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INF5410 Array signal processing. Chapter 2.3 Non-linearity

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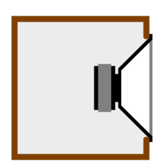
Compliance in closed chamber

- The gas law without heat transfer (adiabatic): $pV^{\gamma}=C$
  - γ is the adiabatic exponent, γ = 1.4 for air
  - p is pressure and V is volume.
- A loudspeaker affects the volume, V
  - Our ears sense the resulting pressure, p.
- In loudspeakers nonlinearity affects the lower frequencies: small subwoofers
  - cone excursion increases with lower frequency
- The nonlinearity of the pressure – volume relationship => nonlinear acoustics.



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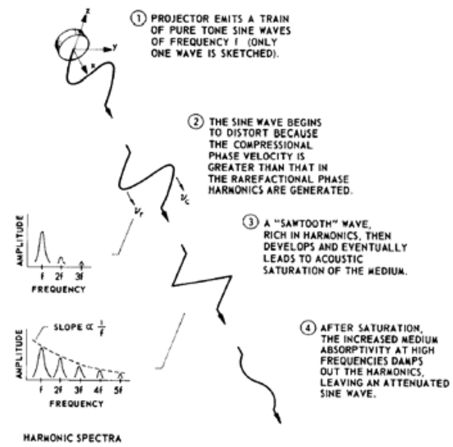
Harmonic generation



- Muir and Carstensen: Prediction of nonlinear acoustic effects at biomedical frequencies and intensities, Ultrasound in Medicine & Biology, 1980

State equation for gas

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^\gamma$$

- Taylor series for pressure variation:

$$p - p_0 = A\frac{\rho - \rho_0}{\rho_0} + \frac{B}{2!}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \cdots$$

- $A=\rho_0\gamma$, $B=\rho_0\gamma(\gamma-1)$; $B/A = \gamma-1$
- A nonlinear spring: replaces Hooke’s law
- Similiar approach for fluids

Non-linear acoustics PDE

From Eqs. (5) and (6) we also obtain the "classical" equation in the single variable ϕ:

$$\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} + \frac{D}{c_0^2}\nabla^2\frac{\partial\phi}{\partial t} = \frac{1}{c_0^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + \frac{1}{c_0^2}\frac{B}{2A}\left(\frac{\partial\phi}{\partial t}\right)^2\right]. \quad (11)$$

- Aanonsen, Barkved, Tjøtta, Tjøtta: Distortion and harmonic generation in the nearfield of a finite amplitude sound beam, JASA, 1984

- Notice:
  - Φ is velocity potential
  - Squaring on r.h.s. implies nonlinearity, B/A is nonlinearity coefficient
  - R.h.s: 1. term = local generation, 2. term is cumulative effect
  - D is viscous absorption term, i.e. ~water $\Rightarrow$ attenuation $\propto \omega^2$

Simplified equations for simulations

- Westerveld equation (1963)
  - Right-hand side: 1. term (local term) is dropped
  - OK >λ away from source (quasi-plane wave)
- KZK-equation (Khoklov-Zabolotskaya-Khoklov, 1969, 1971)
  - Weak nonlinearity: Dissipation and nonlinearity cause slow changes of the beam in space
  - For a directed sound beam where variations across beam are more rapid than along the beam
  - Bergen code: http://folk.uib.no/nmajb/Bergencode.html
- Burgers' equation (1948)
  - Like KZK
  - +1-D = plane waves = no diffraction
  - v = fluid velocity
  - β= 1+B/2A
  - b – related to viscous absorption

$$\frac{\partial v}{\partial x} - \frac{\beta}{c_0^2}\frac{\partial v}{\partial t} - \frac{b}{2c_0^3\rho_0}\frac{\partial^2 v}{\partial \tau^2} = 0$$

Nonlinearity parameter

Material	B/A
Blood	6.1
Brain	6.6
Fat	10
Liver	6.8
Muscle	7.4
Water	5.2

- Wikipedia

Non-linear acoustics

c, varies with the particle displacement, u, or pressure p*:*

$$c(t) = c_0 + \frac{B}{2A}u(t) = c_0 + \frac{B}{2A}\frac{p(t)}{\rho_0 c_0}$$

- p_1(t) = pressure = p_0 + p(t)
- p_0 = 1 atmosphere
- p(t) = applied pressure variation (= ”signal”)

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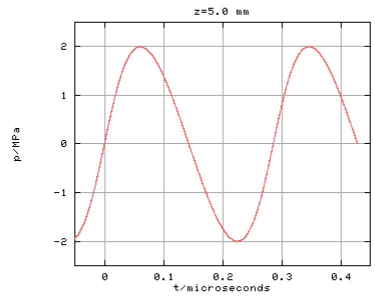
Nonlinearity and plane wave

- A plane wave in water,
- Initial amplitude: 2 MPa (20 atmospheres)
- Frequency of 3.5 MHz
- Propagates for 100 mm.
- Starts to deform immediately,
- Peak-to-peak amplitude and power decrease only slowly, following the usual exponential attenuation of water.
- Beyond 35 mm, however, a shock wave has formed, and the amplitude decreases relatively rapidly.
- By 100 mm, the amplitude has halved, and 80% of the beam's power has been lost.
- Generated by the "Bergen code" written at the University of Bergen in Norway.
- http://www.bath.ac.uk/~pyscmd/acoustics/nonlin.htm



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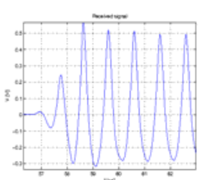
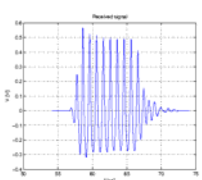
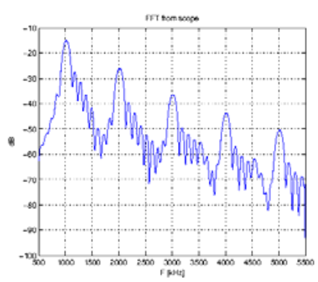
Nonlinear pulse shape measured in water tank in our lab







Fabrice Prieur, Sept. 2009

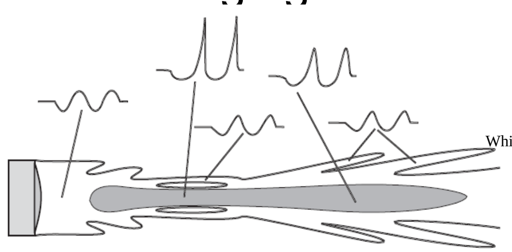
Harmonic vs intermodulation distortion

Transmit f_1 and f_2

1. Harmonic distortion $2f_1$, $2f_2$, $3f_1$, $3f_2$, ...
2. Intermodulation distortion f_1-f_2, f_1+f_2, $2f_1$-f_2, ...

1. Harmonic imaging:
   - Transmit f
   - Generate 2f, 3f, 4f, ...
     - » Usually 2f is the most important one
2. Parametric sonar, parametric sound source:
   - Transmit f_1 and f_2
   - Use difference frequency f_1-f_2,

1. Harmonic imaging



Whittingham, 2007

- Positive effect on images:
  - 2. harmonic beam is narrower => better resolution
  - Is not generated in sidelobes of 1. harmonic beam => less sidelobes
  - Is generated inside medium => avoids some of the reverberations from chest wall
- Negative effect:
  - 2. harmonics attenuates faster => less penetration

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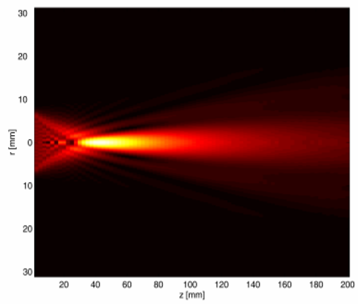
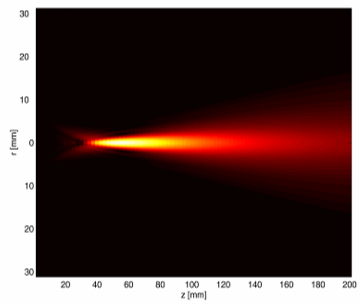
Circular symmetric (1-D) simulation - J.F.Synnevåg
Burgers equation (Christopher & Parker k-space)



1. harmonic



2. harmonic

Focus 60 mm, f=2.275 MHz

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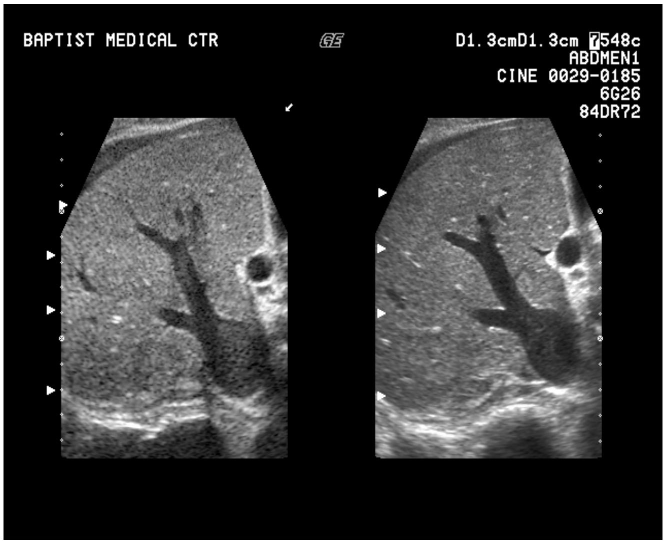
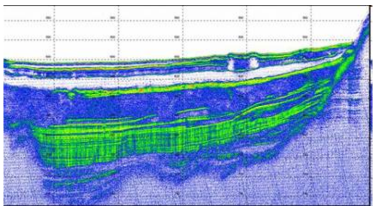
Liver
2 harm



2a. Parametric sonar



- Topas: Kongsberg Defense & Aerospace
- Parametric sub-bottom profilers
- Low frequency sound generation due to non-linear interaction in the water column from two high intensity sound beams at higher frequencies.
- The resulting signal has a high relative bandwidth (~80%), narrow beam profile
- Penetration ~100 m, 150 ms

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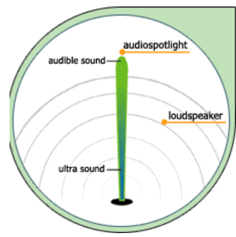
Topas: Parametric profilers

	TOPAS PS18	TOPAS PS40
Secondary frequency	0.5-6kHz	1-10kHz
Primary frequencies	15-21 kHz or 30-42 kHz	35-45 kHz or 70-90 kHz
Source levels	Secondary: 208 Primary: 242/225 dB	Secondary: 207 Primary: 241/226 dB
Hor. resolution	<5 x 5 deg	3 x 6 deg
Signatures	CW, Chirp, Ricker	

2b. Parametric audio sound source

- Non-linear interaction
- Holosonics: Audio Spotlight
  - http://www.holosonics.com/index.html
- American Technology Corporation: HyperSonic Sound technology:
  - http://www.atcsd.com/site/content/view/13/104/
  - http://www.prosoundweb.com/install/tech_corner/parametric.php



Mad Labs: Audio Spotlight

- Youtube demo: http://www.youtube.com/watch?v=veDk2Vd-9oQ&feature=related
- Mad Labs from the National Geographic Channel presents the Audio Spotlight, focused loudspeaker technology, 3 min 12 sec
- See http://www.audiospotlight.com for more.

Array Processing Implications

- Nonlinearity may create new frequencies that were not present in the source
  - Harmonics
  - Intermodulation: [Sum]/Difference frequencies

- Harmonics: harmonic (octave) imaging in medical ultrasound
- Difference frequency: Parametric sonar, directive audio source