## INF5830-2015 FALL NATURAL LANGUAGE PROCESSING

Jan Tore Lønning, Lecture 12, 2.11

## Today

$\square$ Feature selection 1 (Oblig 2)
$\square$ Scikit-Learn from NLTK
$\square$ Linear classifiers
$\square$ Naive Bayes is log linear
$\square$ Logistic Regression
$\square$ Multinomial Logistic Regression $=$ Maximum Entropy Classifiers

## Machine Learning



Selecting
Cleaning
Tokenization
Lemmatizing?
"Munging"

Feature
Selection
Arguably the
most important
step for the
results

## Example: Word Sense Disambiguation

An electric guitar and bass player stand off to one side, not really part of the scene,
just as a sort of nod to gringo expectations perhaps.
"Bag of words"-features
$\square$ Features: [fishing, big, sound, player, fly, rod, pound, double, runs, playing, guitar, band]
$\square$ The example: $[0,0,0,1,0,0,0,0,0,0,1,0]$
$\square$ Which words as features? How many?

- Many features
- Boolean values
$\square$ NLTK, initially: The most frequent ones
$\square$ There might be better ways to select (we return to this later)


## Hard-line-serve

| Number of <br> word fectures | Hard |  |  |
| :--- | :--- | :--- | :--- |
| 0 | 0.802 |  |  |
| 10 | 0.768 |  |  |
| 20 | 0.764 |  |  |
| 50 | 0.774 |  |  |
| 100 | 0.800 |  |  |
| 200 | 0.812 |  |  |
| 500 | 0.830 |  |  |
| 1000 | 0.842 |  |  |
| 2000 | 0.846 |  |  |
| 5000 | 0.844 |  |  |

## Hard-line-serve

| Number of <br> word features | Hard | Serve |  |
| :--- | :--- | :--- | :--- |
| 0 | 0.802 | 0.350 |  |
| 10 | 0.768 | 0.550 |  |
| 20 | 0.764 | 0.622 |  |
| 50 | 0.774 | 0.692 |  |
| 100 | 0.800 | 0.728 |  |
| 200 | 0.812 | 0.766 |  |
| 500 | 0.830 | 0.784 |  |
| 1000 | 0.842 | 0.794 |  |
| 2000 | 0.846 | 0.802 |  |
| 5000 | 0.844 | 0.804 |  |

## Hard-line-serve

| Number of <br> word features | Hard | Serve | Line |
| :--- | :--- | :--- | :--- |
| 0 | 0.802 | 0.350 | 0.528 |
| 10 | 0.768 | 0.550 | 0.528 |
| 20 | 0.764 | 0.622 | 0.534 |
| 50 | 0.774 | 0.692 | 0.576 |
| 100 | 0.800 | 0.728 | 0.688 |
| 200 | 0.812 | 0.766 | 0.706 |
| 500 | 0.830 | 0.784 | 0.744 |
| 1000 | 0.842 | 0.794 | 0.774 |
| 2000 | 0.846 | 0.802 | 0.802 |
| 5000 | 0.844 | 0.804 | 0.826 |

## Collocational features

An electric guitar and bass player stand off to one side, not really part of the scene, just as a sort of nod to gringo expectations perhaps.
$\square$ With tags:
$\square\left[w_{i-2}, \mathrm{POS}_{i-2}, w_{i-1}, \mathrm{POS}_{i-1}, w_{i+1}, \mathrm{POS}_{i+1}, w_{i+2}, \mathrm{POS}_{i+2}\right]$
$\square$ Example: [guitar, NN, and, CC, player, NN, stand, VB]
$\square$ Without tags:
$\square\left[w_{i-2}, w_{i-1}, w_{i+1}, w_{i+2}\right]$

- Few features
- Many possible values
$\square$ Example: [guitar, and, player, stand]


## Window size (without tags)

| Words on each <br> side | Hard | Serve | Line |
| :--- | :--- | :--- | :--- |
| 0 | 0.802 | 0.350 | 0.528 |
| 1 | 0.898 | 0.742 | 0.734 |
| 2 | 0.886 | 0.818 | 0.772 |
| 3 | 0.868 | 0.856 | 0.776 |
| 4 | 0.864 | 0.856 | 0.782 |
| 5 | 0.854 | 0.858 | 0.768 |

## Both BoW and Colloc. features

## Line.pos

 BOW features| 0 | 0.528 | 0.734 | 0.772 | 0.776 | 0.782 | 0.768 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 10 | 0.528 | 0.724 | 0.788 | 0.780 | 0.780 | 0.772 |
| 20 | 0.534 | 0.758 | 0.772 | 0.770 | 0.796 | 0.776 |
| 50 | 0.576 | 0.764 | 0.800 | 0.800 | 0.804 | 0.796 |
| 100 | 0.688 | 0.788 | 0.814 | 0.834 | 0.818 | 0.816 |
| 200 | 0.706 | 0.784 | 0.814 | 0.828 | 0.830 | 0.814 |
| 500 | 0.744 | 0.814 | 0.848 | 0.836 | 0.852 | 0.842 |
| 1000 | 0.774 | 0.836 | 0.860 | 0.864 | 0.854 | 0.848 |
| 2000 | 0.802 | 0.846 | 0.866 | 0.864 | 0.872 | 0.874 |
| 5000 | 0.826 | 0.872 | 0.886 | 0.886 | 0.894 | 0.890 |

## Today

$\square$ Feature selection 1 (Oblig 2)
$\square$ Scikit-Learn from NLTK
$\square$ Linear classifiers
$\square$ Naive Bayes is log linear
$\square$ Logistic Regression
$\square$ Multinomial Logistic Regression = Maximum Entropy Classifiers

## Other ML algorithms in NLTK

$\square$ Included:
$\square$ Naive Bayes (Bernoulli)
$\square$ Decision trees
$\square$ Import from Scikit-Learn
$\square$ Example:

- from sklearn.linear_model import LogisticRegression

■ sk_classifier = SklearnClassifier(LogisticRegression())

- sk_classifier.train(train_set)
- Instead of:

■ classifier $=$ nltk.NaiveBayesClassifier.train(train_set)
$\square$ Then use the same set-up as in the oblig

## Scikit-Learn

$\square$ A large set of various ML classification algorithms
$\square$ They can be imported into NLTK
$\square$ In general faster than NLTK's algorithms
$\square$ Beware how the features are selected/formulated:
$\square$ They may be reformulated/altered when translated into Scikit
$\square$ Example:
$\square$ SklearnClassifier(BernoulliNB()) performed inferior to nltk.NaiveBayesClassifier when we used the NLTK-features

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## Geometry: lines

$\square$ Descartes

- (1596-1650)
$\square$ Line:
$\square a x+b y+c=0$
$\square$ If $b \neq 0$ :
$\square y=m x+n$
$\square \mathrm{n}=-\mathrm{c} / \mathrm{b}$ is
the intercept with the $y$ -
 axis
$\square m=-a / b$ is the slope
$\square$ A point $=$ intersection of two lines

$$
\begin{aligned}
& y=-2 x+5 \\
& 4 x+2 y-10=0
\end{aligned}
$$

## Normal vector of a line

$\square \cos (\pi / 2)=0$
$\square$ If P passes through $(0,0)$ there is an $\mathbf{n}=$ $\left(x_{n}, y_{n}\right)$ s.t.
$\square(x, y)$ is on P iff
$\square(x, y) \bullet\left(x_{n}, y_{n}\right)=0$
$\square x \times x_{n}=-y \times y_{n}$
$\square$ If $(a, b) \neq(0,0)$ is on $P$ :
n $=s \times(b,-a)$ for some S


Vector $(2,5)$ is normal to the line $y=-2 x / 5$

## Example:

$$
\begin{aligned}
& y=-2 x / 5 \\
& 2 x+5 y=0 \\
& (x, y) \bullet(2,5)=0
\end{aligned}
$$

## Lines not through $(0,0)$

$\square y=-2 x+5$
$\square 2 x+y-5=0$
$\square(x, y) \bullet(2,1)=5$


## Geometry: planes

$\square$ Plane:
$\square a x+b y+c z+d=0$
$\square$ If $c \neq 0$ :
$\square \mathrm{z}=\mathrm{mx}+\mathrm{ny}+\mathrm{n}$
$\square$ A line is the intersection of two planes


http://www.univie.ac.at/future.media/mo
e/galerie/geom2/geom2.html\#eb

## Normal vector of a plane

$\square$ All points $(x, y, z)$ where
$\square\left((x, y, z)-\left(x_{0}, y_{0}, z_{0}\right)\right) \bullet(a, b, c)=0$
$\square(x, y, z) \bullet(a, b, c)=d$
$\square\left(d=a x_{0}+b y_{0}+c z_{0}\right)$
$\square$ Hyperplane
$\square \mathrm{w}_{0}+\mathrm{w}_{1} \mathrm{x}_{1}+\mathrm{w}_{2} \mathrm{x}_{2}+\ldots+\mathrm{w}_{\mathrm{n}} \mathrm{x}_{\mathrm{n}}=0$
$\square\left(w_{1}, w_{2}, \ldots, w_{n}\right) \bullet\left(x_{1}, x_{2}, \ldots x_{n}\right)=-w_{0}$
$\square$ Sometimes ( $\mathrm{n}+1$ dimensions):
$\square\left(w_{0}, w_{1}, w_{2}, \ldots, w_{n}\right) \bullet\left(1, x_{1}, x_{2}, \ldots x_{n}\right)=0$


## Hyperplanes

$\square$ Generalizes to higher dimensions
$\square \ln n$-dimensional space ( $x_{1}, x_{2}, \ldots, x_{n}$ ):

- Points satisfying:
$\square w_{0}+w_{1} x_{1}+w_{2} x_{2}+\ldots+w_{n} x_{n}=0$
- for any choice of $w_{0}, w_{1}, w_{2}, \ldots w_{n}$
- where not all of $w_{1}, w_{2}, \ldots w_{n}=0$
$\square$ is called a hyper-plane
$\square$ (In machine learning) the same as the intersection of two hyper-planes in $n+1$ dimensional space:
$\square w_{0} x_{0}+w_{1} x_{1}+w_{2} x_{2}+\ldots+w_{n} x_{n}$
$\square \mathrm{x}_{0}=1$


## Linear classifiers

$\square$ Assume:

- All features are numerical (including Boolean)
- Two classes
$\square$ The two classes are linearly separable if they can be separated by a hyperplane
$\square$ In 2 dimensions that is a line:
$\square a x+b y<c$ for red points
$\square a x+b y>c$ for blue points




## Linear classifiers

$\square$ A linear classifier introduces a
hyperplane and
classifies accordingly
$\square$ (If the data aren't linearly separable, the classifier will make mistakes).



## Linear classifiers - general case

$\square$ Try to separate the classes by a hyperplane

$$
\sum_{i=1}^{M} w_{i} x_{i}=\theta
$$

$\square$ (equivalently $\vec{w} \bullet \vec{x}=\sum_{i=0}^{M} w_{i} x_{i}=0$

- taking $w_{0}=-\theta$ and $x_{0}=1$ )
$\square$ The object represented by
$\left(x_{1}, x_{2}, \ldots, x_{n}\right)$
$\square$ is in C if and only if $\sum_{i=1}^{M} w_{i} x_{i}>\theta$

$\square$ and in -C if $\sum_{i=1}^{M} w_{i} x_{i}<\theta$


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## Naive Bayes is a log linear classifier

$$
\begin{aligned}
& \hat{C}=\underset{c \in\left\{c_{1}, c_{2}\right\}}{\arg } \max P(C) \prod_{j=1}^{n} P\left(f_{j} \mid C\right) \\
& P\left(C_{1}\right) \prod_{j=1}^{n} P\left(f_{j} \mid C_{1}\right)>P\left(C_{2}\right) \prod_{j=1}^{n} P\left(f_{j} \mid C_{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{P\left(c_{1}\right) \prod_{j=1}^{n} P\left(f_{j} \mid c_{1}\right)}{P\left(c_{2}\right) \prod_{j=1}^{n} P\left(f_{j} \mid c_{2}\right)}>1 \\
& \frac{P\left(c_{1}\right)}{P\left(c_{2}\right)} \prod_{j=1}^{n} \frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}>1
\end{aligned}
$$

$$
\begin{aligned}
& \log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)} \prod_{j=1}^{n} \frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0 \\
& \log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)+\sum_{j=1}^{n} \log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0 \\
& \sum_{i=1}^{M} w_{i} x_{i}=\theta \quad w_{j}=\log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right) \\
& \theta=-w_{0}=-\log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)
\end{aligned}
$$

## A closer look: The Bernoulli model

$$
\log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)+\sum_{j=1}^{n} \log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0 \quad \sum_{i=1}^{M} w_{i} x_{i}=\theta \quad w_{j}=\log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)
$$

$\square$ A feature $x_{i}$ equals 0 or 1 and corresponds to the combination of
$\square$ what we earlier registered as a feature, and
$\square$ the value of such a feature
Example 1 (gender of names, NLTK), where one feature registers the last letter of the name
$\square$ Original view:
$\square$ One (categorical) feature f1

- 26 possible different values: $a, b, c, \ldots, z$
- Current view:
- 26 different features $\times 1, \times 2, \ldots, \times 26$
- Each takes as value 0 or 1
- Exactly one equals 1 , the rest equals 0


## A closer look: The Bernoulli model

$$
\log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)+\sum_{j=1}^{n} \log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0 \quad \sum_{i=1}^{M} w_{i} x_{i}=\theta \quad w_{j}=\log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)
$$

$\square$ A feature $x_{i}$ equals 0 or 1 and corresponds to the combination of
$\square$ what we earlier registered as a feature, and
$\square$ the value of such a feature

## Example 2: text categorization:

$\square$ Original view: one feature $f_{i}$ for a term $t_{i}$ :
$\square f_{i}=1$ if $t_{i}$ is present, $f_{i}=0$ if $t_{i}$ isn't present
$\square$ Current view
$\square$ one term $x_{2 i}$ corresponding to $t_{i}$ being present and one term $x_{2 i+1}$ corresponding to $t_{i}$ being absent
$\square$ One of these equals 1, the other equals 0

## A closer look: the multinomial model

$\square$ The multinomial does not strictly fit the NB-model:

$$
\log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)+\sum_{j=1}^{n} \log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0
$$

$\square$ But it fits the linear model $\sum_{i=1}^{M} w_{j} x_{j}=\theta$

- If
$\square \mathrm{i}$ is the index of a feature term (lexeme) $\mathrm{t}_{\mathrm{i}}$ (not a particular occurrence in a document)
$\square x_{i}$ is the number of occurrrences of $t_{i}$ in the document
$\square$ and $w_{i}$ is

$$
w_{j}=\log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)
$$

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## NB and logistic regression

$\square$ The NB uses a linear expression to decide
$\log \left(\frac{P\left(c_{1} \mid \vec{f}\right)}{P\left(c_{2} \mid \vec{f}\right)}\right)=\log \left(\frac{P\left(c_{1} \mid \vec{f}\right)}{1-P\left(c_{1} \mid \vec{f}\right)}\right)=\vec{w} \bullet \vec{f}=\sum_{i=0}^{M} w_{i} x_{i}=w_{0} x_{0}+\sum_{i=1}^{M} w_{i} x_{i}>0$
$\square$ where

$$
w_{j}=\log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)
$$

$\square$ Are these the best choices for the $\mathrm{w}_{\mathrm{i}}$ ?
$\square$ Logistic regression instead faces the question directly:
$\square$ Which $w_{i}$ s make the best classifier of the form
$\operatorname{logit}\left(P\left(c_{1} \mid \vec{f}\right)\right)=\ln \left(\frac{P\left(c_{1} \mid \vec{f}\right)}{1-P\left(c_{1} \mid \vec{f}\right)}\right)=\vec{w} \bullet \vec{f}=\sum_{i=0}^{M} w_{i} x_{i}=w_{0} x_{0}+\sum_{i=1}^{M} w_{i} x_{i}>0$

## Logistic regression - learning

$\square$ Conditional maximum likelihood estimation:
Choose the model that fits the training data best!

$$
\hat{w}=\underset{w}{\arg \max } \prod_{i=1}^{m} P\left(c^{i} \mid \vec{f}^{i}\right)=\underset{w}{\arg \max } \sum_{i=1}^{m} \log P\left(c^{i} \mid \vec{f}^{i}\right)
$$

$\square$ where:
$\square$ There are $m$ many training data
$\square c^{i}$ is the class of observation i, i.e. $c_{1}$ or $c_{2}$.
$\square$ The feature vector for observation $i$ is: $\vec{f}^{i}=\left(f_{1}^{i}, f_{2}^{i}, \ldots f_{n}^{i}\right)$

## Furthermore

$\square$ To estimate

$$
\hat{w}=\underset{w}{\arg \max } \prod_{i=1}^{m} P\left(c^{i} \mid \vec{f}^{i}\right)=\underset{w}{\arg \max } \sum_{i=1}^{m} \log P\left(c^{i} \mid \vec{f}^{i}\right)
$$

$\square$ we must find the relationship between $w$ and $P\left(c^{i} \mid f^{i}\right)$

$$
\begin{aligned}
& \ln \left(\frac{P\left(c_{1} \mid \vec{f}\right)}{1-P\left(c_{1} \mid \vec{f}\right)}\right)=\vec{w} \bullet \vec{f} \\
& \frac{P\left(c_{1} \mid \vec{f}\right)}{1-P\left(c_{1} \mid \vec{f}\right)}=e^{\bar{w} \bullet \vec{f}} \\
& P\left(c_{1} \mid \vec{f}\right)=\frac{e^{\bar{w} \bullet \vec{f}}}{1+e^{\bar{w} \bullet \vec{f}}} \\
& P\left(c_{1} \mid \vec{f}\right)=\frac{1}{1+e^{-\vec{w} \cdot \vec{f}}}
\end{aligned}
$$



## Learning algorithms

$\square$ There is no analytic solution to

$$
\hat{w}=\underset{w}{\arg \max } \sum_{i=1}^{m} \log P\left(c^{i} \mid \vec{f}^{i}\right) \quad \text { where } \quad P\left(c_{1} \mid \vec{f}\right)=\frac{1}{1+e^{-\vec{w} \bullet \vec{f}}}
$$

$\square$ Use some numeric method which runs through a series of iterations
$\square$ e.g. gradient ascent (hill climbing)

- There are partial derivatives (gradient) which points out the direction of the ascent
- There is a global optimum: convergence
- But we cannot predict how far to go.
$\square$ There is a tendency to overfitting, hence regularization

$$
\hat{w}=\underset{w}{\arg \max } \sum_{i=1}^{m} \log P\left(c^{i} \mid \vec{f}^{i}\right)-\alpha R(w)
$$

$\square$ Don't try this at home! Use a package

## Gradient ascent



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## A slight reformulation

$\square$ We saw that for NB

$$
P\left(c_{1} \mid \vec{f}\right)>P\left(c_{2} \mid \vec{f}\right) \quad P\left(c_{1}\right) \prod_{j=1}^{n} P\left(f_{j} \mid c_{1}\right)>P\left(c_{2}\right) \prod_{j=1}^{n} P\left(f_{j} \mid c_{2}\right)
$$

$\square$ iff

$$
\log \left(\frac{P\left(c_{1}\right)}{P\left(c_{2}\right)}\right)+\sum_{j=1}^{n} \log \left(\frac{P\left(f_{j} \mid c_{1}\right)}{P\left(f_{j} \mid c_{2}\right)}\right)>0
$$

$\square$ This could also be written

$$
\left(\log P\left(c_{1}\right)-\log P\left(c_{2}\right)\right)+\sum_{j=1}^{n}\left(\log P\left(f_{j} \mid c_{1}\right)-\log P\left(f_{j} \mid c 2\right)\right)>0
$$

$$
\log P\left(c_{1}\right)+\sum_{j=1}^{n} \log P\left(f_{j} \mid c_{1}\right)>\log P\left(c_{2}\right)+\sum_{j=1}^{n} \log P\left(f_{j} \mid c 2\right)
$$

## Reformualtion, contd.

$\square \quad \log P\left(c_{1}\right)+\sum_{j=1}^{n} \log P\left(f_{j} \mid c_{1}\right)>\log P\left(c_{2}\right)+\sum_{j=1}^{n} \log P\left(f_{j} \mid c 2\right)$
$\square$ has the form

$$
\vec{w}^{1} \bullet \vec{f}=\sum_{i=0}^{M} w_{i}^{1} x_{i}>\sum_{i=0}^{M} w_{i}^{2} x_{i}=\vec{w}^{2} \bullet \vec{f}
$$

$\square$ where

- $w_{j}^{1}=\log \left(P\left(f_{j} \mid c_{1}\right)\right)$
- $w_{j}^{2}=\log \left(P\left(f_{j} \mid c_{2}\right)\right)$
$\square$ and our earlier $w_{j}=w_{j}^{1}-w_{j}^{2}$
$\square$ So the probability in this notation

$$
P\left(c_{1} \mid \vec{f}\right)=\frac{e^{\overline{\bar{\omega}} \bullet \vec{f}}}{1+e^{\overline{\bar{\omega}} \bullet \vec{f}}}=\frac{e^{\left(\bar{w}^{1}-\bar{w}^{2}\right) \bullet \vec{f}}}{1+e^{\left(\bar{w}^{1}-\vec{w}^{2}\right) \bullet \vec{f}}}=\frac{e^{\overline{\bar{w}}^{1} \bullet \vec{f}}}{e^{\overline{\bar{w}}^{2} \bullet \vec{f}}+e^{\bar{w}^{1} \bullet \vec{f}}}
$$

$\square$ and similarly for $\mathrm{P}\left(\mathrm{c}_{2} \mid \mathbf{f}\right)$

## Multinomial logistic regression

$\square$ We may generalize this to more than two classes
$\square$ For each class $c^{i}$ for $i=1, . ., k$

- a linear expression $\quad \vec{w}^{j} \bullet \vec{f}=\sum_{i=0}^{M} w_{i}^{j} x_{i}$
- and the probability of belonging to class c :

$$
P\left(c^{j} \mid \vec{f}\right)=\frac{1}{Z} \exp \left(\vec{w}^{j} \bullet \vec{f}\right)=\frac{1}{Z} e^{\bar{w}^{j} \cdot \vec{f}}=\frac{1}{Z} e^{\sum_{i} w_{i}^{j} f_{i}}=\frac{1}{Z} \prod_{i}\left(e^{w_{i}^{j}}\right)^{f_{i}}=\frac{1}{Z} \prod_{i} a_{i}^{f_{i}}
$$

- where $Z=\sum_{j=1}^{k} \exp \left(\vec{w}^{j} \bullet \vec{f}\right)$
and $a_{i}=e^{w_{i}}$

$$
\frac{\text { Multinomial regression }}{\text { Logistic regression }} \approx \frac{\text { Naive Bayes (Bernoulli) }}{\text { Binary NB as linear classifier }}
$$

## Footnote: Alternative formulation

$\square$ (In case you read other presentations, like Mitchell or Hastie et. al.:
$\square$ They use a slightly different formulation, corresponding to
$\square$ where for $i=1,2, \ldots, k-1$ :
$P\left(c^{i} \mid \vec{f}\right)=\frac{1}{Z} \exp \left(\vec{w}^{i} \bullet \vec{f}\right)=\frac{1}{Z} e^{\vec{w}^{i} \bullet \vec{f}}=\frac{1}{Z} e^{\sum_{j} j_{j}^{i} f_{j}}=\frac{1}{Z} \prod_{j}\left(e^{W_{j}^{i}}\right)^{f_{j}}=\frac{1}{Z} \prod_{j} a_{j}^{f_{j}}$
But $Z=1+\sum_{i=1}^{k-1} \exp \left(\vec{w}^{i} \bullet \vec{f}\right)$ and $P\left(c^{k} \mid \vec{f}\right)=\frac{1}{1+\sum_{i=1}^{k-1} \exp \left(\vec{w}^{i} \bullet \vec{f}\right)}$
$\square$ The two formulations are equivalent though:

- In the J\&M formulation, divide the numerator and denominator in each $P\left(c^{i} \mid f\right)$ with

$$
\exp \left(\vec{w}^{k} \bullet \vec{f}\right)
$$

- and you get this formulation (with adjustments to $Z$ and $\mathbf{w}$.)


## Indicator variables

$$
P\left(c^{j} \mid \vec{f}\right)=\frac{1}{Z} \exp \left(\bar{w}^{j} \bullet \vec{f}\right)=\frac{\exp \left(\vec{w}^{j} \bullet \vec{f}\right)}{\sum_{l=1}^{k} \exp \left(\bar{w}^{\prime} \bullet \vec{f}\right)}=\frac{\exp \left(\sum_{i=0}^{n} w_{i}^{j} f_{i}\right)}{\sum_{l=1}^{k} \exp \left(\sum_{i=0}^{n} w_{i}^{\prime} f_{i}\right)}=\frac{\exp \left(\sum_{i=1}^{m} w_{i} f_{i}\left(c^{j}, x\right)\right)}{\sum_{i=1}^{k} \exp \left(\sum_{i=0}^{m} w_{i} f_{i}\left(c^{\prime}, x\right)\right)}
$$

$\square$ Already seen: categorical variables represented by indicator variables, taking the values 0,1
$\square$ Also usual to let the variables indicate both observation and class

## Examples - J\&M

We would like to know whether to assign the class $V B$ to race (or instead assign some other class like $N N$ ). One useful feature, we'll call it $f_{1}$, would be the fact that the current word is race. We can thus add a binary feature which is true if this is the case:

$$
f_{1}(c, x)= \begin{cases}1 & \text { if } \text { word }_{i}=" \text { race" } \& ~ c=\mathrm{NN} \\ 0 \text { otherwise }\end{cases}
$$

Another feature would be whether the previous word has the tag TO:

$$
f_{2}(c, x)= \begin{cases}1 & \text { if } t_{i-1}=\mathrm{TO} \& c=\mathrm{VB} \\ 0 & \text { otherwise }\end{cases}
$$

Two more part-of-speech tagging features might focus on aspects of a word's spelling and case:

$$
f_{3}(c, x)=\left\{\begin{array}{l}
1 \text { if suffix }\left(\text { word }_{i}\right)=\text { "ing" \& } c=\mathrm{VBG} \\
0 \text { otherwise }
\end{array}\right.
$$

## Why called "maximum entropy"?

| NN | JJ | NN | V | NN | IN | M | U | SYM | VB | P | PR | CC | CD |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ | $\frac{1}{45}$ |  |


$P(N N)+P(N N S)=0.8$

| NN | JJ | NNS | VB | NNP | $\ldots$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{4}{10}$ | $\frac{1}{10}$ | $\frac{4}{10}$ | $\frac{1}{10}$ | 0 | $\ldots$ |

$$
P(V B)=1 / 20
$$

| NN | JJ | NNS | VB |
| :---: | :---: | :---: | :---: |
| $\frac{4}{10}$ | $\frac{3}{20}$ | $\frac{4}{10}$ | $\frac{1}{20}$ |

See NLTK book for a further example

## Why called "maximum entropy"?

$\square$ The multinomial logistic regression yields the probability distribution which
$\square$ Gives the maximum entropy
$\square$ Given our training data

## Learning

$\square$ Similarly to the binary logistic regression,
$\square$ Regularization

NLTK: Some iterative optimization techniques are much faster than others.

When training Maximum Entropy models, avoid the use of

- Generalized Iterative Scaling (GIS) or
$\square$ Improved Iterative Scaling (IIS),
which are both considerably slower than the
$\square$ Conjugate Gradient (CG) and
$\square$ the BFGS optimization methods.


## Line - Most frequen BoW-features

| Number of <br> word features | NaiveBayes | SklearnClassifi <br> er(LogisticRegr <br> ession()) |
| :--- | :--- | :--- |
| 0 | 0.528 | 0.528 |
| 10 | 0.528 | 0.528 |
| 20 | 0.534 | 0.546 |
| 50 | 0.576 | 0.624 |
| 100 | 0.688 | 0.732 |
| 200 | 0.706 | 0.752 |
| 500 | 0.744 | 0.804 |
| 1000 | 0.774 | 0.838 |
| 2000 | 0.802 | 0.846 |
| 5000 | 0.826 | 0.850 |

