# OPL-CPLEX exercises for INF-MAT 3370

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## 1 Airline company

Solve exercise 1.2 in Vanderbei and use OPL-CPLEX to find the solution to the LP problem you formulated.

## 2 Unboundedness

Consider the following LP problem:
Maximize $x_1 + x_2$
Subject to:

$$3x_1 + x_2 \geq 0$$

$$4 * x_1 \leq 7$$

$$x_1 \geq 0$$

Can you find a feasible solution?

Solve the problem with CPLEX. What do you find? Explore the tab "Statistics". Try changing the .ops-file: Under "Preprocessing", change "Primal and dual reduction type" to "No primal and dual reductions". What happens if you run the problem again? Why?

Note: If your OPL-project does not contain an .ops-file, you may add one if you right-click on the project name and choose New→Settings.

## 3 Asset investment

You want to invest in assets, and have come up with 5 interesting companies. Now you want to find the optimal way to distribute your money between these companies. You check today's value of their assets and you create 6 possible

| Company | Now | State 1 | State 2 | State 3 | State 4 | State 5 | State 6 |
|---|---|---|---|---|---|---|---|
| Algeta | 62 | 75 | 60 | 62 | 63 | 61 | 62 |
| Clavis | 54 | 56 | 52 | 55 | 54 | 56 | 54 |
| Statoil | 146 | 147 | 151 | 143 | 149 | 141 | 147 |
| Acergy | 90 | 91 | 93 | 88 | 95 | 87 | 91 |
| SAS | 3.25 | 3.20 | 3.14 | 3.50 | 3.18 | 3.20 | 3.23 |

Table 1: The asset values.

| States | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Probability | 5 | 20 | 5 | 25 | 15 | 30 |

Table 2: The probabilities of the states.

cases for their prices tomorrow. Each case is associated with a probability, and the probabilities sum to 1. This means you are certain that one of the cases will be true. The data is shown in tables 1 and 2.

**a)**

Use what you learned in compulsory project 1 to check whether it is possible to invest money such that you are guaranteed to gain money.

**b)**

Consider the following LP problem

$$\begin{array}{ll} \max & v - \lambda \sum_{t=1}^{T} p_t u_t \\ \text{subject to} & \\ & v = \sum_{j=1}^{n} \sum_{t=1}^{T} r_{jt} p_t x_j \\ & u_t \geq \sum_{j=1}^{n} r_{jt} x_j - v \quad (t \leq T) \\ & u_t \geq v - \sum_{j=1}^{n} r_{jt} x_j \quad (t \leq T) \\ & \sum_{j=1}^{n} x_j = 1 \\ & x_j \geq 0 \quad (j \leq n). \end{array}$$

The (continuous, real) variables in (P) are $u_1, u_2, \ldots, u_T$, $v$, and $x_1, x_2, \ldots, x_n$. $\lambda$ is a nonnegative real number.

The interpretation of this problem is as follows. Given assets $1, 2, \ldots, n$ (asset means investment possibility, like stocks (norsk: aksjer) or putting money in the bank). The returns on these assets are unknown and considered as random variables $R_1, R_2, \ldots, R_n$. (The return is nonnegative: if $R = 0$ the money is lost, if $R = 1$ you have the same amount as invested, and if

$R > 1$ you gain money, ...). We assume a discrete probability model: with probability $p_t$ the returns are given by

$$R_1 = r_{1t},\ R_2 = r_{2t},\ \ldots,\ R_n = r_{nt}.$$

Thus, there are $T$ possible outcomes; they correspond to $T$ different scenarios for the future development. The probabilities $p_1, p_2, \ldots, p_T$ describe our belief in each of these scenarios, so therefore $\sum_t p_t = 1$.

Consider the *investment vector* $x = (x_1, x_2, \ldots, x_n)$ where $x_j$ represents the fraction of your capital that you decide to invest in asset $j$. The (random) *return* on your investment is then the random variable

$$R(x) = \sum_{j=1}^{n} R_j x_j$$

which clearly depends on $x$.

Now, let $p$ be a vector containing the probabilities in table 2. The matrix $R$ is computed from table 1 in the following way: For each company, the value in each state is divided by the now-value for the company. Each column in R contains the ratios for one company, while each line in R contains the ratios in one state. Use $\lambda = 1$ and solve the LP problem to find the optimal investment strategy. Also try with a higher and a lower value for $\lambda$. What is the interpretation of $\lambda$?

If you want to understand more of the theory behind this LP problem, you are welcome to study the compulsory problem from 2009.

**c)**

Either repeat the exercise using your own table of your favorite companies with cases you consider likely and with suitable probabilities, or change the tables 1 and 2 to what you consider more (or less) likely and observe the changes in the solution.