INF-MAT 5360: Obligatory project 2

To be handed in by November 23

1 Sudoku

1.1 Introduction

Sudoku is a logic-based puzzle that first appeared in the U.S. in 1979. Sudoku most commonly appears in its 9×9 matrix form. The rules are simple: fill in the matrix so that every row, column, and certain 3×3 submatrix (see Figure 1) contains the digits 1 through 9 exactly once. Each puzzle appears with a certain number of *givens*. The number and location of these determines the games level of difficulty. This puzzle idea can accommodate games of other sizes. Of course, a 4×4 puzzle would be easier and a 16×16 puzzle, harder. In general, any $n \times n$ game can be created, where $n = m^2$ and m is any positive integer.

Sudoku puzzles elicit the following interesting mathematical questions:

- How can these puzzles be solved efficiently mathematically?
- When does a puzzle have a solution, and when is it unique?
- What mathematical techniques can be used to create these puzzles?

1.2 Solving the puzzle

Exercise 1. Formulate the problem of solving a Suduko as a binary integer problem using the variables

$$x_{ijk} = \begin{cases} 1, & \text{if element } (i,j) \text{ of the Sudoko matrix has value } k \\ 0, & \text{otherwise} \end{cases}$$

Explain the meaning of your objective function and each of your constraints.

Exercise 2. Use your binary integer problem to create an AMPL model to solve the Sudoku problem. Create a data set for the Sudoku problem shown in Figure 1, and find the solution.

2					8			6
	3	4	9					
				5			9	
		1	8					5
8								3
6					9	4		
	2			4				
					3	6	1	
5			6					4

Figure 1: Sudoku puzzle

1.3 Multiple solutions

In normal Sudoku puzzles there is a unique solution, but in general this may not always be the case as it depends upon the givens. It is still an open question what is the minimum number of givens that will produce a unique solution. The current record is 17 givens. No puzzle with 16 or less givens, and producing a unique solution, has been discovered.

Exercise 3. Show that a 4×4 Sudoku with 4 givens satisfying the condition that each given appears in its own row, own column, own submatrix, and has its own integer value has a unique solution.

1.4 Creating puzzles

A Sudoku matrix is a $n \times n$ matrix satisfying the requirements given in the introduction. There are several ways to create a new Sudoku matrix from an existing one.

Exercise 4. Show that if S is a Sudoku matrix, then S^T is also a Sudoku matrix.

2 Slitherlink

Slitherlink (also known as Fences, Takegaki, Loop the Loop, Ouroboros and Dotty Dilemma) is a logic puzzle. Slitherlink is played on a rectangular lattice of dots. Some of the squares formed by the dots have numbers inside them. The objective is to connect horizontally and vertically adjacent dots so that the lines form a single cycle with no loose ends. In addition, the number inside a square represents how many of its four sides are segments in the loop. An example of a puzzle is shown in Figure 2.

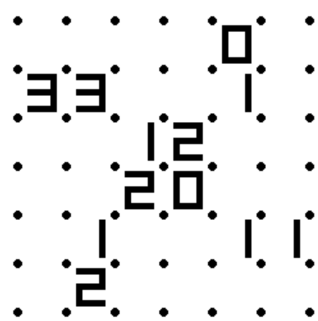

Figure 2: Slitherlink puzzle.

Form a graph $G = (V, E)$ by introducing a vertex v for each dot in the lattice and an edge e between all pairs of horiontal and vertical neighbors. For each square $k = 1, \dots, K$ with a number in it, let $B(k)$ be the set of surrounding edges and b_k the number inside. The following in an attempt at modelling the Slitherlink problem as an integer problem.

$$
\begin{array}{lrl}
\max & 0 & \\
 & & \\
\text{(i)} & \sum_{e \in B(k)} y_e = b_k, & k = 1, \dots, K \\
\text{(ii)} & \sum_{e \in \delta(v)} y_e = 2x_v, & \text{for all } v \in V \\
\text{(iii)} & y_e = x_v, & \text{for } v \in e \\
\text{(iv)} & x, y & \text{binary.}
\end{array}
$$

Exercise 5. Interpret this integer programming by giving a meaning to the variables and the constraints. Explain why this formulation in general will not produce legal solutions to the Slitherlink puzzle?

Extra challenge. This exercise is not compulsory. Formulate an integer problem that solves the Slitherlink problem and a corresponding AMPL model. Test your model on the problem in Figure 2.

Please send the answers to the exercises and commented versions of your AMPL models (and data) to *Truls.Flatberg@sintef.no*.

Good luck!