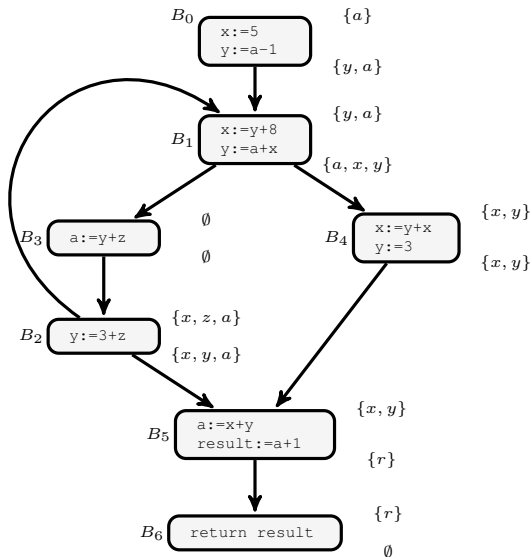
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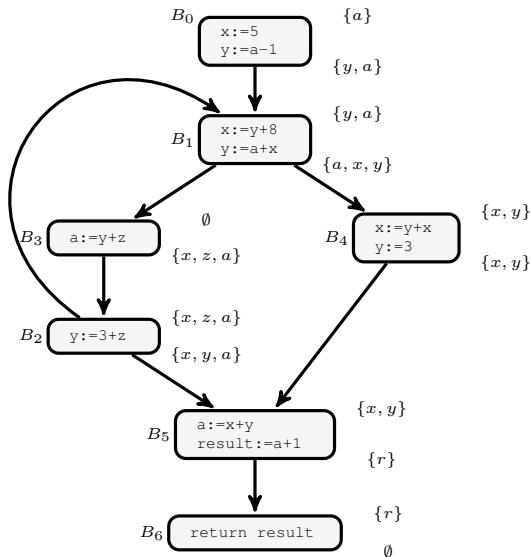
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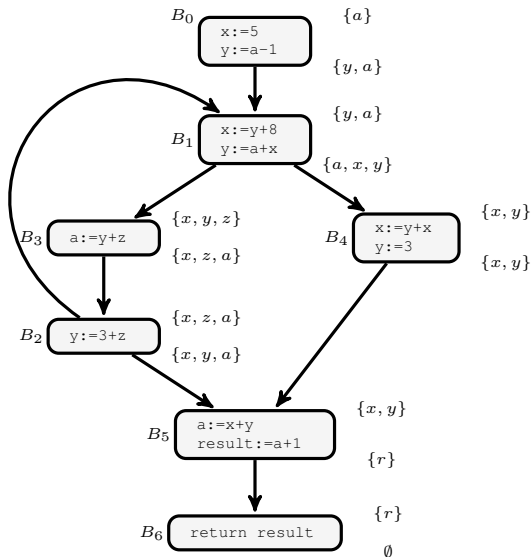
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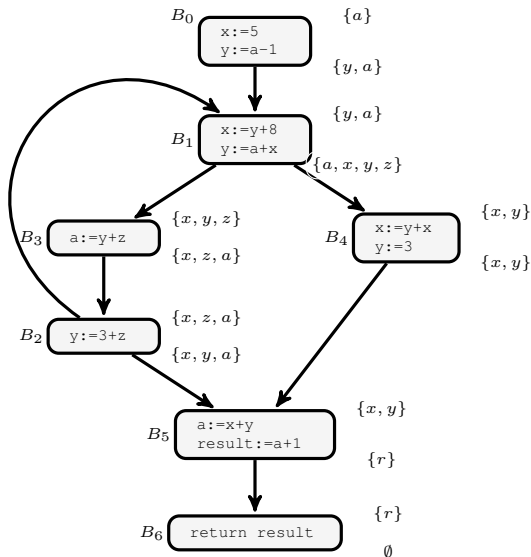
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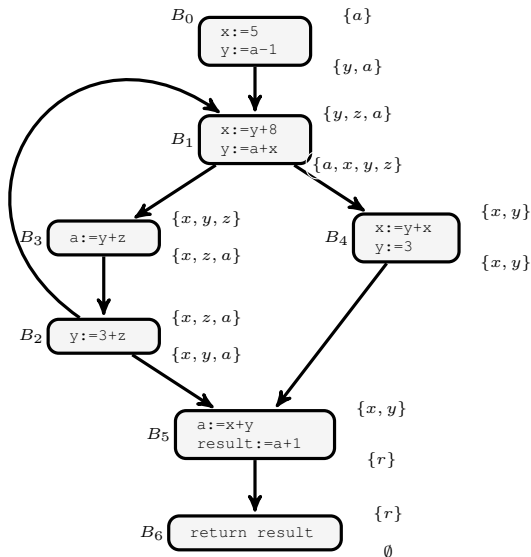
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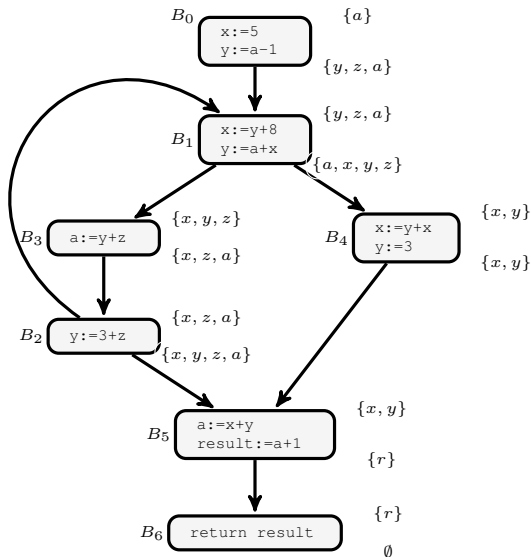
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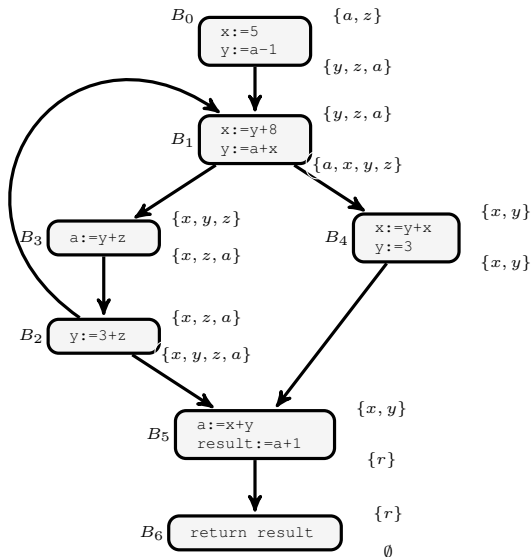
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# Example remarks

- loop: this time: updating estimation more than once
- evaluation order not chosen ideally (but it's not generally solvable)



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# Precomputing the block-local “liveness effects”

- *precomputation* of the relevant info: efficiency
- traditionally: represented as *kill* and *generate* information
- here (for liveness)
  1. **kill**: variable instances, which are overwritten
  2. **generate**: variables used in the block (before overwritten)
  3. **rests**: all other variables won't change their status

## Constraint per basic block (transfer function)

$$inLive = outLive \setminus kill(B) \cup generate(B)$$

- note:
  - order of kill and generate in above's equation
  - a variable killed in a block may be “revived” in a block
- simplest (one line) example:  $x := x + 1$



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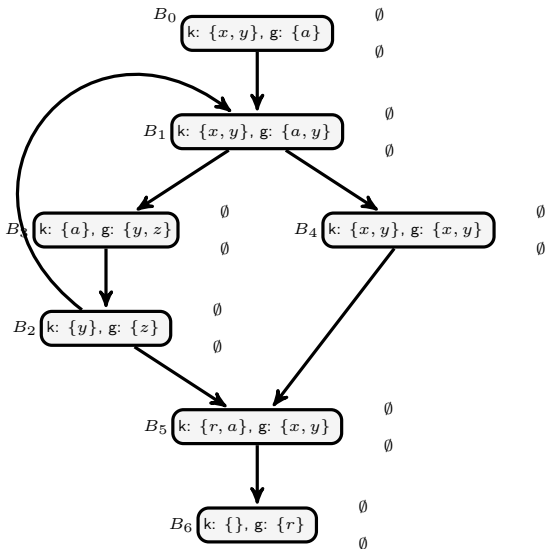
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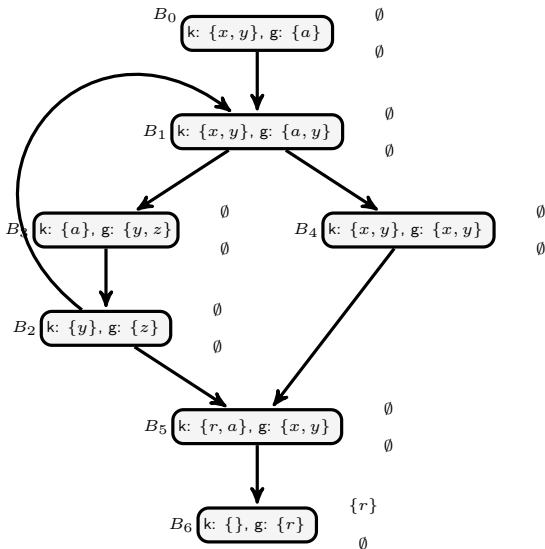
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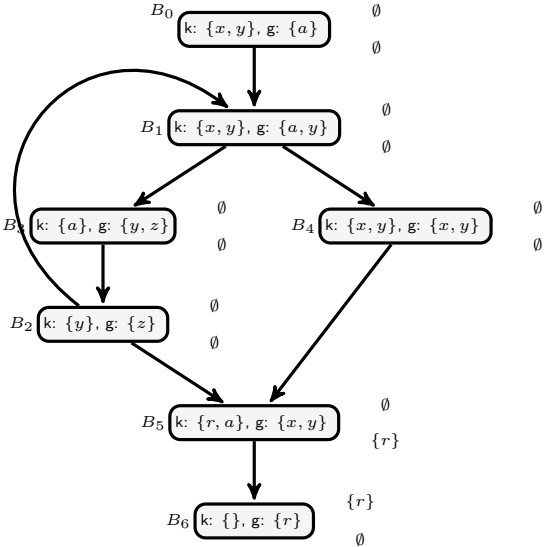
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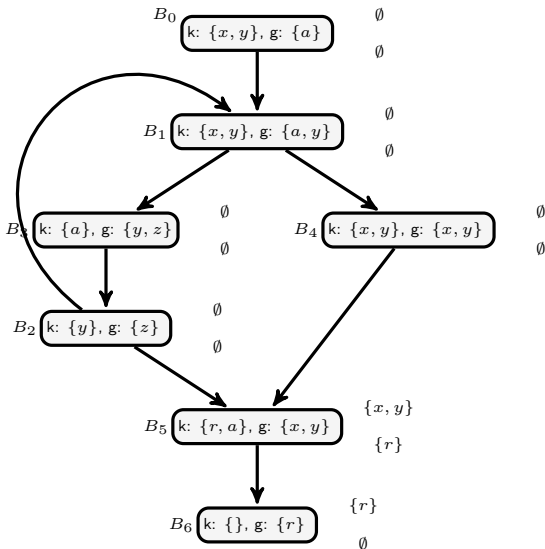
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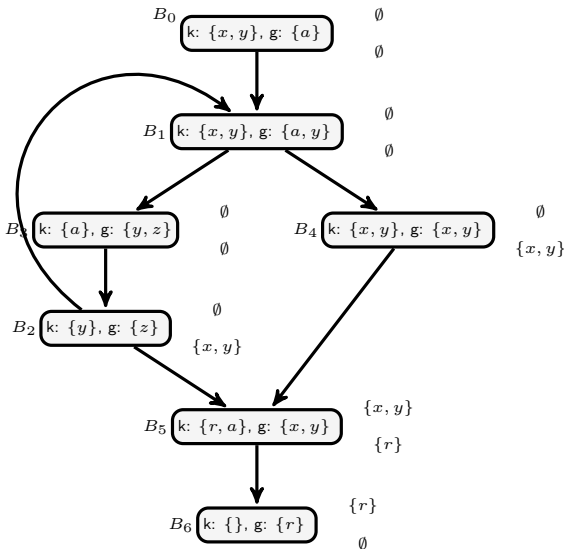
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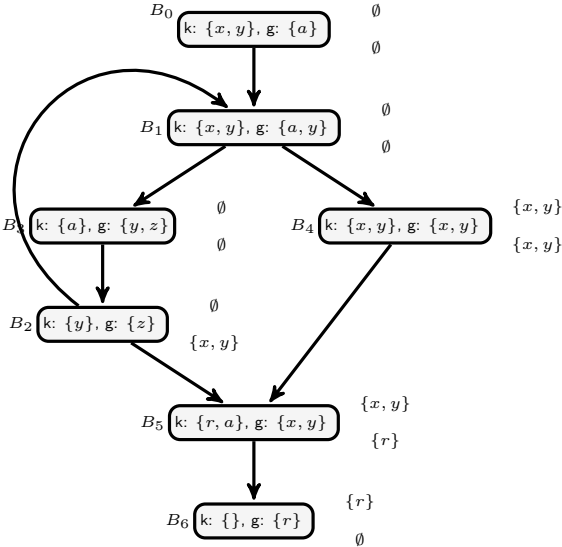
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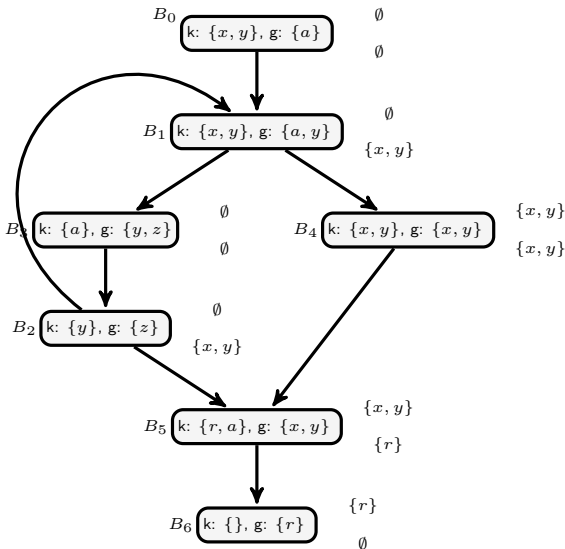
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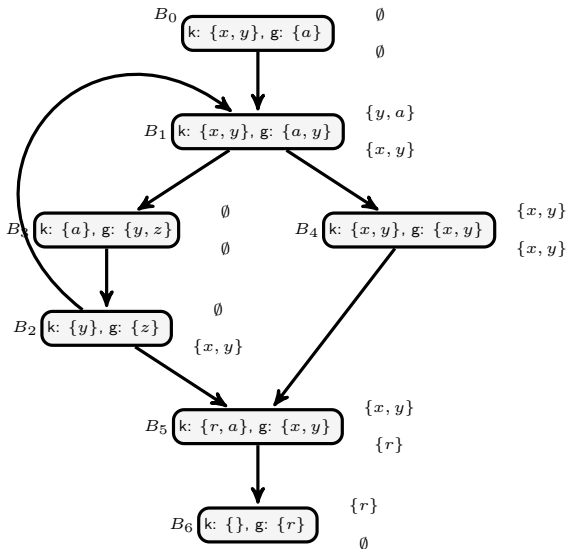
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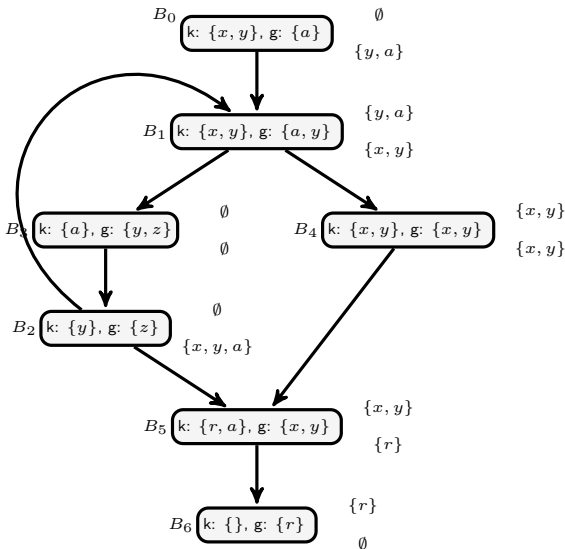
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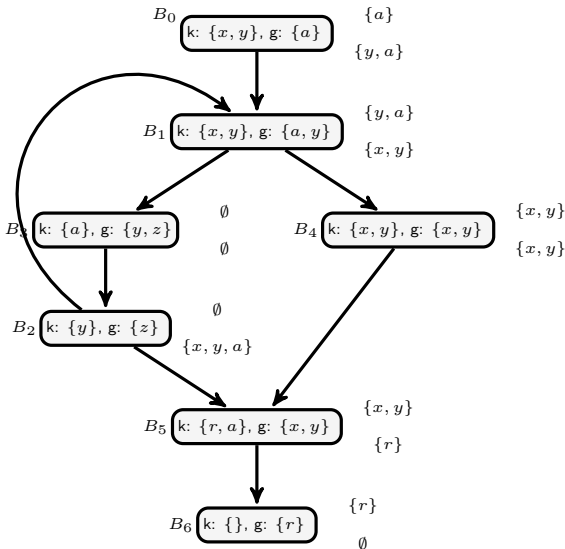
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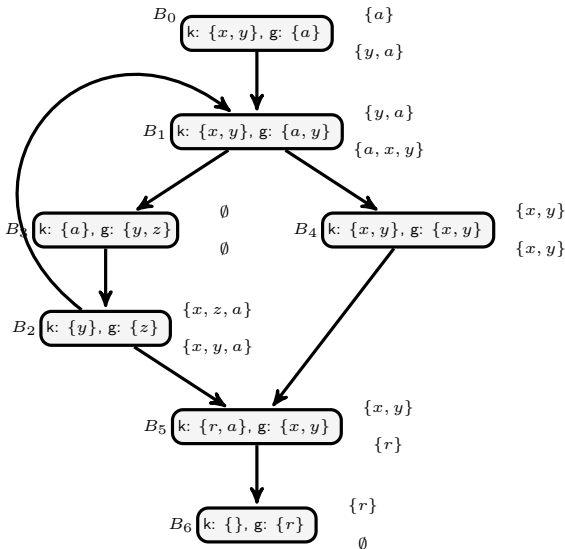
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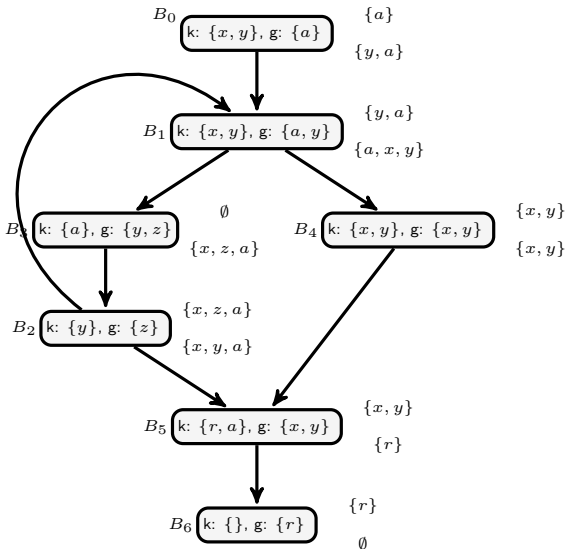
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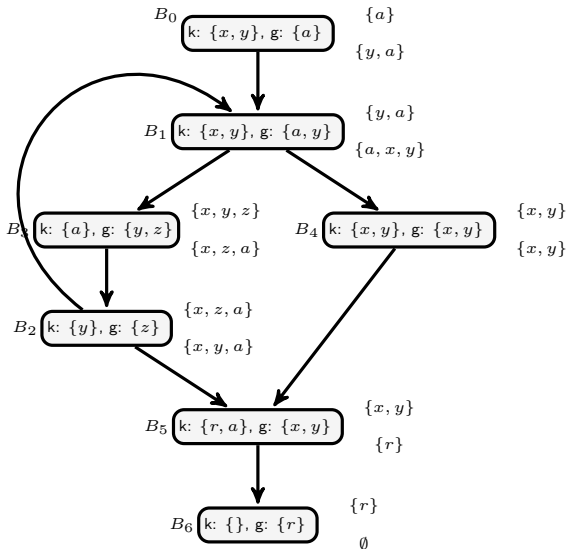
### Local liveness: dead or alive

### Local liveness<sup>++</sup>: Dependence graph

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# Example once again: kill and gen



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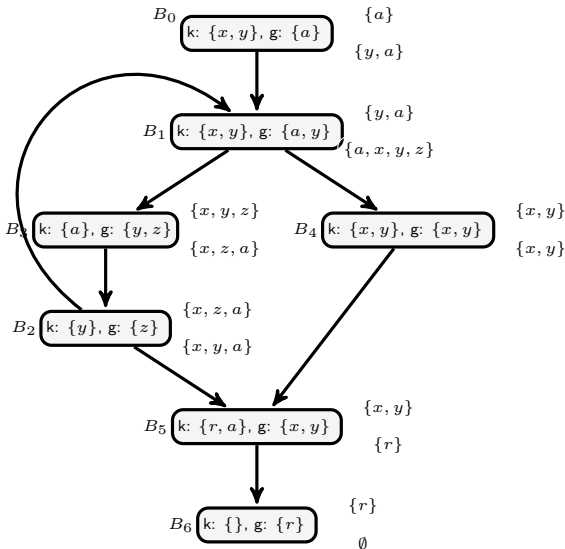
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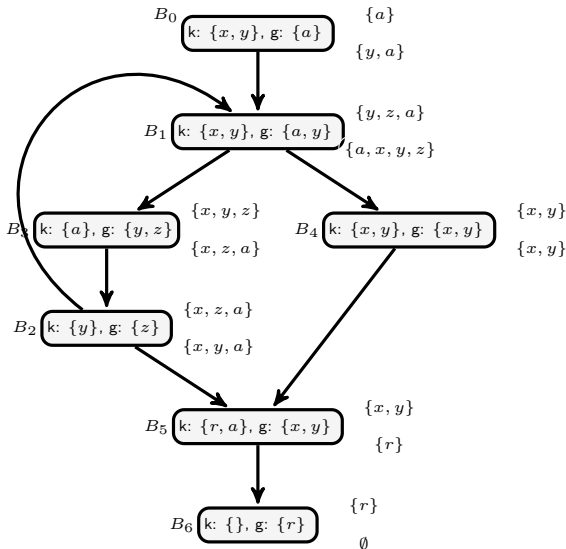
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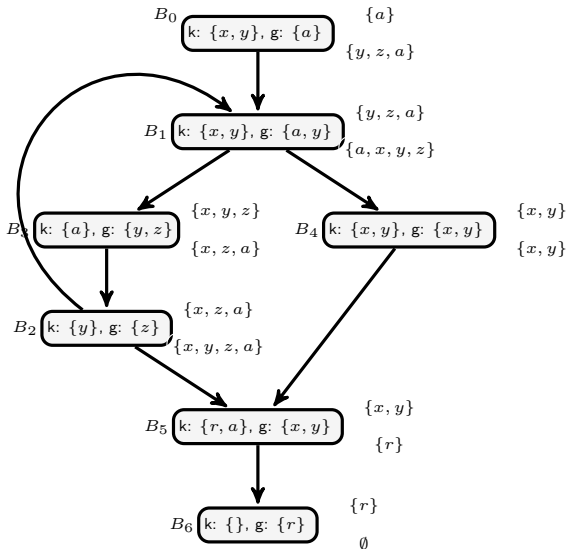
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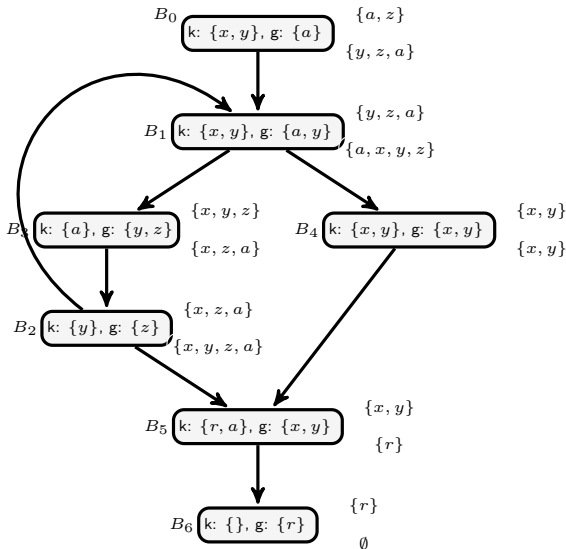
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# Section

## Code generation algo

Chapter 10 “Code generation”  
Course “Compiler Construction”  
Martin Steffen  
Spring 2024



# Simple code generation algo

- simple algo: *intra-block* code generation
- core problem: **register use**
- register allocation & assignment
- hold calculated values in registers longest possible
- intra-block only  $\Rightarrow$  at exit:
  - all *variables* stored back to main memory
  - all temps assumed “lost”
- remember: assumptions in the intra-block liveness analysis



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# Limitations of the code generation

- local **intra block**:
  - no analysis across blocks
  - no procedure calls, etc.
- no complex data structures
  - arrays
  - pointers
  - ...

## some limitations on how the algo itself works for one block

- for read-only variables: never put in registers, even if variable is *repeatedly* read
  - algo works only with the temps/variables given and does not come up with new ones
  - for instance: DAGs could help
- no *semantics* considered
  - like *commutativity*:  $a + b$  equals  $b + a$



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# Purpose and “signature” of the *getreg* function



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- one *core* of the code generation algo
- simple code-generation here  $\Rightarrow$  simple *getreg*

## *getreg* function

available: *liveness/next-use* info

**Input:** TAIC-instruction  $x := y \text{ op } z$

**Output:** return *location* where  $x$  is to be stored

- **location:** register (if possible) or memory location

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# Code generation invariant



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it should go without saying ... :

## Basic safety invariant

At each point, “live” variables (with or without next use in the current block) must exist in at least one location

- another invariant: the location returned by `getreg`: the one where the result of a 3AIC assignment ends up

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# Register and address descriptors

- code generation/*getreg*: keep track of
  1. register contents
  2. addresses for names

## Register descriptor

- tracking current “content” of reg’s (if any)
- consulted when new reg needed
- as said: at block entry, assume all regs unused

## Address descriptor

- tracking location(s) where current value of name can be found
- possible locations: register, stack location, main memory
- > 1 location possible (but not due to over-approximation, exact tracking)



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## Code generation algo for $x := y \text{ op } z$

1. determine location (preferably register) for result

```
l = getreg( ``x := y op z``)
```

2. make sure, that the value of  $y$  is in  $l$  :

- consult address descriptor for  $y \Rightarrow$  current locations  $l_y$  for  $y$
- choose the best location  $l_y$  from those (preferably register)
- if value of  $y$  *not* in  $l$ , generate

```
MOV  $l_y$ , l
```

3. generate

```
OP  $l_z$ , l //  $l_z$ : a current location of  $z$  (prefer reg's)
```

- update address descriptor  $[x \mapsto_{\cup} l]$
- if  $l$  is a reg: update reg descriptor  $l \mapsto x$

4. exploit liveness/next use info: update register descriptors

# Skeleton code generation algo for

$x := y \text{ op } z$

```
l = getreg(``x:= y op z'') // target location for x
if l  $\notin$   $T_a(y)$  then let  $l_y \in T_a(y)$  in emit ("MOV  $l_y, l$ ");
let  $l_z \in T_a(z)$  in emit ("OP  $l_z, l$ ");
```

- “skeleton”
  - *nondeterministic*: we ignored how to choose  $l_z$  and  $l_y$
  - we ignore *bookkeeping* in the *name* and *address* descriptor tables ( $\Rightarrow$  step 4 also missing)
  - details of *getreg* hidden.



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```

l = getreg(` `x:= y op z'') // generate target location for x
if l ∉ Ta(y)
then let ly ∈ Ta(y) // pick a location for y
    in emit (MOV ly, l)
else skip;
let lz ∈ Ta(z) in emit ("OP lz, l");
Ta := Ta[x ↦ ∪ l];
if l is a register
then Tr := Tr[l ↦ x]

```



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# Exploit liveness/next use info: recycling registers

- register descriptors: don't update themselves during code generation
- once set (e.g. as  $R_0 \mapsto t$ ), the info stays, unless reset
- thus in step 4 for  $z := x \text{ op } y$ :



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## Code generation algo for $x := y \text{ op } z$

```
l = getreg ("i: x := y op z") // i for instructions line number/label
if l  $\notin$   $T_a(y)$ 
then let  $l_y = \text{best}$  ( $T_a(y)$ )
      in emit ("MOV  $l_y, l$ ")
else skip;
let  $l_z = \text{best}$  ( $T_a(z)$ )
in emit ("OP  $l_z, l$ ");
 $T_a := T_a \setminus (\_ \mapsto l)$ ;
 $T_a := T_a[x \mapsto l]$ ;
if l is a register
then  $T_r := T_r[l \mapsto x]$ ;

if  $\neg T_{live}[i, y]$  and  $T_a(y) = r$  then  $T_r := T_r \setminus (r \mapsto y)$ 
if  $\neg T_{live}[i, z]$  and  $T_a(z) = r$  then  $T_r := T_r \setminus (r \mapsto z)$ 
```

### Updating and exploit liveness info by recycling reg's

if  $y$  and/or  $z$  are currently

- *not live* and are
- in registers,

$\Rightarrow$  "wipe" the info from the corresponding register descriptors

*getreg* algo:  $x := y \text{ op } z$

- goal: return a location for  $x$
- basically: check possibilities of register uses
- starting with the “cheapest” option

### Do the following steps, in that order

1. **in place**: if  $x$  is in a register already (and if that’s fine otherwise), then return the register
2. **new register**: if there’s an unused register: return that
3. **purge filled register**: choose more or less cleverly a filled register and save its content, if needed, and return that register
4. **use main memory**: if all else fails



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## getreg algo: $x := y \text{ op } z$ in more details

### 1. if

- $y$  in register  $R$
- $R$  holds *no alternative names*
- $y$  is *not live* and has no next use after the 3AIC instruction
- $\Rightarrow$  return  $R$

### 2. else: if there is an **empty** register $R'$ : return $R'$

### 3. else: if

- $x$  has a next use [or operator requires a register]  $\Rightarrow$ 
  - find an **occupied** register  $R$
  - store  $R$  into  $M$  if needed ( $\text{MOV } R, M$ )
  - don't forget to update  $M$ 's address descriptor, if needed
  - return  $R$

### 4. else: $x$ not used in the block *or* no suitable occupied register can be found

- return  $x$  as location  $l$
- choice of purged register: *heuristics*
- remember (for step 3): registers may contain value for  $> 1$  variable  $\Rightarrow$  *multiple*  $\text{MOV}'\text{s}$



# Sample TAIC

$$d := (a-b) + (a-c) + (a-c)$$

```
t := a - b
u := a - c
v := t + u
d := v + u
```

line	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>t</i>	<i>u</i>	<i>v</i>
[0]	<i>L</i> (1)	<i>L</i> (1)	<i>L</i> (2)	<i>D</i>	<i>D</i>	<i>D</i>	<i>D</i>
1	<i>L</i> (2)	<i>L</i> (⊥)	<i>L</i> (2)	<i>D</i>	<i>L</i> (3)	<i>D</i>	<i>D</i>
2	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>D</i>	<i>L</i> (3)	<i>L</i> (3)	<i>D</i>
3	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>D</i>	<i>D</i>	<i>L</i> (4)	<i>L</i> (4)
4	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>L</i> (⊥)	<i>D</i>	<i>D</i>	<i>D</i>



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# Code sequence

	3AIC	2AC	reg. descr.		addr. descriptor						
			$R_0$	$R_1$	a	b	c	d	t	u	v
[0]			$\perp$	$\perp$	a	b	c	d	t	u	v
1	t := a - b	MOV a, R0 SUB b, R0	[a]		[ <del><math>R_0</math></del> ]				$R_0$		
2	u := a - c	MOV a, R1 SUB c, R1	.	[a]	[ <del><math>R_0</math></del> ]					$R_1$	
3	v := t + u	ADD R1, R0	v	.					<del><math>R_0</math></del>		$R_0$
4	d := v + u	ADD R1, R0 MOV R0, d	d				$R_0$				<del><math>R_0</math></del>
			$R_j$ : unused		all var's in "home position"						

- address descr's: "home position" not explicitly needed.
- e.g. variable  $a$  to be found "at  $a$ " (if not stale), as indicated in line "0".
- in the table: only *changes* (from top to bottom) indicated
- after line 3:
  - $t$  **dead**
  - $t$  resides in  $R_0$  (and nothing else in  $R_0$ )
 → **reuse  $R_0$**
- Remark: info in [brackets]: "ephemeral"



# References I



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## Bibliography

- [1] Aho, A. V., Lam, M. S., Sethi, R., and Ullman, J. D. (2007). *Compilers: Principles, Techniques and Tools*. Pearson, Addison-Wesley, second edition.
- [2] Aho, A. V., Sethi, R., and Ullman, J. D. (1986). *Compilers: Principles, Techniques, and Tools*. Addison-Wesley.

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