INF5410 Array signal processing. Chapter 2.3 Non-linearity

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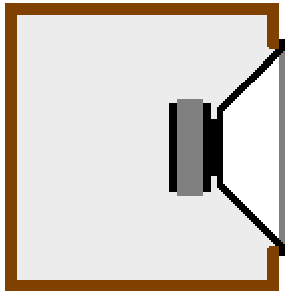
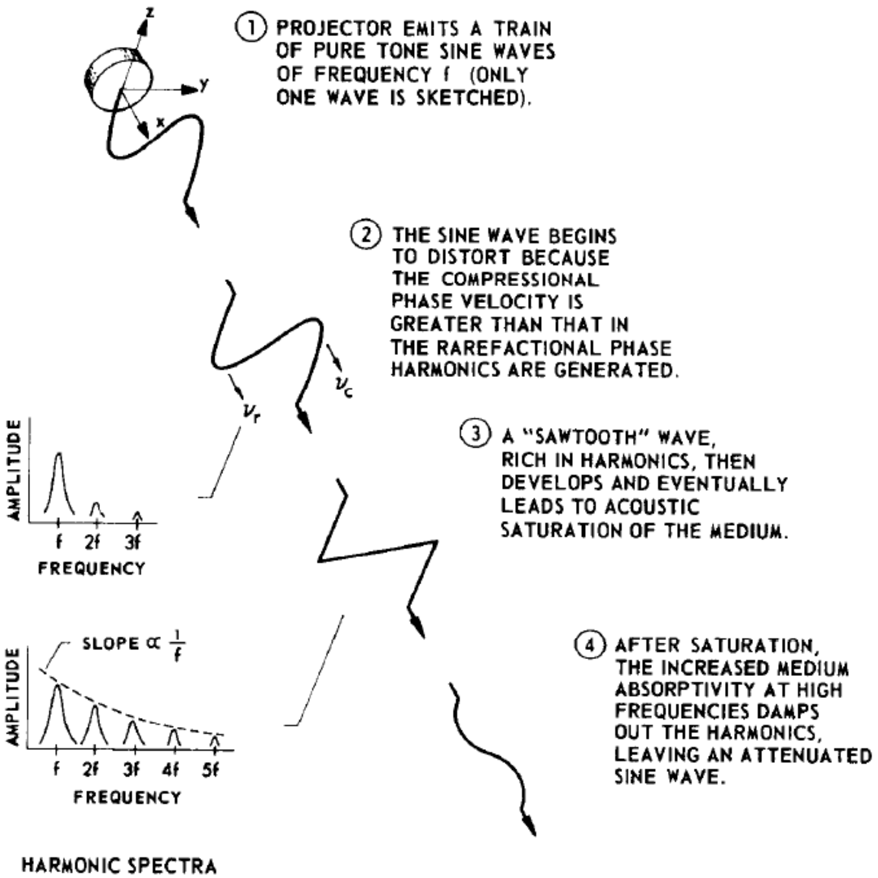
Compliance in closed chamber

- The gas law without heat transfer (adiabatic): $pV^{\gamma}=C$
  - γ is the adiabatic exponent, γ = 1.4 for air
  - p is pressure and V is volume.
- A loudspeaker affects the volume, V
  - Our ears sense the resulting pressure, p.
- In loudspeakers nonlinearity affects the lower frequencies: small subwoofers
  - cone excursion increases with lower frequency
- The nonlinearity of the pressure – volume relationship => nonlinear acoustics.



Harmonic generation



- Muir and Carstensen: Prediction of nonlinear acoustic effects at biomedical frequencies and intensities, Ultrasound in Medicine & Biology, 1980

State equation for gas

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

- Taylor series for pressure variation:

$$p - p_0 = A\frac{\rho - \rho_0}{\rho_0} + \frac{B}{2!}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2 + \cdots$$

- $A=\rho_0\gamma$, $B=\rho_0\gamma(\gamma-1)$; $B/A = \gamma-1$
- A nonlinear spring: replaces Hooke's law
- Similiar approach for fluids

Non-linear acoustics PDE

From Eqs. (5) and (6) we also obtain the "classical" equation in the single variable ϕ:

$$\nabla^2\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} + \frac{D}{c_0^2}\nabla^2\frac{\partial\phi}{\partial t} = \frac{1}{c_0^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + \frac{1}{c_0^2}\frac{B}{2A}\left(\frac{\partial\phi}{\partial t}\right)^2\right]. \quad (11)$$

- Aanonsen, Barkved, Tjøtta, Tjøtta: Distortion and harmonic generation in the nearfield of a finite amplitude sound beam, JASA, 1984

- Notice:
  - Φ is velocity potential
  - Squaring on r.h.s. implies nonlinearity, B/A is nonlinearity coefficient
  - R.h.s: 1. term = local generation, 2. term is cumulative effect
  - D is viscous absorption term, i.e. ~water $\Rightarrow$ attenuation $\propto \omega^2$

Simplified equations for simulations

- Westerveld equation (1963)
  - Right-hand side: 1. term (local term) is dropped
  - OK >λ away from source (quasi-plane wave)
- KZK-equation (Khoklov-Zabolotskaya-Khoklov, 1969, 1971)
  - Weak nonlinearity: Dissipation and nonlinearity cause slow changes of the beam in space
  - For a directed sound beam where variations across beam are more rapid than along the beam
  - Bergen code: http://folk.uib.no/nmajb/Bergencode.html
- Burgers' equation (1948)
  - Like KZK
  - +1-D = plane waves = no diffraction
  - v = fluid velocity
  - β= 1+B/2A
  - b – related to viscous absorption

$$\frac{\partial v}{\partial x} - \frac{\beta}{c_0^2}\frac{\partial v}{\partial t} - \frac{b}{2c_0^3\rho_0}\frac{\partial^2 v}{\partial \tau^2} = 0$$

Nonlinear acoustic wave equations with fractional loss operators

- Fabrice Prieur, Sverre Holm, J. Acoust. Soc. Am, Sept 2011

Fractional derivatives are well suited to describe wave propagation in complex media. When introduced in classical wave equations, they allow a modelling of attenuation and dispersion that better describes sound propagation in biological tissues.

Traditional constitutive equations from solid mechanics and heat conduction are modified using fractional derivatives. They are used to derive a nonlinear wave equation which describes attenuation and dispersion laws that match observations.

This wave equation is a generalization of the Westervelt equation, and also leads to a fractional version of the Khokhlov-Zabolotskaya-Kuznetsov and Burgers’ equations.

Nonlinearity parameter

Material	B/A
Blood	6.1
Brain	6.6
Fat	10
Liver	6.8
Muscle	7.4
Water	5.2

- Wikipedia

Non-linear acoustics

c, varies with the particle displacement, u, or pressure p*:*

$$\frac{dx}{dt} = c(t) = c_0 + (1 + \frac{B}{2A})u(t) = c_0 + (1 + \frac{B}{2A})\frac{p(t)}{\rho_0 c_0}$$

- $p_1(t)$ = pressure = p_0 + p(t)
- p_0 = 1 atmosphere
- p(t) = applied pressure variation (= ”signal”)

- Two sources of nonlinearity:
  - Inherent in the material’s properties (equation of state): B/2A
  - Due to convection: the ’1’, exists even if material nonlinearity B/2A = 0

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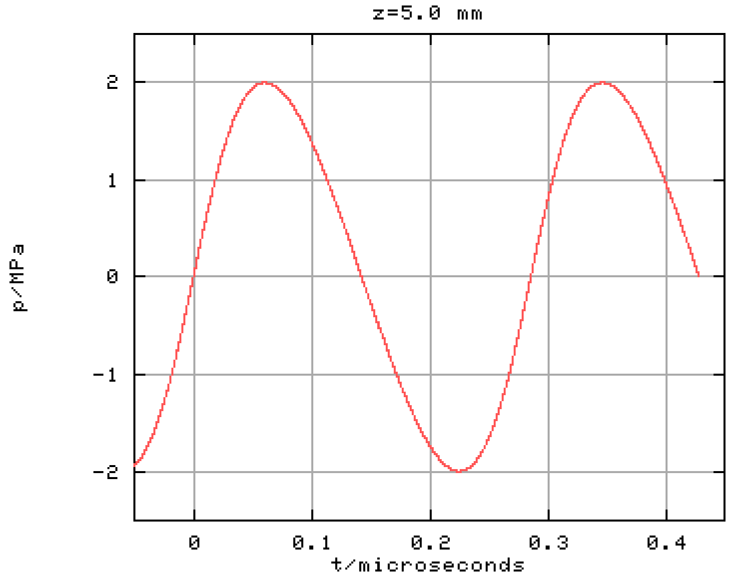
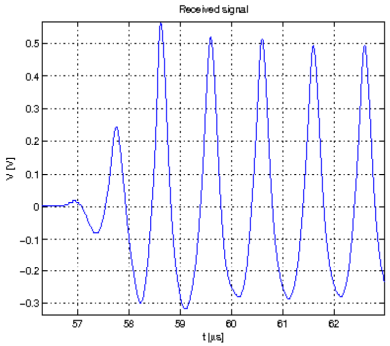
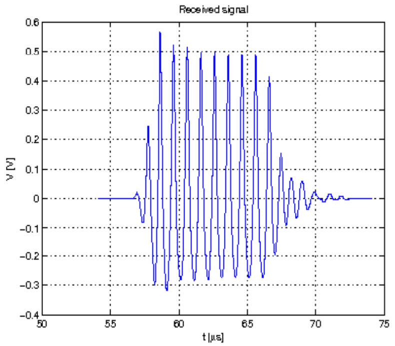
Nonlinearity and plane wave

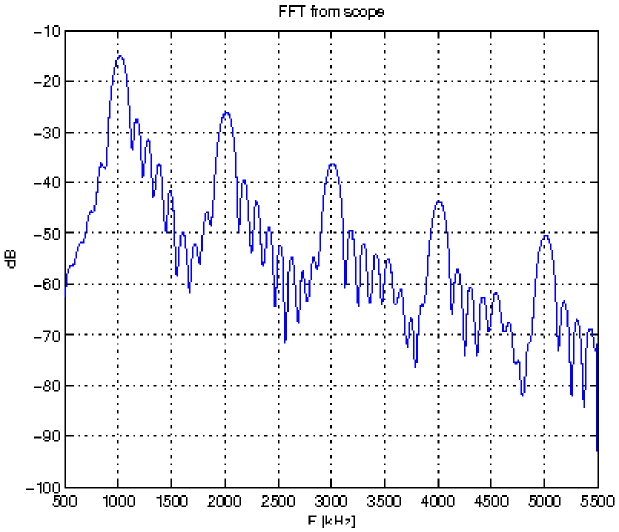
- A plane wave in water,
- Initial amplitude: 2 MPa (20 atmospheres)
- Frequency of 3.5 MHz
- Propagates for 100 mm.
- Starts to deform immediately,
- Peak-to-peak amplitude and power decrease only slowly, following the usual exponential attenuation of water.
- Beyond 35 mm, however, a shock wave has formed, and the amplitude decreases relatively rapidly.
- By 100 mm, the amplitude has halved, and 80% of the beam's power has been lost.
- Generated by the "Bergen code" written at the University of Bergen in Norway.
- http://www.bath.ac.uk/~pyscmd/acoustics/nonlin.htm



Nonlinear pulse shape measured in water tank in our lab







Fabrice Prieur, Sept. 2009

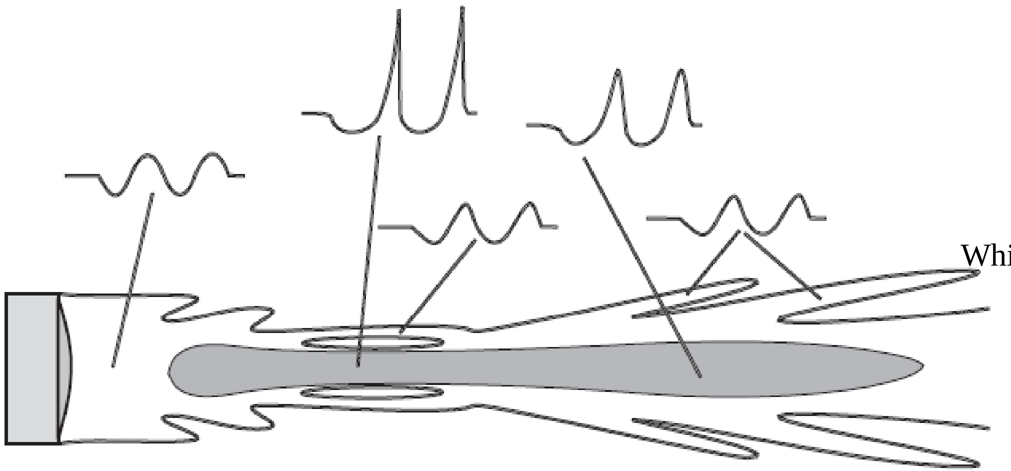
Harmonic vs intermodulation distortion

Transmit f_1 and f_2

1. Harmonic distortion $2f_1$, $2f_2$, $3f_1$, $3f_2$, ...
2. Intermodulation distortion f_1-f_2, f_1+f_2, $2f_1$-f_2, ...

1. Harmonic imaging, medical ultrasound:
   - Transmit f
   - Generate 2f, 3f, 4f, ...
     - » Usually 2f is the most important one
2. Parametric sound source, parametric sonar:
   - Transmit f_1 and f_2
   - Use difference frequency f_1-f_2,

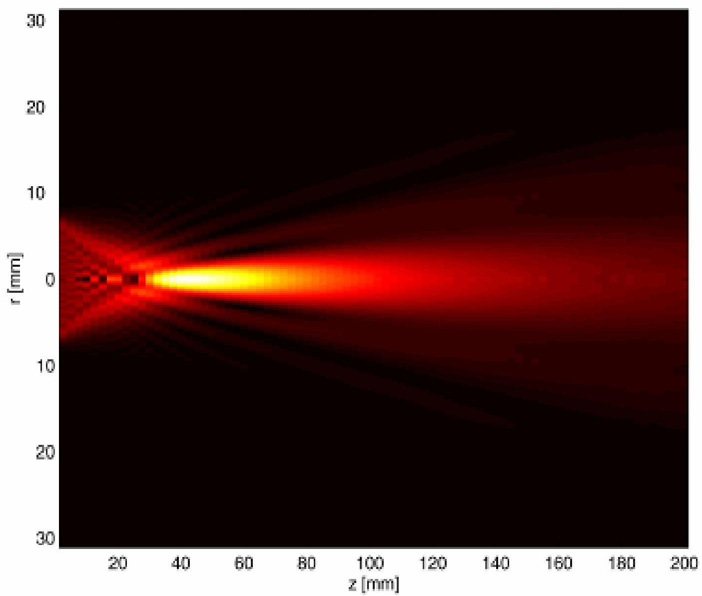
1. Harmonic imaging



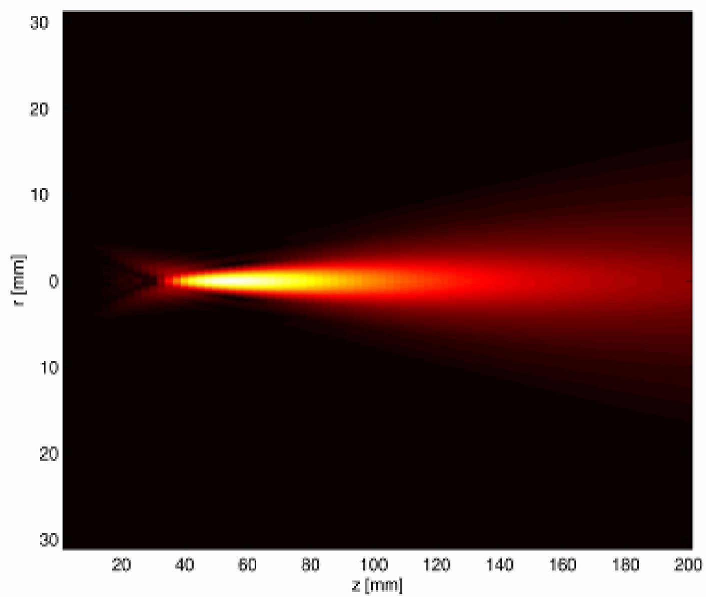
Whittingham, 2007

- Positive effect on images:
  - 2. harmonic beam is narrower => better resolution
  - Is not generated in sidelobes of 1. harmonic beam => less sidelobes
  - Is generated inside medium => avoids some of the reverberations from chest wall
- Negative effect:
  - 2. harmonics attenuates faster => less penetration

Circular symmetric (1-D) simulation - J.F.Synnevåg
Burgers equation (Christopher & Parker k-space)



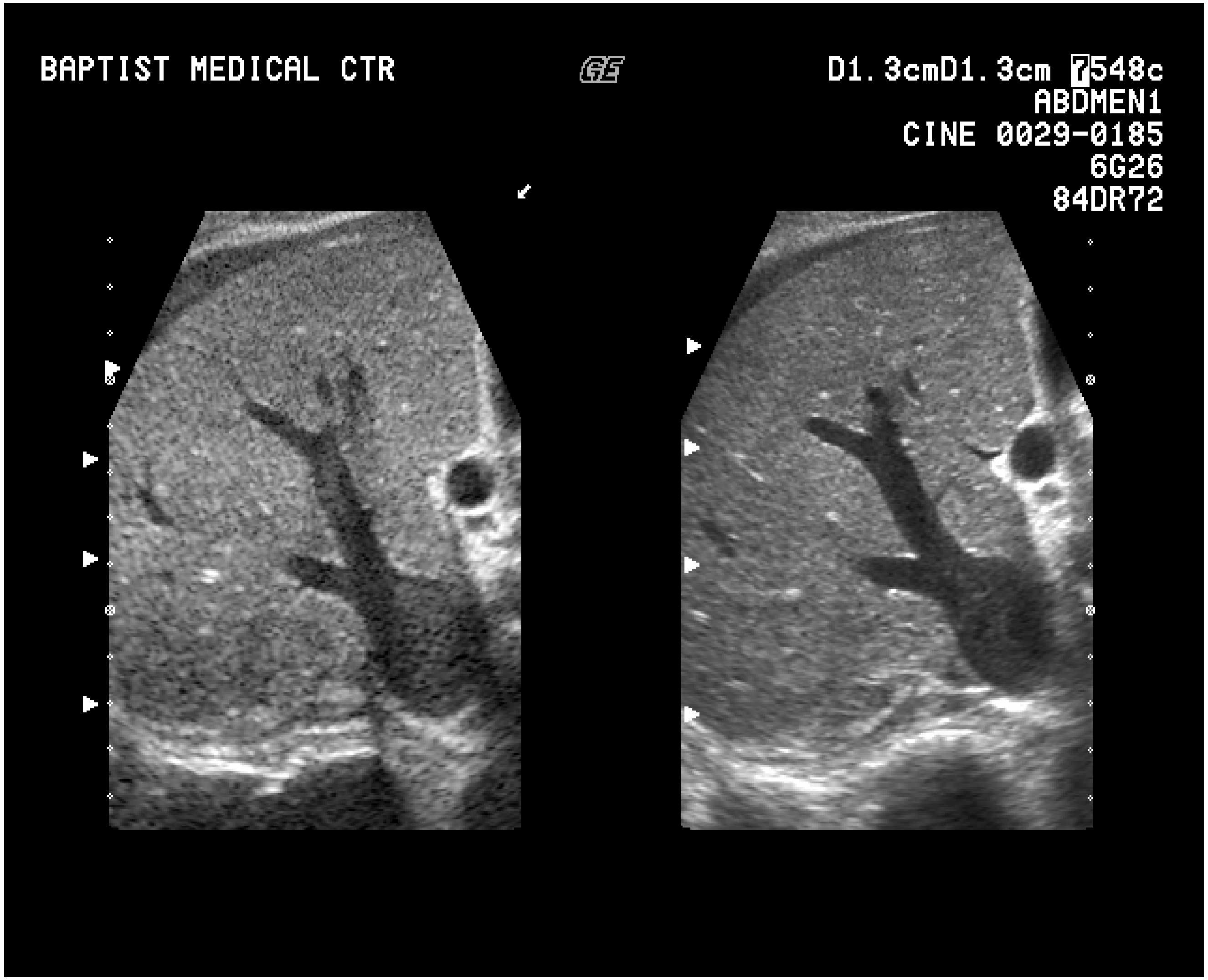
1. harmonic



2. harmonic

Focus 60 mm, f=2.275 MHz

Liver
2 harm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ABDMEN1
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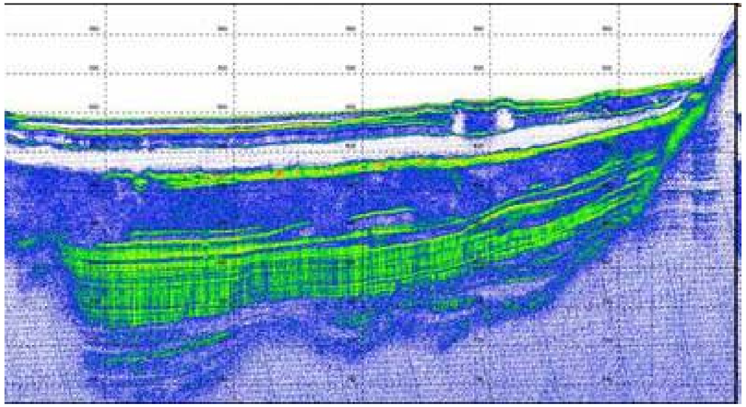
2a. Parametric sonar



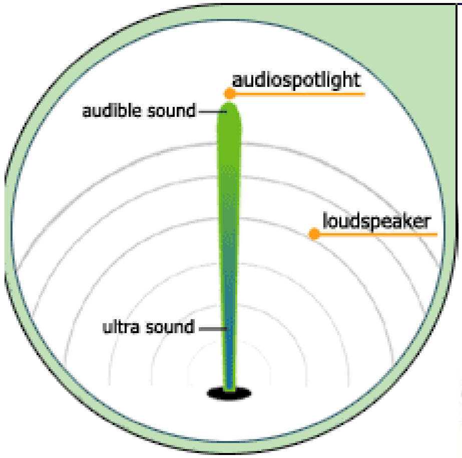
- Topas: Kongsberg Defense & Aerospace
- Parametric sub-bottom profilers
- Low frequency sound generation due to non-linear interaction in the water column from two high intensity sound beams at higher frequencies.
- The resulting signal has a high relative bandwidth (~80%), narrow beam profile
- Penetration ~100 m, 150 ms

Topas: Parametric profilers

	TOPAS PS18	TOPAS PS40
Secondary frequency	0.5-6kHz	1-10kHz
Primary frequencies	15-21 kHz or 30-42 kHz	35-45 kHz or 70-90 kHz
Source levels	Secondary: 208 Primary: 242/225 dB	Secondary: 207 Primary: 241/226 dB
Hor. resolution	<5 x 5 deg	3 x 6 deg
Signatures	CW, Chirp, Ricker	

2b. Parametric audio sound source

- Non-linear interaction
- Holosonics: Audio Spotlight
  - http://www.holosonics.com/index.html
- American Technology Corporation: HyperSonic Sound technology:
  - http://www.atcsd.com/site/content/view/13/104/
  - http://www.prosoundweb.com/install/tech_corner/parametric.php



Mad Labs: Audio Spotlight

- Youtube demo: http://www.youtube.com/watch?v=veDk2Vd-9oQ&feature=related
- Mad Labs from the National Geographic Channel presents the Audio Spotlight, focused loudspeaker technology, 3 min 12 sec
- See http://www.audiospotlight.com for more.

Array Processing Implications

- Nonlinearity may create new frequencies that were not present in the source
  - Harmonics
  - Intermodulation: [Sum]/Difference frequencies

- Harmonics: harmonic (octave) imaging in medical ultrasound
- Difference frequency: Parametric sonar, directive audio source