INF5410 Array signal processing. Chapter 2.3 Attenuation

Sverre Holm

Chapters in Johnson & Dungeon

- Ch. 1: Introduction.
- Ch. 2: Signals in Space and Time.
  - Physics: Waves and wave equation.
    - » c, λ, f, ω, k vector,...
    - » Ideal and "real" conditions
- Ch. 3: Apertures and Arrays.
- Ch. 4: Beamforming.
  - Classical, time and frequency domain algorithms.
- Ch. 7: Adaptive Array Processing.

Norsk terminologi

- Bølgeligningen
- Planbølger, sfæriske bølger
- Propagerende bølger, bølgetall
- Sinking/sakking: $\vec{\alpha}$
- Dispersjon
- Attenuasjon eller demping
- Refraksjon
- Ikke-linearitet
- Diffraksjon; nærfelt, fjernfelt
- Gruppeantenne (= array)

Kilde: Bl.a. J. M. Hovem: ``Marin akustikk'', NTNU, 1999

Deviations from simple media

1. Dispersion: $c = c(\omega)$
   - Group and phase velocity, dispersion equation: $\omega = f(k) \neq c \cdot k$
   - Evanescent (= non-propagating) waves: purely imaginary k
2. Loss: $c = c_{\Re} + jc_{\Im}$
   - Wavenumber is no longer real, imaginary part gives attenuation.
   - Waveform changes with distance
3. Non-linearity: $c = c(s(t))$
   - Generation of harmonics, shock waves
4. Refraction, non-homogenoeus medium: $c=c(x,y,z)$
   - Snell's law

UNIVERSITY
OF OSLO

Dispersion and Attenuation

- Ideal medium: Transfer function is a delay only
- Attenuation: Transfer function contains resistors
- Dispersion: Transfer function is made from capacitors and inductors (and resistors) => phase varies with frequency

DEPARTMENT OF INFORMATICS 5

UNIVERSITY
OF OSLO

2. Attenuation/absorption

1. Absorption in air and water: $\propto f^2$
   - Viscous differential equation
2. Also differential equation for $\propto f^0$
3. Medical ultrasound $\propto f^y$, where $y \approx 1$
4. General differential equation for $0 \leq y \leq 2$?

DEPARTMENT OF INFORMATICS 6

UNIVERSITY
OF OSLO

Viscous wave equation

Additional loss term

- Sound in a viscous fluid, augmented wave eq.:

$$\nabla^2 \vec{s} = \frac{1}{c^2}\frac{\partial^2 \vec{s}}{\partial t^2} - \frac{4\mu}{3\rho_0 c^2}\frac{\partial}{\partial t}\nabla^2 \vec{s} = \frac{1}{c^2}\frac{\partial^2 \vec{s}}{\partial t^2} - \tau\frac{\partial}{\partial t}\nabla^2 \vec{s}$$

  - μ is shear bulk viscocity coefficient
  - τ is a relaxation time
  - Johnson & Dudgeon, problem 2.7
- Approximate solution (low frequency, low loss):

$$k_{\Im} \approx -\frac{\tau}{2c}\omega^2$$

- Attenuation that increases with ω^2

UNIVERSITY
OF OSLO

Dispersion relation

- Viscoelastic wave equation: $\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau\frac{\partial}{\partial t}(\nabla^2 u) = 0$
- Assume 1-D, and u(x,t)=exp(j(ωt-kx)):

$$(-jk)^2 u(x,t) - \frac{(j\omega)^2}{c_0^2}u(x,t) + \tau(j\omega(-jk)^2)u(x,t) = 0$$

$$k^2 - \frac{\omega^2}{c_0^2} + j\omega\tau k^2 = 0$$

- $k=k_{\Re}+jk_{\Im}=\beta-j\alpha \Rightarrow u=\exp(-\alpha x)\cdot\exp(j(\omega t-\beta x))$
- Let $\omega\tau \ll 1$ and solve for k: $\alpha \approx \frac{\tau}{2c_0}\omega^2$

See solution of problem 2.7 for more details

Slightly more complex: Viscous + multiple relaxation

$$k_{\Im} = \alpha = A_0\omega^2 + \sum_{n=1}^{N} A_n \frac{\omega_n}{\omega^2 + \omega_n^2}\omega^2$$

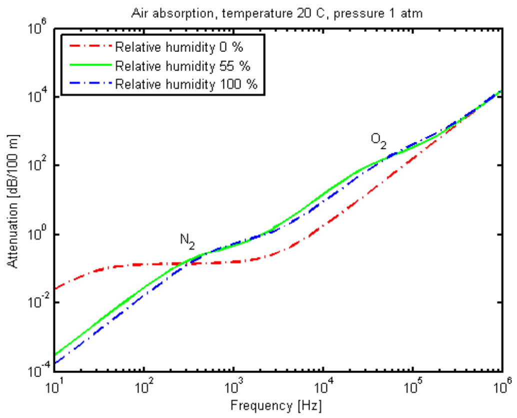
- First term: Classical losses – exchange of energy into heat, primarily viscous losses, also heat conduction, diffusion, and radiation
- Sum: Relaxation losses – change of kinetic or translational energy of the molecules into internal energy
- Nachman et al: An equation for acoustic propagation in inhomogeneous media with relaxation losses, JASA 1990

Viscoelastic case: Air

- Viscous losses dominate the first term (A_0)
- N=2:
  - Nitrogen: f_1<650 Hz
  - Oxygen: f_2<80 kHz

Evans, Bass, Sutherland: Atmospheric absorption of sound: Theoretical predictions, JASA 1972

Bass, Sutherland, Zuckerwar, Blackstock, Hester, Atmospheric absorption of sound: Further developments, JASA, 1995

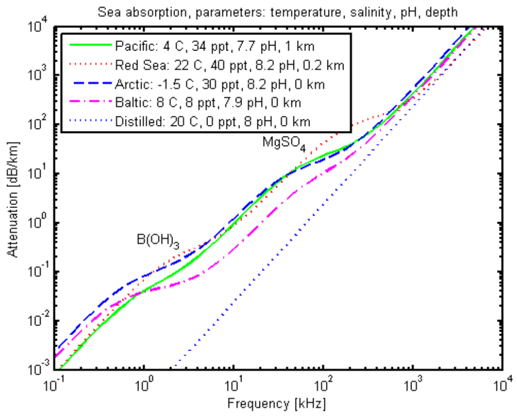


UNIVERSITY
OF OSLO

Viscoelastic case: Sea water

- A_0: Viscous absorption of water molecule
  = distilled water
- N=2:
  - Boron acid: f_1<2 kHz
  - Magnesium sulphate: f_2<150 kHz

Ainslie & McColm, A simplified formula for viscous and chemical absorption in sea water, JASA, 1998



UNIVERSITY
OF OSLO

Attenuation and loss

- Fall in amplitude due to spherical spreading: 20log (R/R_0)
- Additional losses
  - In water for underwater acoustics
  - Can usually neglect it for audible sound
- Combined: 20log $(R/R_0) + \alpha R$
- Plays a role in estimating level i.e. range for
  - long-range sonar
  - Ultrasound in air positioning

UNIVERSITY
OF OSLO

Medical ultrasound

$\alpha = a \cdot f^b$

- Liver:
  - b= 1,...,1.3
  - a=0.35,..., 0.9 dB/MHz/cm at 1 MHz.
- Breast up to b=1.5
- Ex: 5 MHz, 10 cm two-way, b=1: Loss = 5 MHz·20 cm·0.5 dB/MHz/cm = 50 dB
  - Absorption dominates over spherical spreading loss

- Plots from
Kadaba, Bhagat, Wu, “Attenuation and backscattering of ultrasound in freshly excised animal tissues, IEEE Trans. Biomedical Eng., 1980,

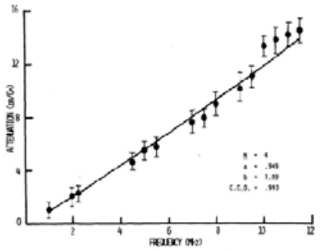


Fig. 6. Attenuation coefficient versus frequency for freshly excised LV muscle tissue.

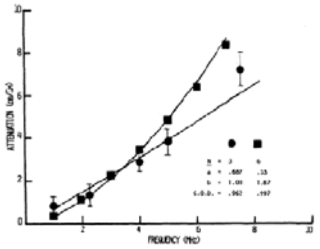
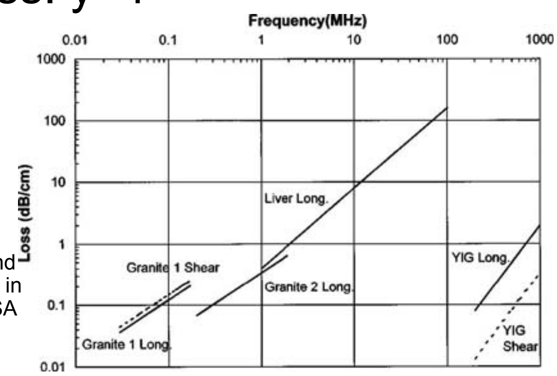


Fig. 7. Attenuation coefficient versus frequency for freshly excised (•) and fixed (▪) spleen tissue.

UNIVERSITY
OF OSLO

P- and S- loss: y>1

- Similar power laws
- Longitudinal, pressure:
  - Granite: y ≈ 1
  - Liver: y ≈ 1.3
- Shear:
  - YIG: y=2
(Yttrium indium garnet)
  - Granite: y ≈ 1
- T. Szabo and J. Wu, “A model for longitudinal and shear wave propagation in viscoelastic media”, JASA (2000).



Data for shear and longitudinal wave loss which show power-law dependence over four decades of frequency.

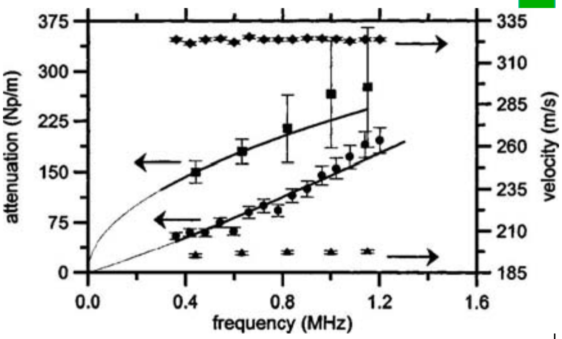
2010.02.03

Silica aerogels, S: 0<y<1

Used for matching layer in aircoupled transducer

Longitudinal (●): y=1.1 ± 0.05
Shear (■) : y=0.5 ± 0.15

T. Gomez Álvarez-Arenas, F. de Espinosa, M. Moner-Girona, E. Rodrıguez, A. Roig, and E. Molins, "Viscoelasticity of silica aerogels at ultrasonic frequencies", Appl. Phys. Lett., 2002



Attenuation vs frequency of longitudinal (●) and shear (■) waves. Solid lines: power fitting. Velocity vs frequency of longitudinal (◊) and shear (▲) waves.

Wave equation – constant loss

- Electric field in a conducting medium or transverse electric waves in a homogeneous isotropic plasma

$$\nabla^2 \vec{H} = \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} + \sigma\mu\frac{\partial \vec{H}}{\delta t}$$

Additional term

- Ch 2.3.2 + prob. 2.6:
  - k_{Im} = $-\sigma\mu c/2$ for small σ (poor conductor) and high ω
- Attenuation is **constant** with frequency

General differential equation?

- Differential equation derived from physics that has power law with other than 0. or 2.order power law?
- Area of research:
  - T. L. Szabo, ”**Time domain** wave equations for lossy media obeying a frequency power law, J. Acoust. Soc. Amer., pp. 491-500, Jul. 1994.
  - W. Chen and S. Holm, "Modified Szabo's wave equation models for lossy media obeying frequency power law," J. Acoust. Soc. Amer., pp. 2570-2574, Nov. 2003.
  - W. Chen and S. Holm, "Fractional Laplacian **time-space** models for linear and nonlinear lossy media exhibiting arbitrary frequency dependency," J. Acoust. Soc. Amer., pp. 1424-1430, Apr. 2004.
  - S. Holm and R. Sinkus, ”A unifying fractional wave equation for compressional and shear waves,” Journ. Acoust. Soc. Am., vol 127, no 1, pp-542-548, 2010.

Attenuation and dispersion are linked

- Causality ⇒ $k(\omega)=k_r(\omega)+jk_i(\omega)$ satisfies Kramers-Kronig relationship.
  - Dispersion can be found from attenuation and vice versa
- Transfer function through medium:
  - $H(\omega) = e^{jk(\omega)\cdot l}$ where l is travel distance
- Kramers-Kronig relation is similar to Hilbert transform in filter theory

UNIVERSITY
OF OSLO

Causal filter: Hilbert transform

- Hilbert transform in the time domain:
  - Impulse response $h(t)=h_r(t)+j\ h_i(t)$

$$h_i(t) = \frac{1}{\pi} P.V. \int_{\infty}^{\infty} \frac{h_r(\tau)}{\tau - t} d\tau$$

    - P.V.: Cauchy principal value
  - A convolution with kernel $x(t) = 1/(\pi t)$
- Hilbert transform in the frequency domain:
  - Kernel: $x(t) = 1/(\pi t) \Leftrightarrow X(\omega)=-\ j \cdot sgn(\omega)$
  - Filter's frequency response $H(\omega) = H_r(\omega)+j\ H_i(\omega)$
    - Transforms of real and imaginary parts of h(t), not real/imag part of H(ω)!
  - To find $H_i(\omega)$: Add 90^0 to $H_r(\omega)$ for $\omega>0$, subtract 90^0 for $\omega<0$

UNIVERSITY
OF OSLO

Attenuation - Dispersion

- Attenuation and dispersion are linked to guarantee causality
- O'Donnell, Jaynes, Miller, `Kramers-Kronig relationship between ultrasonic attenuation and phase velocity,' J. Acoust. Soc. Am., 1981
  - Predicted dispersion in dog myocardium
  - Very small => distortion of pulse form in medical ultrasound is negligible

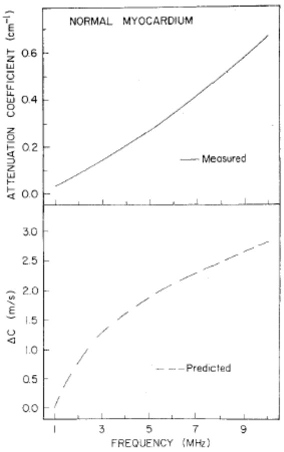


FIG. 5. The attenuation measured in normal dog myocardium is illustrated in the top panel. The lower panel presents the dispersion predicted by applying Eq. (30) to the attenuation data of the top panel.

Self-similarity, long-range correlation, fractality

Constitutive equations

Partial differential equation

Power law attenuation
$\alpha=\alpha_o|\omega|^y$

Approaches:

1. Model the medium from first principles => differential equation
2. Measure the characteristics of the medium, fit to an equation (empirical, descriptive equation or differential equation)

Often hard to unite the two, i.e. find the differential equation that yields an attenuation that can be measured

Even harder to relate such a differential equation to first principles in physics

Lower-left: Descriptive approach

- Find a partial differential equation that gives the proper power law attenuation
- Does not necessarily have its root in fundamental principles in physics:
  - May not be causal, i.e. will not model the proper dispersion
  - Solution can maybe be 'fixed' afterwards

Viscous losses: from spatial to temporal derivatives

$$\nabla^2 s = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2} - \frac{\nu}{c^2}\frac{\partial}{\partial t}\nabla^2 s, \nu = \frac{4\mu}{3\rho_0}$$

$k^2 = \omega^2/c^2 + j(\nu/c^2)\omega k^2 \Rightarrow k^2 = \omega^2/c^2 \,/\, (1-j(\nu/c^2)\,\omega)$

- Approx. 1: $k^2 \approx \omega^2/c^2 \cdot (1+j(\nu/c^2)\omega)$, small ω
- Resulting partial differential equation (time-derivatives only):

$$\nabla^2 s = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2} - \frac{\nu}{c^4}\frac{\partial^3 s}{\partial t^3}$$

← 3. order

- Approx. 2: $k \approx \omega/c \cdot (1+j(\nu/2c^2)\omega)$: quadratic loss: 2. order
- (The last one is the same step as in problem 2.7)

UNIVERSITY
OF OSLO

Loss: temporal/spatial or temporal derivatives

- P- and S-waves:

$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + Lu = 0$$ Lu is a loss operator

- Viscoelastic equation (water, air, ..., Prob 2.7):

$$Lu = \tau \frac{\partial}{\partial t}(\nabla^2 u) \approx \frac{\tau}{c_0^2}\frac{\partial^3 u}{\partial t^3}, \omega\tau \ll 1$$

  – Second order spatial deriv: Invariance wrt. rotation and translation

- For $\omega \cdot \tau \ll 1$:

$$c_0 = \sqrt{\frac{c}{\rho}}, \alpha = \frac{\tau}{2c_0}\omega^2$$ frequency squared

UNIVERSITY
OF OSLO

Szabo, 1994: Order of loss term ⇔ 1+exponent in attenuation term

- Third-order time derivative => quadratic loss:

$$\nabla^2 s = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2} - \frac{\nu}{c^4}\frac{\partial^3 s}{\partial t^3}, k_{\Im} \approx \frac{\nu}{2c^3}\omega^2$$

- First-order time derivative => constant loss (ex, p. 26)

$$\nabla^2 s = \frac{1}{c^2}\frac{\partial^2 s}{\partial t^2} + \sigma\mu\frac{\partial s}{\delta t}, k_{\Im} \approx -\sigma\mu c/2$$

- Observation by Szabo: Exponent of loss term in differential equation is one more than exponent in k_{Im}
  – T. L. Szabo, ”Time domain wave equations for lossy media obeying a frequency power law, J. Acoust. Soc. Amer., 1994.

UNIVERSITY
OF OSLO

Modified Szabo equation

- A fractional derivative interpretation of Szabo 1994:
  - Implicit Riemann-Liouville fractional derivative
  - Hypersingular, improper integral for y≠0 or y≠2
- Modification
  - Caputo fractional operator instead, in principle equal to:

(sign change for y=2)

$$Lu \propto -\frac{\partial^{y+1} u}{\partial t^{y+1}}$$

  - W. Chen and S. Holm, "Modified Szabo's wave equation models for lossy media obeying frequency power law," JASA, Nov. 2003.

UNIVERSITY
OF OSLO

Fractional derivative – a simple approach

- Fourier property, *n* integer:

$$FT\left(\frac{d^n x(t)}{dt}\right) = (i\omega)^n X(\omega)$$

- Fractional derivative:
  - Generalize *n* to any real number

UNIVERSITY OF OSLO

Fractional spatial derivative

- Fractional Laplacian $(-\nabla^2)^{y/2}$:

$$Lu \propto -\frac{\partial}{\partial t}(-\nabla^2)^{y/2}u \qquad F_{-}(-\nabla^2)^{y/2}u = k^2U, 0 < y < 2$$

- Agrees with viscous wave eq, not just $\partial^3/\partial t^3$ approx
- But still not causal as it does not have the right dispersion
  - Actually no dispersion for low ω
  - W. Chen and S. Holm, "Fractional Laplacian time-space models for linear and nonlinear lossy media exhibiting arbitrary frequency dependency," JASA, Apr. 2004.

DEPARTMENT OF INFORMATICS 30

UNIVERSITY OF OSLO

Self-similarity, long-range correlation, fractality	Constitutive equations
Partial differential equation	Power law attenuation $\alpha=\alpha_0\|\omega\|^y$

DEPARTMENT OF INFORMATICS 31

UNIVERSITY
OF OSLO

Upper-right: Constitutive equations

- Find constitutive equations that will result in a partial differential equation that gives the proper power law attenuation
- Rooted in physics
- Ensures causality, i.e. will model the proper dispersion
- Need to go back to the roots of the viscoelastic (lossy) wave equation

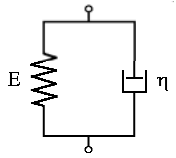
UNIVERSITY
OF OSLO

Constitutive equation

- Hooke's law: $T = cS$
  - Stress T, strain S, displacement u: $S = \frac{\partial u}{\partial x}$
  - c = stiffness
    - » Compressional wave: c=K is the bulk modulus or the inverse of the compressibility
    - » Shear wave: c44 = μ is the shear modulus.
- Include a damper:

$$T = cS + \eta \frac{\partial S}{\partial t}$$

  - η: viscosity coefficient



Voigt model
(Wikipedia Commons)

Viscoelasticity: standard

- Constitutive eq.: $T = cS + \eta \frac{\partial S}{\partial t}$ S
- Wave equation: $\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau \frac{\partial}{\partial t}(\nabla^2 u) = 0$ P
- Power law: $\alpha \approx \frac{\tau}{2c_0}\omega^2$ P (S)
- Can we find a description with a general power law for both P- and S-waves?

Stress-strain: fractional viscous term

- Stress, T, as a function of strain, S: $T = cS + \eta \frac{\partial^{z_0} S}{\partial t^{z_0}}$
- $z_0 \in (0,1]$, we extend it to (0,2)
- Introduces memory to the loss mechanism
- Wave equation:

$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau^{z_0} \frac{\partial^{z_0}}{\partial t^{z_0}}(\nabla^2 u) = 0$$

  - Zero-frequency propagation speed, $c_0^2 = c/\rho$,
  - Relaxation time $\tau^{z_0} = \eta/c$.

UNIVERSITY OF OSLO

Fractional wave equation

$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau^{z_0}\frac{\partial^{z_0}}{\partial t^{z_0}}(\nabla^2 u) = 0$$

- Introduced in 1967:
  - M. Caputo, “Linear models of dissipation whose Q is almost frequency independent-II,” Geophys J. R. Astron. Soc, 1967.
- Rediscovered; not rooted in constitutive eq., only to fit measurement
  - M. Wismer, “Finite element analysis of broadband acoustic pulses through inhomogenous media with power law attenuation”, JASA, 2006
- Link between Caputo and Wismer:
  - J. F. Kelly and R. J. McGough, “Fractal ladder models and power-law wave equations”, JASA, 2009
- Analyzed both for low and high $\omega\tau$ cases
  - S. Holm and R. Sinkus, ”A unifying fractional wave equation for compressional and shear waves,” Journ. Acoust. Soc. Am., 2010.

DEPARTMENT OF INFORMATICS 37

UNIVERSITY OF OSLO

Lossy wave equation

$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + Lu = 0$$

- Physics-based:
  - Viscoelastic, y=2:

$$Lu = \tau\frac{\partial}{\partial t}(\nabla^2 u) \approx \quad \propto \partial^3 u/\partial t^3$$

  - E-field in conductor, y=0

$$Lu \propto -\partial u/\partial t$$

- Fractional derivatives:
  - Szabo94 (Chen,Holm03):

$$Lu \propto -\frac{\partial^{y+1} u}{\partial t^{y+1}}$$

  - Chen, Holm 04:

$$Lu \propto -\frac{\partial}{\partial t}(-\nabla^2)^{y/2} u$$

  - Wismer 06, Caputo 67:

$$Lu = \tau^{z_0}\frac{\partial^{y-1}}{\partial t^{y-1}}(\nabla^2 u)$$

DEPARTMENT OF INFORMATICS

UNIVERSITY
OF OSLO

Fractional Caputo wave equation

- Dispersion relation:

$$k^2 - \frac{\omega^2}{c_0^2} + (i\omega\tau)^{z_0} k^2 = 0$$

$$k^2 = \frac{\omega^2}{c_0^2} \frac{1}{1 + (i\omega\tau)^{z_0}} = \frac{\omega^2}{c_0^2} \frac{1}{1 + (\omega\tau)^{z_0} \left(\cos(\frac{\pi z_0}{2}) + i \sin(\frac{\pi z_0}{2})\right)}$$

$$c_0 = \sqrt{\mu/\rho}, \tau = \eta/\mu$$

DEPARTMENT OF INFORMATICS

UNIVERSITY
OF OSLO

High- and low-frequency approx.

1. Low frequency: $(\omega\tau)^{z_0} << 1$
   - P-waves: Air: $\tau = 1.7 \cdot 10^{-10}$ sec, water: $\tau = 6 \cdot 10^{-13}$ sec, air: at least 100 MHz, water: at least 1 GHz
   - YIG, Shear and pressure waves up to 100’s of MHz.

1. High frequency: $(\omega\tau)^{z_0} >> 1$
   - Ex: Shear waves in tissue, dynamic elastography

DEPARTMENT OF INFORMATICS

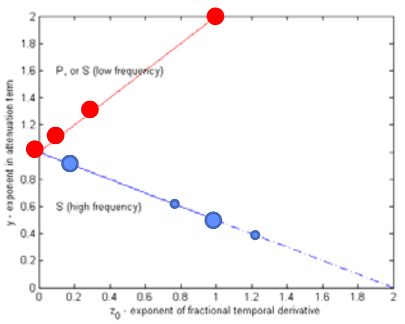
Summary Caputo equation

- Stress-strain: $T = c_{ii} S + \eta_{ii} \frac{\partial^{z_0} S}{\partial t^{z_0}}$
- Wave eq.:

$$\nabla^2 u - \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2} + \tau^{z_0}\frac{\partial^{z_0}}{\partial t^{z_0}}(\nabla^2 u) = 0, \tau = \frac{\eta_{ii}}{c_{ii}}, c_0^2 = c_{ii}/\rho$$

- Low-f (P-waves, low-f S): $\alpha = \alpha_0 |\omega|^y, \alpha_0 = \frac{\tau^{y-1}}{2c_0}|\cos\frac{\pi y}{2}|$
  - $y = z_0+1$, $y \in (1,2]$, $z_0 \in (0,1]$
- Hi-f, S-waves: $c_0 = \sqrt{\mu/\rho}, \tau = \eta/\mu$ $\alpha = \alpha_0 |\omega|^y, \alpha_0 = \frac{\tau^{y-1}}{c_0}\cos\frac{\pi y}{2}$
  - $y = 1-z_0/2$, $y \in [0,1)$, $z_0 \in (0,2]$

z_0, fract. deriv. – y, exp in power law



y=2: Water,air (P), YIG (P, S)

y=1.3: Liver (P)
y=1.1: Aerogels (P)
y=1: Granite (P, S)
z_0=0.2: Living cells (S)
- 0.16-18: cortical
- 0.26-0.29: intracellular

y=0.5±0.15 : Aerogels (S)

UNIVERSITY
OF OSLO

Parallel development of wave equations with memory term

- Convolution term as loss operator

$$Lu \propto \frac{\partial}{\partial t}(\nabla^2\{h \otimes u\})$$

- The relaxation function and its Fourier transform are

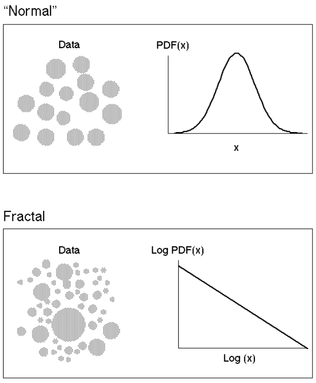
$$h(t) = \frac{1}{t^{z_0}}, t > 0 \Leftrightarrow H(\omega) = \Gamma(1 - z_0)(i\omega)^{z_0 - 1}$$

- Can show that it can be transformed to a fractional derivative and thus is of the same form (Holm, Sinkus, 2010)
- Buckingham, "Theory of acoustic attenuation, dispersion, and pulse propagation in unconsolidated granular materials including marine sediments," J. Acoust. Soc. Am.,1997.

DEPARTMENT OF INFORMATICS 43

UNIVERSITY
OF OSLO

Normal vs fractal distribution of scatterers



Top: Usual assumption: PDF is a "normal" distribution.

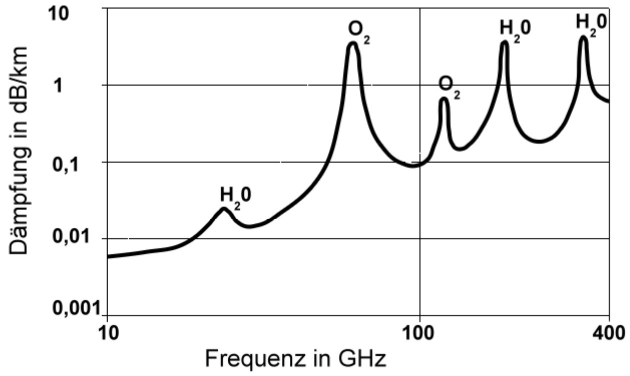
Bottom: Much data from the natural world consists of an ever larger number of ever smaller values. The PDF is a fractal distribution.

Liebovitch and Scheurle,Two lessons from fractals and chaos, Complexity, 2000

DEPARTMENT OF INFORMATICS 44

UNIVERSITY
OF OSLO

Electromagnetic, atmosphere ?



Wikipedia

UNIVERSITY
OF OSLO

Array Processing Implications

- Lossy media cause signals to decay more rapidly than predicted by ideal wave equation
  - Limits range
  - Ultrasound imaging: low frequency ⇔ deeper penetration, but poorer resolution
- Attenuation and dispersion are coupled
  - Attenuation $\propto f^2 \Rightarrow$ dispersion is zero