INF5410 Array signal processing.
Chapter 2.4 Refraction and diffraction

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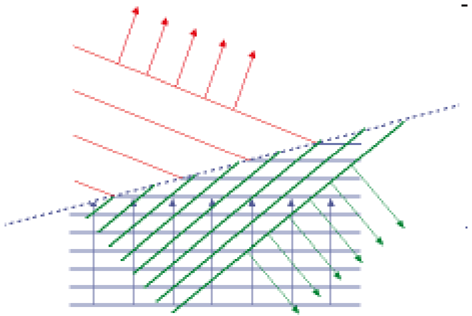
Deviations from simple media

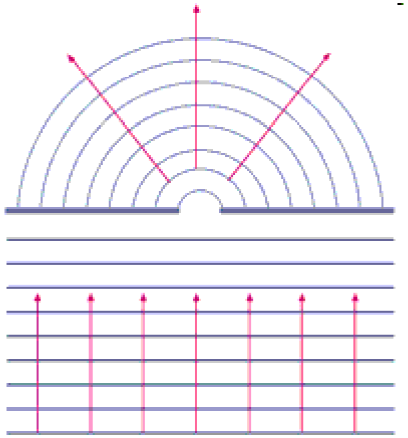
1. Dispersion: $c = c(\omega)$
   - Group and phase velocity, dispersion equation: $\omega = f(k) \neq c \cdot k$
   - Evanescent (= non-propagating) waves: purely imaginary k
2. Attenuation: $c = c_{\Re} + jc_{\Im}$
   - Wavenumber is no longer real, imaginary part gives attenuation.
   - Waveform changes with distance
3. Non-linearity: $c = c(s(t))$
   - Generation of harmonics, shock waves
4. Refraction: $c=c(x,y,z)$
   - Snell's law

Reflection, refraction, diffraction

- Click http://lectureonline.cl.msu.edu/~mmp/kap13/cd372.htm



When a wave meets the interface, it is either refracted or reflected.



A wave is diffracted by an aperture.

4. Refraction - avbøyning

- Unchanged phase on interface:

$$\overrightarrow{k_i} \cdot \overrightarrow{x} = \overrightarrow{k_r} \cdot \overrightarrow{x} = \overrightarrow{k_t} \cdot \overrightarrow{x}$$

- Fig 2.10:

$$|\overrightarrow{k_i}| \cdot \sin\theta_i = |\overrightarrow{k_r}| \cdot \sin\theta_r = |\overrightarrow{k_t}| \cdot \sin\theta_t$$

- Reflected (same c) :
  $k=\omega/c \Rightarrow \theta_r = \theta_i$
- Refracted, Snell's law:
  $\sin\theta_i/c_i=\sin\theta_t/c_t$
- Willebrand Snell von Royen, NL 1591-1626

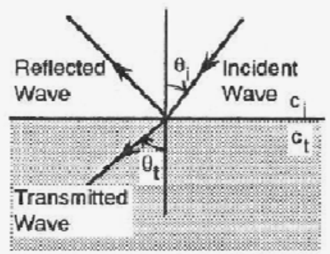


Figure 2.10 An incident wave striking a discontinuity in the medium results in a reflected wave and a transmitted wave. The angle of reflection equals the angle of incidence, and the angle of refraction of the transmitted wave obeys Snell's Law. In this example, the propagation speed in the lower medium is greater than in the upper.

Critical Angle – total reflection

- $\sin\theta_i/c_i=\sin\theta_t/c_t$
- Total reflection for all θ_i which result in $\theta_t>90^0$
- Critical angle $\sin\theta_i=c_i/c_t$
- Ex: steel c_t=5800 m/s, water c_i=1490 m/s, $\theta_i<16.5^0$
- Important for containing 100% of transmitted energy inside optical fibers

Simple model for the sea

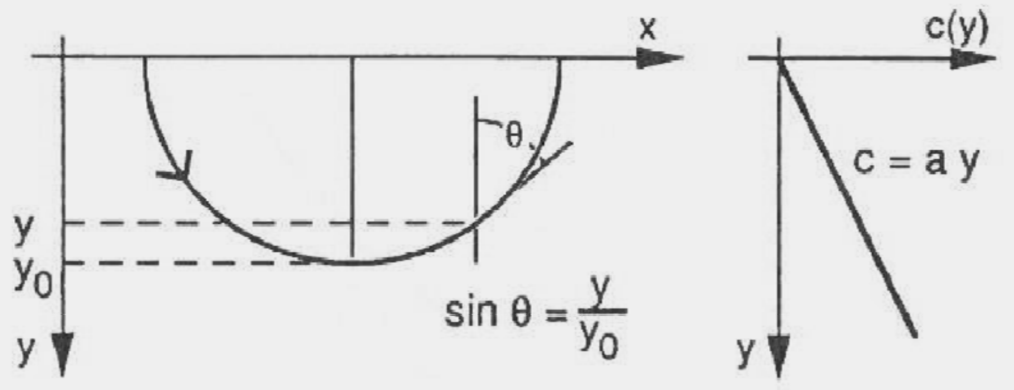


Figure 2.11 A linear change in the speed of sound with depth results in circular rays. The linear sound profile is shown in the right panel.

Problem 2.8. More general: $c=c_0(1+ay)$

Snell's law on differential form

- Snell's law: $$\frac{\sin\theta(y)}{c(y)} = \frac{\sin\theta(y-\delta y)}{c(y-\delta y)}$$
- Multiply by c(y-δy) and subtract sinθ(y)

$$\sin\theta(y-\delta y) - \sin\theta(y) = \frac{\sin\theta(y)}{c(y)} \cdot [c(y-\delta y) - c(y)]$$

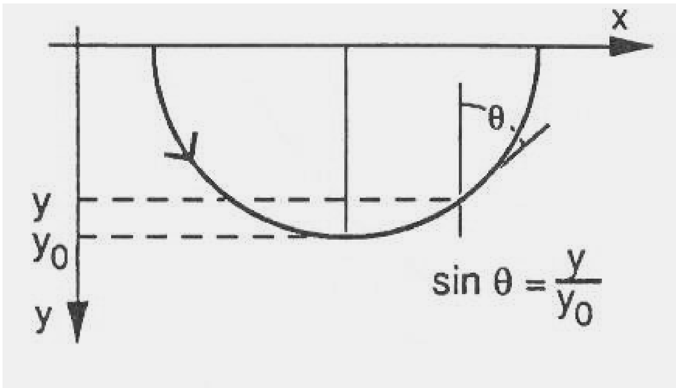
- In the limit:

$$\frac{1}{\sin\theta(y)}\frac{d\sin\theta}{dy} = \frac{1}{c(y)}\frac{dc}{dy}$$

- Solution:

$$\ln(\sin\theta) = \ln(c) + C_0$$

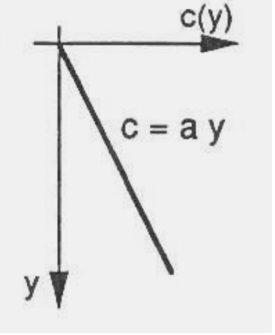
$$\sin\theta(y) = C_1 c(y)$$



Linear variation of c with depth

$$\sin\theta(y) = C_1 c(y)$$

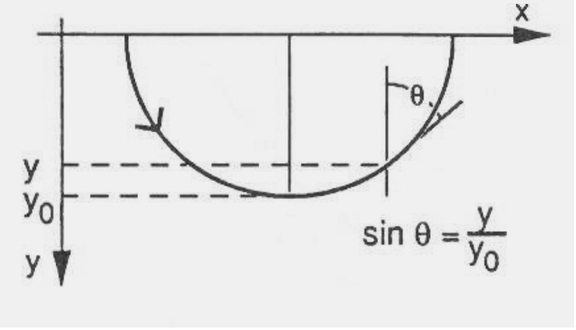
- Linear variation of c with depth. Find $\theta(y)$ when wave is horizontal at depth y_0
  - Linear variation: $c(y)=ay$
  - Boundary condition: $\sin\theta(y_0)=1 \Rightarrow C_1=1/ay_0$



- Solution:

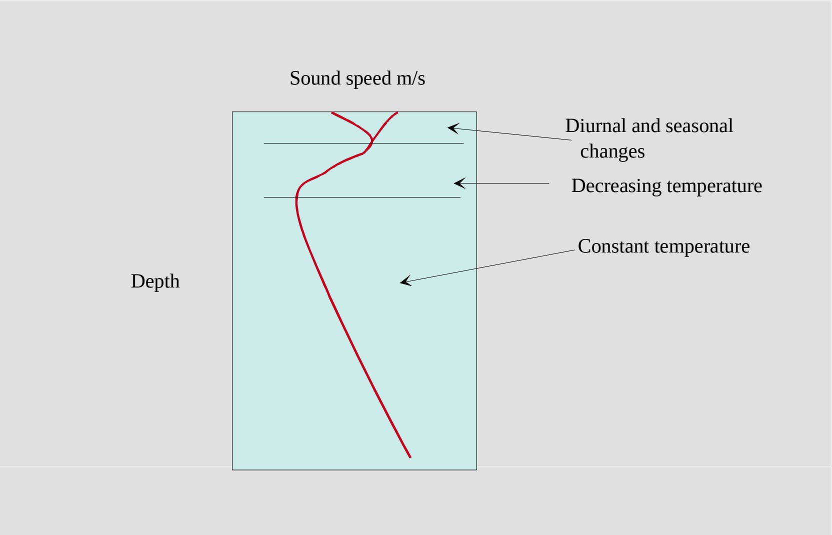
$$\sin\theta = C_1 c(y) = ay/ay_0 = y/y_0$$

- A circle with radius y_0



Underwater acoustics: Sound speed profiles

$$c = 1448.6 + 4.618T - 0.0523T^2 + 1.25(S - 35) + 0.017D \quad .$$



Empirical law:

- c = velocity of sound (m/s),
- T = temperature (^{o}C),
- S = salinity (per thousand, promille),
- D = depth (m).

Fra: J Hovem, TTT4175
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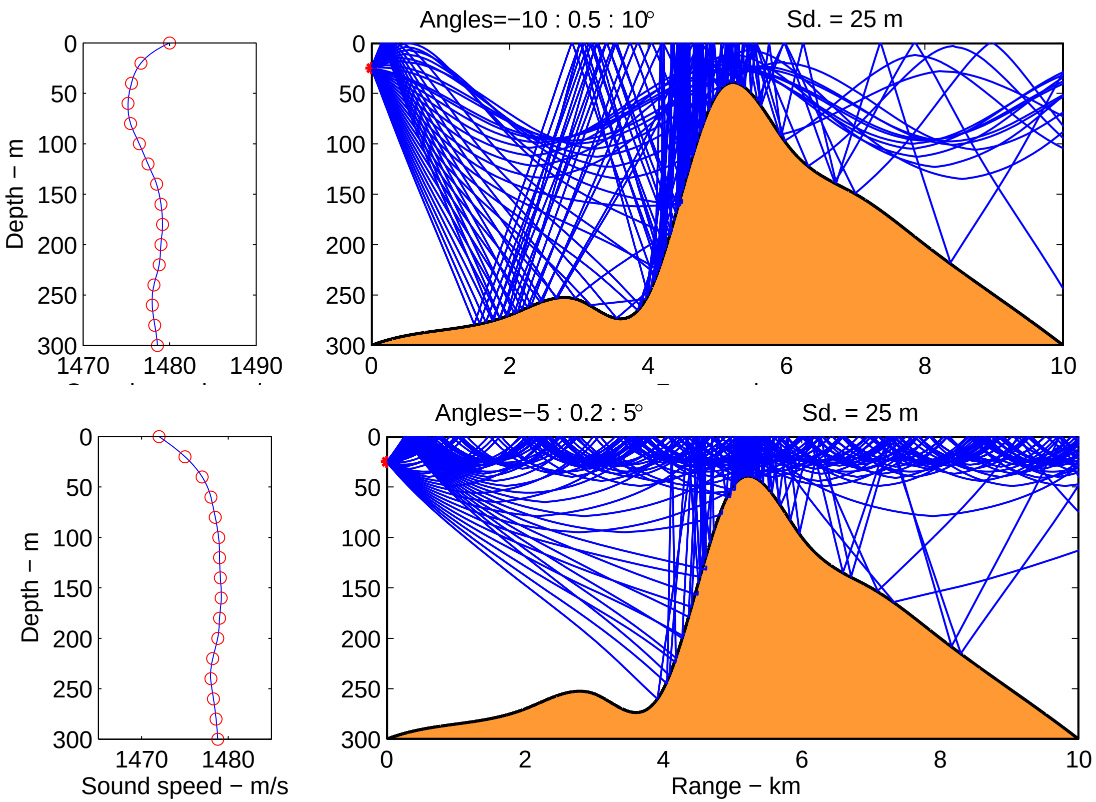
Sound propagation: underwater peak



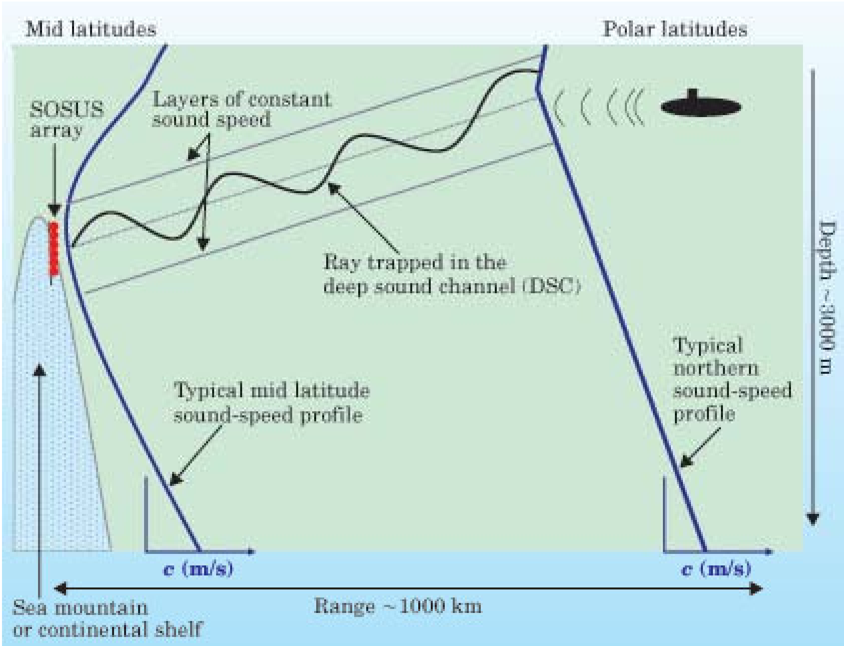
Summer

Winter

From: J Hovem, TTT4175
Marin akustikk, NTNU

Deep sound channel

- c decreases as the water cools but increases with depth.
- Deep sound channel (DSC)
- From the cold surface at the poles to ~1300 m at the equator
- Sound can propagate thousands of kilometers
- 1950s: US Navy SOSUS (Sound Ocean Surveillance System) network to monitor Soviet submarines.
- Kuperman and Lynch, “Shallow-water acoustics”, Physics Today, 2004



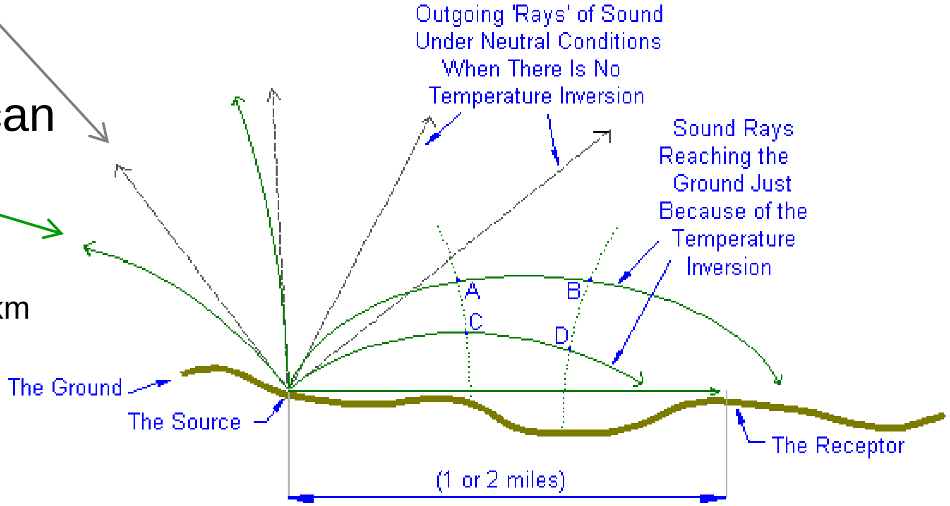
Bending of sound in air

- c ≈ 331.4 + 0.6·T
  around room temperature
- Ex: T=20 C => c=343,4 m/s

- Usually T falls with height
  - Sound is bent out into space.

- Inversion: opposite => Sound can be heard over much longer distances
  - Elephants at sunrise and dawn: range of infrasound increases from 1-2 km to 10 km
  - Inversion layer also makes polluted air visible

http://www.aftenposten.no/nyheter/iriks/article4223421.ece



Sound refraction in wind

v_{wind}

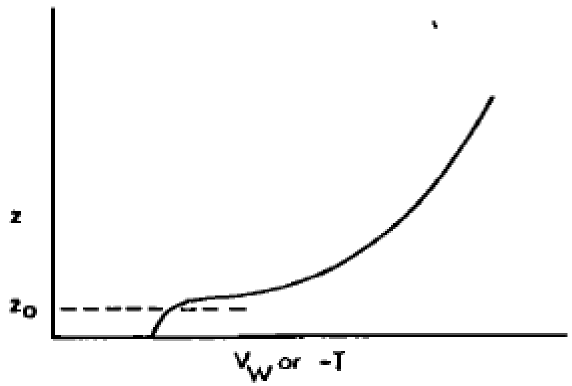


FIG. 12. Variation of wind velocity and temperature in the vicinity of a flat ground surface ($z < \sim 10$ m).

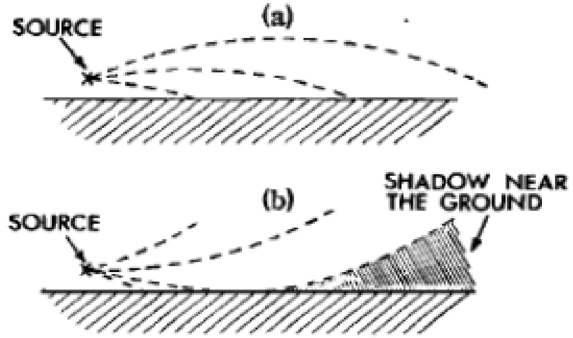
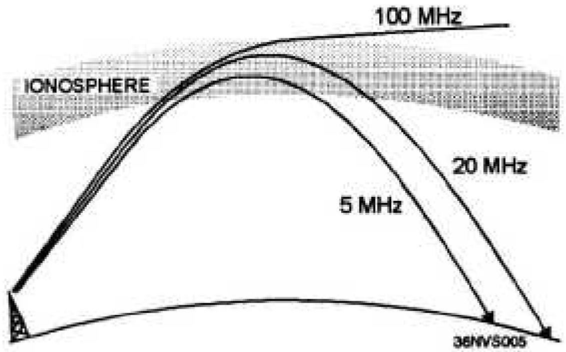
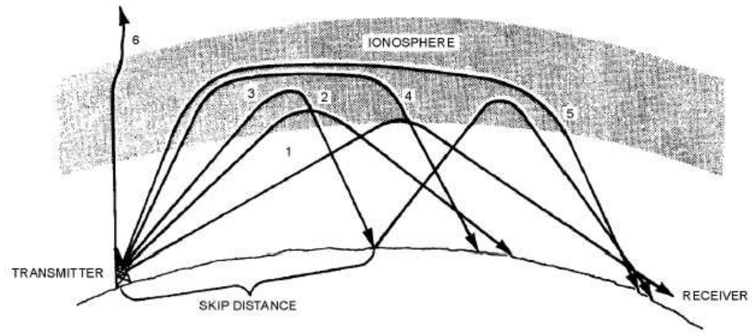


FIG. 14. (a) Refraction downward—inversion or downwind propagation. (b) Refraction upwards—lapse or upwind propagation.

- Effective $c = c_0 + v_{wind}(z)$
  - Downwind, tailwind (medvind): higher c for higher height, z: refraction towards ground: louder traffic noise from highway (like inversion)
  - Upwind, headwind (motvind): lower c for higher height, z: refraction away from ground: weaker noise
  - Piercy et al., Review of noise propagation in the atmosphere, JASA, 1977

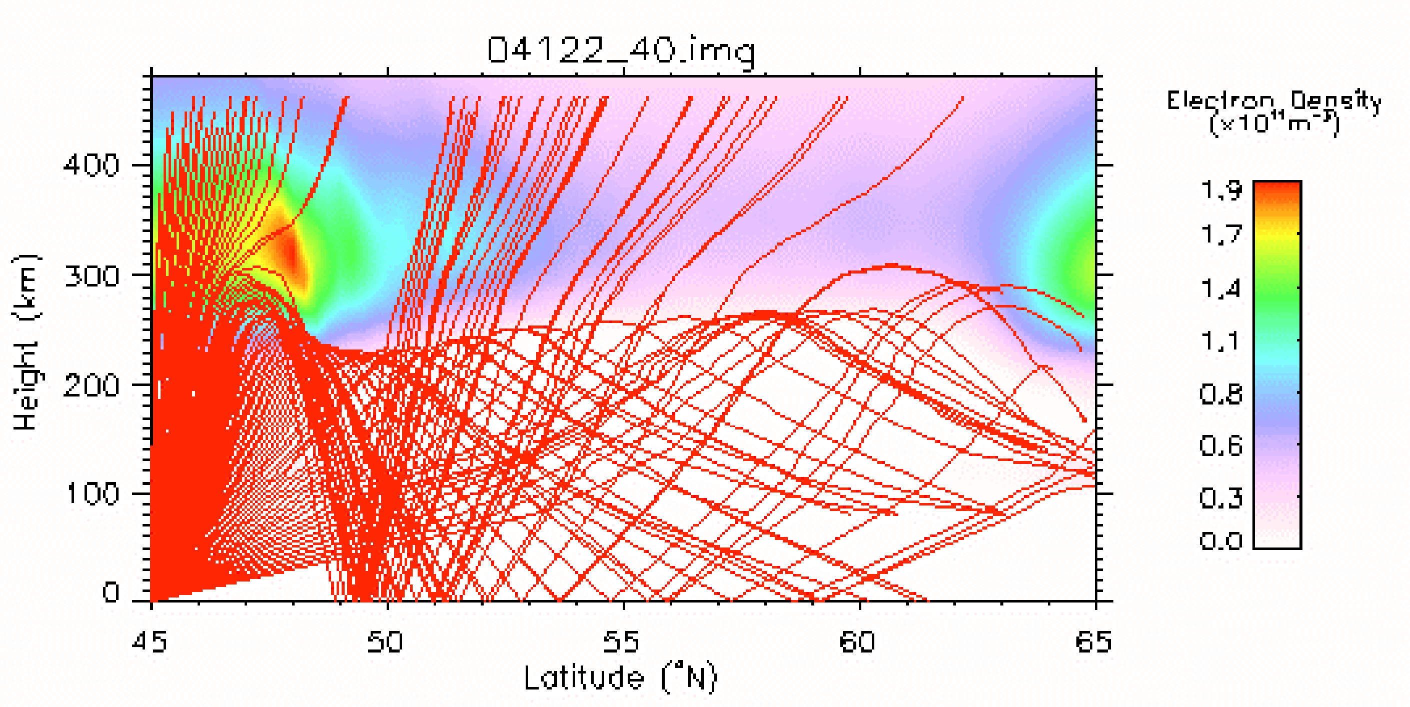
Radio waves – bending in ionosphere





- Refraction and dispersion
- Index of refraction: $n=c/v_p < 1$, i.e. $v_p > c$ ($v_p \cdot v_g = c^2$)

Ray tracing - ionosphere



http://www.cpar.qinetiq.com/raytrace.html

2.4.2 Ray Theory

- Method for finding ray path based on geometry alone = high frequency approximation
- Read the details in the book if you need to understand better underwater acoustics or modeling of the ionosphere!

Periodic media

250-mm-long steel cylinder rods
lattice constant *a=2.5 mm*
radius of cylinders: *R=1.0 mm*
Csteel =6100 m/s, *cair=334.5 m/ s*
f = 41.2 to 48.0 kHz

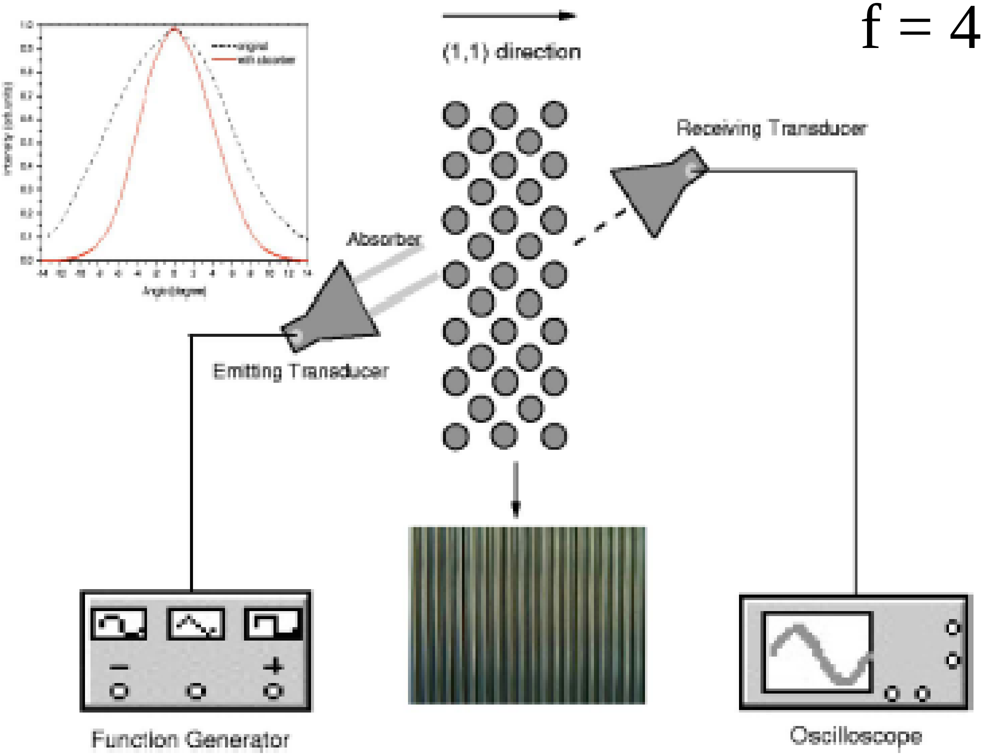
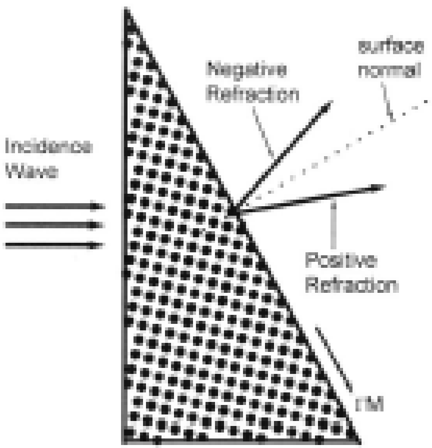


FIG. 2. (Color online) Schematic of the experimental setup used to measure the transmission of ultrasonic wave in a SC, consisting of two transducers, a flat rectangular slab of steel cylinders, a function generator, and an oscilloscope. The SC, the rectangular slab of steel cylinders shown as the middle-bottom inset, is placed between two transducers and the left-top inset shows measurements of the amplitude comparisons with the absorber (solid) to without the absorber (dashed).

Feng, Liu, Chen, Huang, Mao, Chen, Li, Zhu, Negative refraction of acoustic waves in two-dimensional **sonic crystals**, Physical Review B, 2005

Periodic media



- Zhanga, Liu: Negative refraction of acoustic waves in two-dimensional **phononic crystals**, Applied Physics Letters, 2004.

- **Acoustic metamaterials** can manipulate sound waves in surprising ways, which include collimation, focusing, cloaking, sonic screening and extraordinary transmission.
- Recent theories suggested that imaging below the diffraction limit using passive elements can be realized by acoustic superlenses or magnifying hyperlenses. These could markedly enhance the capabilities in underwater sonar sensing, medical ultrasound imaging and non-destructive materials testing.
- Li et al, Experimental demonstration of an acoustic magnifying hyperlens, Nature 2009

Array Processing Implications

- Spatial inhomogeneities must be taken into account by array processing algorithms
  - The essence of matched field processing
- Waves propagating in an inhomogenous medium rarely travel in a straight line
  - Makes array processing/beamforming much harder
- Refraction can lead to multipath
  - Can be modeled as a low-pass filter, e.g. loss of high frequencies

Diffraction

- Ray theory: Geometrical model of optics
- High-frequency – small wavelength model
- Diffraction:
  - Wavelength comparable to structure size
  - Edges of shadows are not perfectly sharp
  - Can hear around corners
- In this course: mainly consequences of diffraction

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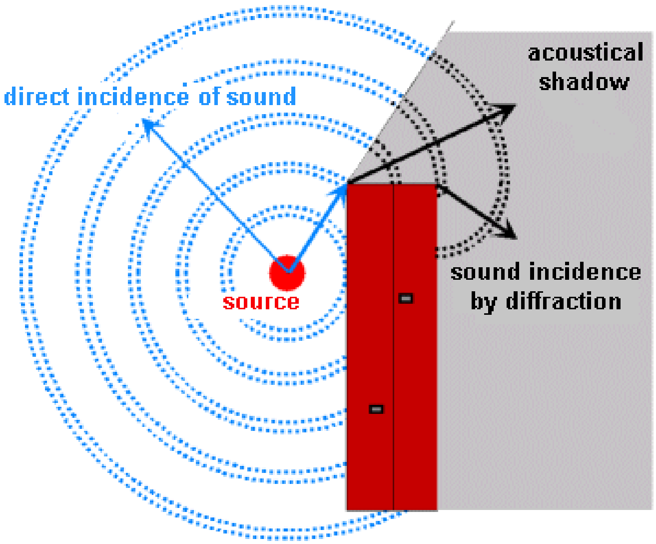
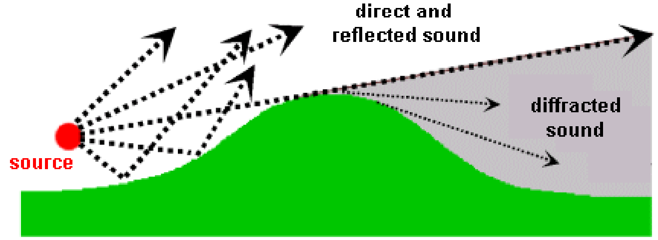
Diffraction – (spredning)





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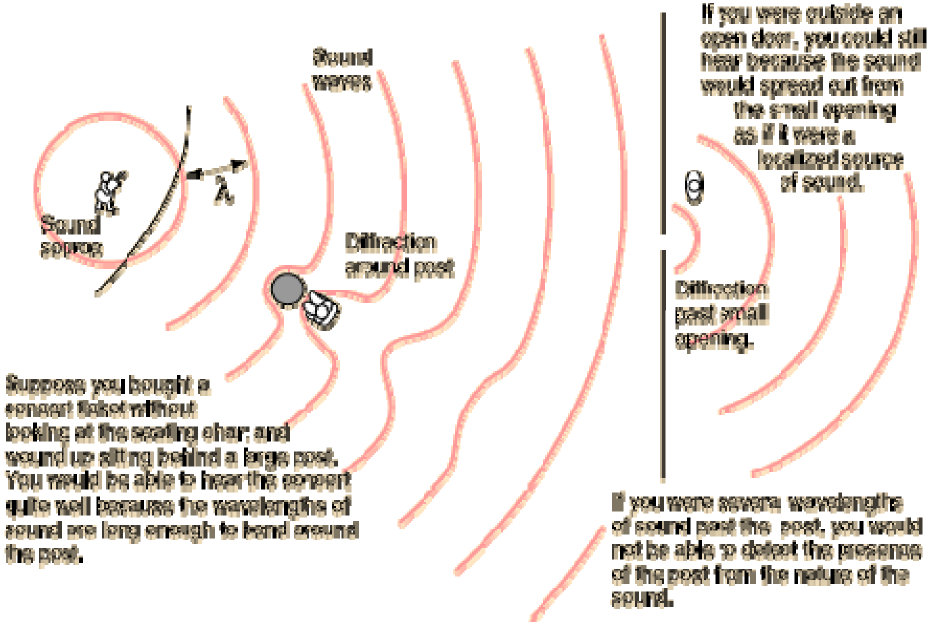
Diffraction



- Geometric acoustic is OK for dimensions > 1 wavelength

Huygens' principle

- Christian Huygens, NL, 1629-1695
- Each point on a travelling wavefront can be considered as a secondary source of spherical radiation
- Also a model for an oscillating piston = acoustic source

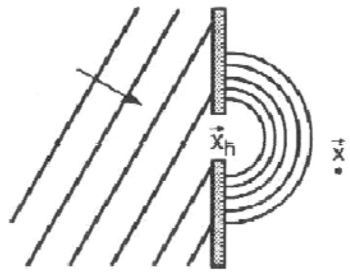


Figure 2.13 A wave is shown impinging on a hole in a planar screen. The Rayleigh-Sommerfeld diffraction formula tells us what the wavefield at the point $\vec{x}$ is in terms of the wavefield at the aperture.

Mathematical formulation of diffraction

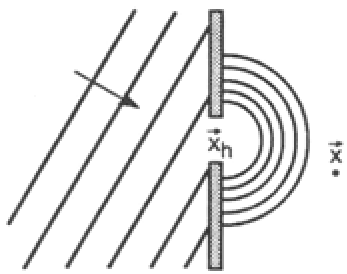
- Augustin Jean Fresnel (F) 1788 – 1827
- Gustav Robert Kirchhoff (D) 1824 – 1887
- Lord Rayleigh, John William Strutt (GB) 1842 – 1919, Nobel prize physics, 1904.
- Arnold Johannes Wilhelm Sommerfeld (D) 1868 – 1951
- Joseph von Fraunhofer (D) 1787 - 1826

Diffraction: deviation from geometrical model

- Rayleigh-Sommerfeld diffraction formula from a hole with aperture A:

$$s(\overrightarrow{x}) = \frac{1}{j\lambda} \int\int_A s(\overrightarrow{x}_h) \frac{\exp\{jkr\}}{r} \cos\theta dA$$

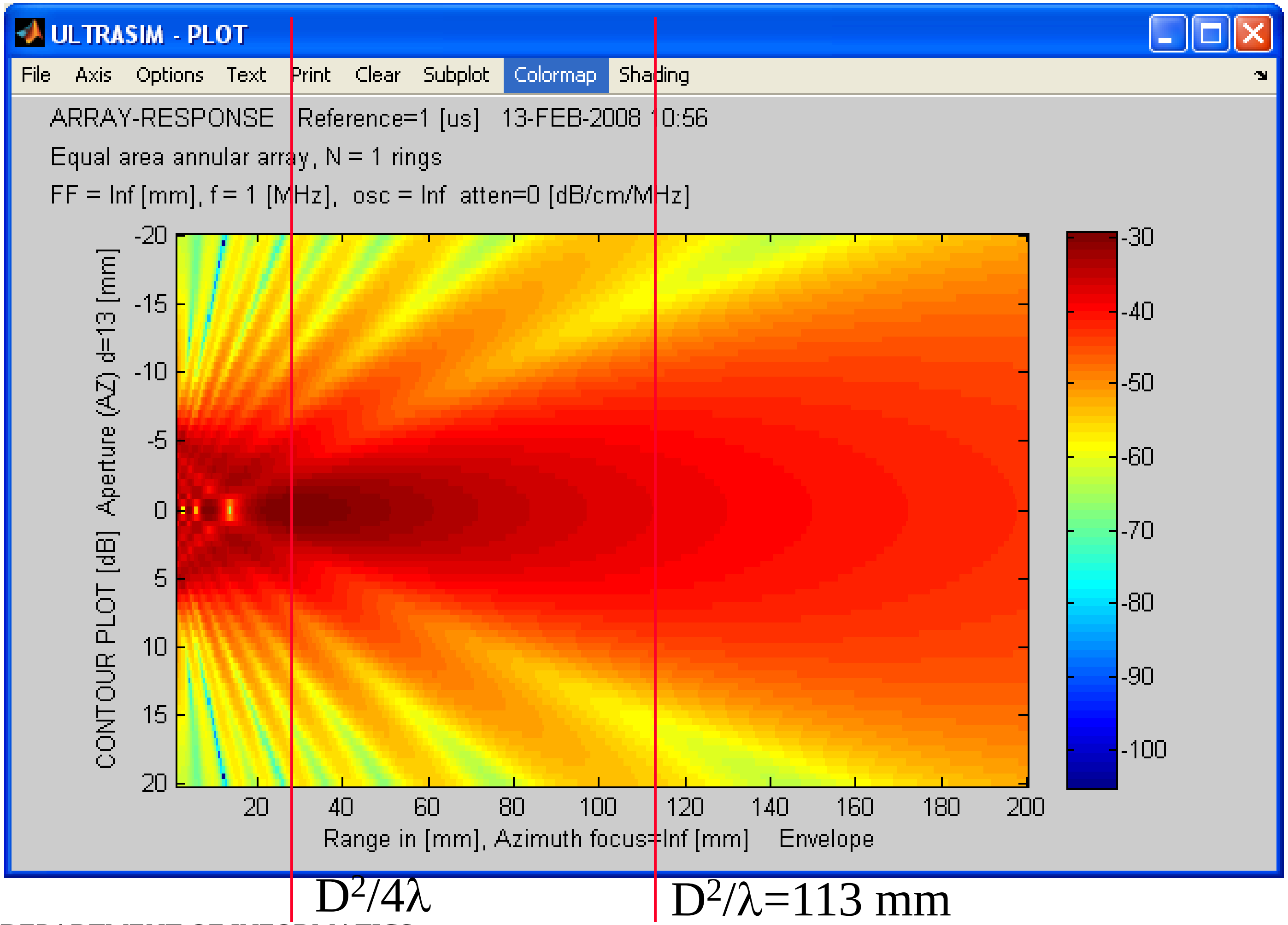
  - Wave at x is a superposition of fields from the hole, due to linearity of wave equation
  - Weighted by a spherical spreading function exp{jkr}/r
  - Also weighted by 1/λ
  - Obliquity factor cosθ
  - Phase shift of π/2 due to 1/j



Two approximations

- Fresnel, nearfield, (but not quite near)
- Fraunhofer, farfield
- Leads to
  - important estimates for nearfield – farfield transition distance
  - Fourier relationship between aperture excitation and field

1 MHz 13 mm, unfocused xdcr



Olympus-Panametrics A303S
(in our lab)

Simulation:
http://www.ifi.uio.no/~ultrasim

Fresnel approximation

$$s(\overrightarrow{x}) = \frac{1}{j\lambda}\int\int_A s(\overrightarrow{x}_h)\frac{\exp\{jkr\}}{r}\cos\theta dA$$

- cos θ ≈ 1, r ≈ d for amplitude
- Phase:
  - spherical surfaces ≈ quadratic
  - parabolic approximation

Fresnel derivation

- Point in the hole $(\tilde{x}, \tilde{y}, 0)$, in observation plane **(x,y,z=d)**
- Distance: $r = [(x-\tilde{x})^2 + (y-\tilde{y})^2 + d^2]^{1/2}$

$$r = d[1 + \frac{(x-\tilde{x})^2 + (y-\tilde{y})^2}{d^2}]^{1/2}$$

- Approximate $(1+x)^{1/2} \approx 1+x/2$, i.e. small x/D ⇔ small angles

$$r \approx d + \frac{(x-\tilde{x})^2 + (y-\tilde{y})^2}{2d}$$

- Use the above expression for the phase and $r \approx d$ for the amplitude in Rayleigh-Sommerfeld integral

Fresnel approximation

$$s(x,y) = \frac{exp\{jkd\}}{j\lambda d} \int\int_A s(\tilde{x},\tilde{y}) \exp\{\frac{jk[(x-\tilde{x})^2 + (y-\tilde{y})^2]}{2d}\} d\tilde{x} d\tilde{y}$$

- Nearfield approximation & within ≈15º of z-axis
- Also called paraxial approximation
- 2D convolution between field in hole and h(x,y):

$$h(x,y) = \frac{exp\{jkd\}}{j\lambda d} \exp\{\frac{jk(x^2+y^2)}{2d}\}$$

- This is a quadratic phase function = the phase shift that a secondary wave encounters during propagation

Fraunhofer approximation

- Expand phase term of Fresnel approximation

$$s(x,y) = \frac{exp\{jkd\}}{j\lambda d} \int\int_A s(\tilde{x},\tilde{y}) \, \mathrm{exp}\,\{\frac{jk[(x-\tilde{x})^2 + (y-\tilde{y})^2]}{2d}\} d\tilde{x} d\tilde{y}$$

- and neglect quadratic phase term variation over hole

$$(x-\tilde{x})^2 + (y-\tilde{y})^2 = x^2 + y^2 - 2x\tilde{x} - 2y\tilde{y} + \tilde{x}^2 + \tilde{y}^2 \approx x^2 + y^2 - 2x\tilde{x} - 2y\tilde{y}$$

- If D = max linear dimension of hole, this is equivalent to assuming (d=dist. from source):

$$\frac{\tilde{x}^2}{2d} \leq \frac{(D/2)^2}{2d} << \lambda/2 => d >> \frac{D^2}{4\lambda}$$ $\leftarrow$ Fresnel limit

Fraunhofer approximation

$$s(x,y) = \frac{exp\{jkd\}}{j\lambda d} \exp\{\frac{jk(x^2+y^2)}{2d}\} \int\int_A s(\tilde{x},\tilde{y}) \exp\{\frac{jk(x\tilde{x}+y\tilde{y})}{d}\} d\tilde{x} d\tilde{y}$$

- Far-field approximation: valid far away from hole
- s(x,y) = 2D Fourier transform of field in hole

Fourier transform relationship

- Very important result
- *Link* between the physics and the signal processing!
- Basis for simplified expressions like angular resolution $\approx \lambda/D$ etc
- Small hole leads to wide beam and vice versa just like a short time-function has a wide spectrum

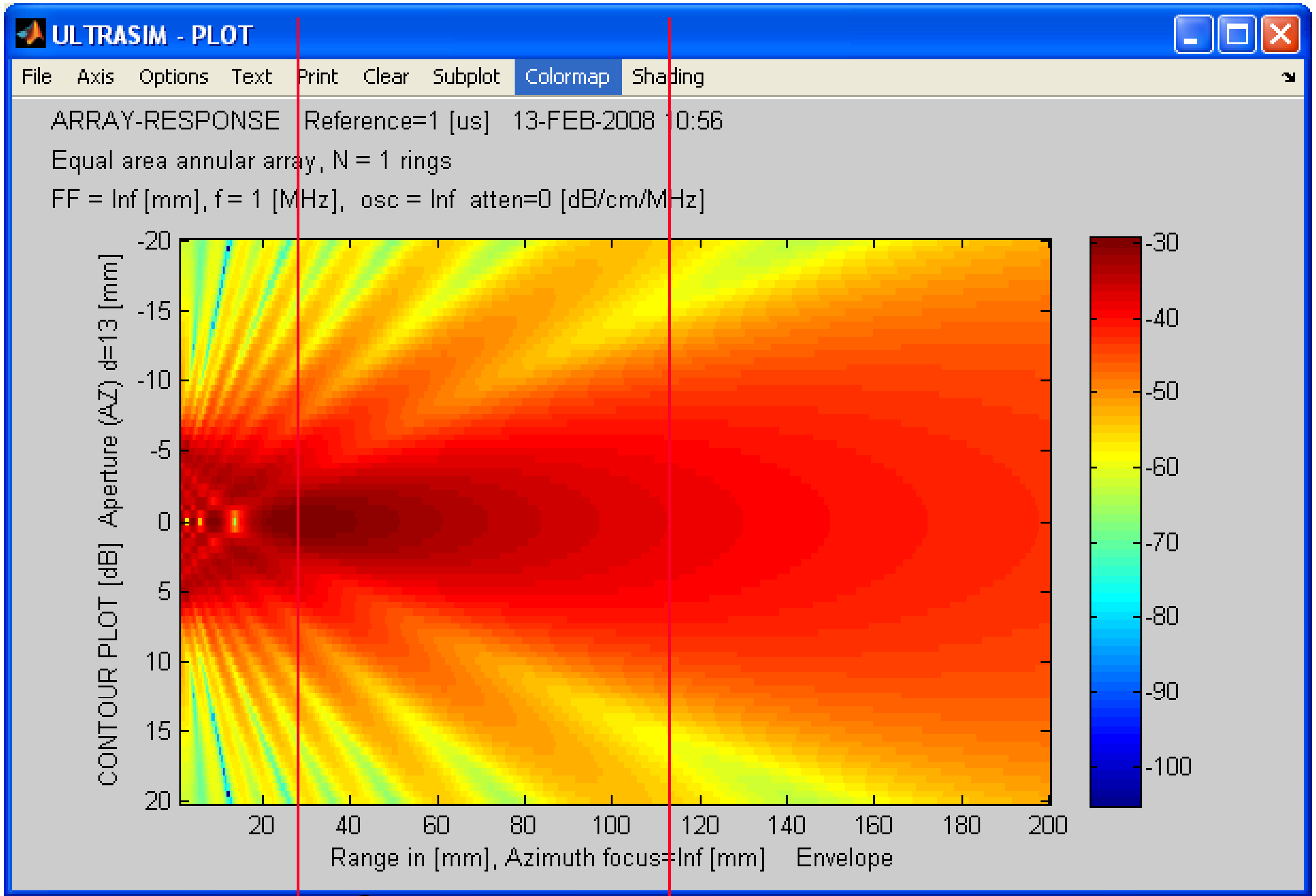
Nearfield-farfield limit

Not a clear transition, several limits are used, in increasing size:

- $d_F = D^2/4\lambda$: Fresnel limit
- $d = \pi r^2/\lambda = \pi/4 \cdot D^2/\lambda$: Diffraction limit
- $d = D^2/\lambda$: max path length difference $\lambda/8$
- $d_R = 2D^2/\lambda$: Rayleigh dist: Δ path = $\lambda/16$

- Proportional to D^2/λ, multiplied by 0.25, 0.79, 1, or 2

1 MHz 13 mm, unfocused xdcr



$D^2/4\lambda$ $D^2/\lambda=113$ mm

Olympus-Panametrics A303S
(in our lab)

Simulation: http://www.ifi.uio.no/~ultrasim

Array Processing Implications

- Diffraction means that opaque objects located between the source and the array can induce complicated wavefields
  - Scattering theory:
    - » Acoustics: Schools of fish
    - » Electromagnetics: rain drops
    - » Complicated, but important to understand

Norsk terminologi

- Bølgeligningen
- Planbølger, sfæriske bølger
- Propagerende bølger, bølgetall
- Sinking/sakking: $\vec{\alpha}$
- Dispersjon
- Attenuasjon eller demping
- Refraksjon
- Ikke-linearitet
- Diffraksjon; nærfelt, fjernfelt
- Gruppeantenne (= array)

Kilde: Bl.a. J. M. Hovem: ``Marin akustikk'', NTNU, 1999