INF5820/INF9820

LANGUAGE TECHNOLOGICAL APPLICATIONS

Jan Tore Lønning, 2014 fall

Probability theory

Exercises

- Khan academy
- http://www.statlect.com/probability_exercises.htm
  - Probability
  - Conditional probability
  - Bayes' rule
  - Independent events
- S1, videregående skole:
  - http://ndla.no/nb/node/57934

Basic concepts

- Random experiment (or trial) (no: forsøk)
  - Observing a chance event
- Outcomes (utfallene)
  - The possible results of the experiment
- Sample space (utfallsrommet)
  - The set of all possible outcomes

Examples

	Experiment	Sample space, Ω
1	Flipping a coin	{H, T}
2	Rolling a dice	{1,2,3,4,5,6}
3	Flipping a coin three times	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
4	Will it rain tomorrow?	{Yes, No}

Examples

	Experiment	Sample space, Ω
1	Flipping a coin	{H, T}
2	Rolling a dice	{1,2,3,4,5,6}
3	Flipping a coin three times	{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
4	Will it rain tomorrow?	{Yes, No}
5	The first word you hear tomorrow	{u | u is a word}
6	Throwing a dice until you get 6	{1,2,3,4, …}
7	The maximum temperature at Blindern for a day	{t | t is a real}

Discrete		Continuous
Finite	Infinite	Infinite
1-4	6	7

Event

- An event (begivenhet) is a set of elementary outcomes

	Experiment	Event	Formally
2	Rolling a dice	Getting 5 or 6	{5,6}
3	Flipping a coin three times	Getting at least two heads	{HHH, HHT, HTH, THH}

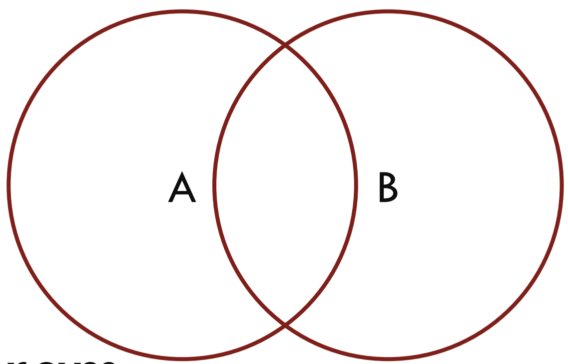
Event

- An event (begivenhet) is a set of elementary outcomes

	Experiment	Event	Formally
2	Rolling a dice	Getting 5 or 6	{5,6}
3	Flipping a coin three times	Getting at least two heads	{HHH, HHT, HTH, THH}
5	The first word you hear tomorrow	You hear a noun	{u | u is a noun}
6	Throwing a dice until you get 6	An odd number of throws	{1,3,5, …}
7	The maximum temperature at Blindern	Between 20 and 22	$\{t \mid 20 \leq t \leq 22\}$

Operations on events

- Union: $A \cap B$
- Intersection (snitt): $A \cap B$
- Complement



- Venn diagram
- http://www.google.com/doodles/john-venns-180th-birthday

Probability measure, sannsynlighetsmål

- A probability measure P is a function from events to the interval [0,1] such that:

1. $P(\Omega) = 1$
2. $P(A) \geq 0$
3. If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$
   - And if A1, A2, A3, … are disjoint, then $P\left(\bigcup_{j=1}^{\infty} A_j\right) = \sum_{j=1}^{\infty} P(A_j)$

Examples

- An event (begivenhet) is a set of elementary outcomes

	Experiment	Event	Probability
2	Rolling a dice	Getting 5 or 6	P({5,6})=2/6=1/3
3	Flipping a coin three times	Getting at least two heads	P({HHH, HHT, HTH, THH}) = 4/8

Examples

- An event (begivenhet) is a set of elementary outcomes

	Experiment	Event	Probability
2	Rolling a dice	Getting 5 or 6	$P(\{5,6\})=2/6=1/3$
3	Flipping a coin three times	Getting at least two heads	$P(\{HHH, HHT, HTH, THH\}) = 4/8$
5	The first word you hear tomorrow	You hear a noun	$P(\{u \mid u \text{ is a noun}\})= 0.43$?
6	Throwing a dice until you get 6	An odd number of throws	$P(\{1,3,5, \ldots\})=$?
7	The maximum temperature at Blindern	Between 20 and 22	$P(\{t \mid 20 \leq t \leq 22\})=0.1$

Some observations

- $P(\emptyset) = 0$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If Ω is discrete, then $P(A) = \sum_{a \in A} P(\{a\})$

Examples

6-sided fair dice

- Two throws: the probability of 2 sixes?
- The probability of getting a six in two throws?
- 5 dices: the probability of getting 5 equal dices?
- 5 dices: the probability of getting 1-2-3-4-5?
- 5 dices: the probability of getting no 6-s?

Counting methods

Given all outcomes equally likely

- P(A) = number of ways A can occur/
  total number of outcomes
- Mutiplication principle:
  if one experiment has m possible outcomes
  and another has n possible outcomes,
  then the two have mn possible outcomes

Sampling

Ordered sequences:

- Choose *k* items from a population of *n* items with replacement: n^k
- Without replacement: n(n-1)(n-2)...(n-k+1)=$\frac{n!}{(n-k)!}$

Unordered sequences

- Without replac.: $\frac{1}{k!}\left(\frac{n!}{(n-k)!}\right) = \left(\frac{n!}{k!(n-k)!}\right) = \binom{n}{k}$
  - = the number of ordered sequences/ The number of ordered sequences containing the same *k* elements

Conditional probability

- $P(A \cap B)$
  - Both A and B happens
- Conditional probability (betinget sannsynlighet)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

  - The probability of A happens if B happens
- Multiplication rule $P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A)$
- A and B are independent iff $P(A \cap B) = P(A)P(B)$

Example

- Throwing two dice
  - A: the sum of the two is 7
  - B: the first dice is 1
    - $P(A) = 6/36 = 1/6$
    - $P(B) = 1/6$
    - $P(A \cap B) = P(\{(1,6)\}) = 1/36 = P(A)P(B)$
    - Independence
  - C: the sum of the two is 5
    - $P(C) = 4/36 = 1/9$
    - $P(C \cap B) = P(\{(1,4)\}) = 1/36$
    - $P(C)P(B) = 1/9 * 1/6 = 1/54$
    - Not independent

Bayes theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Jargon:
  - P(A) – prior probability
  - P(A|B) – posterior probability

Example

- Disease: 1 out of 1000 are infected
- Test:
  - Detects 99% of the infected
  - 2% of the non-infected get a positive test
- Given a positive test: what is the chance you are ill?

What are probabilities?

- Example throwing a dice:

1. Classical view:
   - The six outcomes are equally likely
2. Frequenist:
   - If you throw the dice many, many, many times, the number of 6s approach 16.6666…%
3. Bayesian: subjective beliefs