Assignment 3
INF-MAT5340
Spring 2011

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March 1, 2011

Exercise 1. Write a procedure that will compute the first derivative of a spline function f. Use any of the algorithms discussed in the lecture notes. Test the program by differentiating the two polynomials 1 and x, represented in terms of B-splines.

Exercise 2.

a) Write a program that implements the Schoenberg variation diminishing spline approximation, assuming that the spline degree p and the knot vector $\boldsymbol{t}$, as well as the function f to be approximated, are given.

b) Test your program by computing a cubic spline approximation to the function $f(x) = \sin x$ on the interval $[0, 8\pi]$. Use a knot vector with 20 uniformly spaced knots in the interior of the interval.

c) Use your procedure from exercise 1 to compute the first derivative of the approximation in (b) and plot this derivative and the derivative of $\sin x$ together.

d) Can you find a knot vector $\boldsymbol{t}$ such that the cubic Schoenberg approximation to $\sin x$ on the interval $[0, 3\pi]$ is identically zero?

Exercise 3.

a) Write a program that implements spline interpolation for a given spline space of degree p with knots $\boldsymbol{t}$. Assume that the data $(x_i, y_i)_{i=1}^n$ to be interpolated are given as two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, where n is the dimension of the spline space.

b) Test your program by computing a cubic spline interpolant to the function $f(x) = \sin x$ on the interval $[0, 8\pi]$. Use a knot vector with 20 uniformly spaced knots in the interior of the interval, and let the interpolation points be uniformly spaced in the same interval.

c) Compute the first derivative of the approximation in (b) and plot this derivative and the derivative of $\sin x$ together. Compare the error to that of the Schoenberg approximation in exercise 2.

Exercise 4. Prove that the Schoenberg variation diminishing spline approximation to a function f gives error 0 if f is a straight line.