

INF-MAT 5360 Mathematical optimization

Summary

The following summarizes the main topics and results in the course, and it can be useful when preparing for the oral exam.

General advice

- focus on ideas and concepts
- know the precise definitions of concepts and problems
- know the main theorems and algorithms
- learn the proofs, or at least main steps, for main results
- proofs for less central results may be omitted
- practice on presenting concepts/theorems etc. with a fellow student

The ability to apply the theory in solving/discussing more concrete problems is also welcome and will pay off at the exam!

Summary of main topics/results

Alexander Schrijver : *A Course in Combinatorial Optimization*, 2007.

- Chapter 1
 - Spanning tree: methods and correctness of these, running time
 - Shortest paths: theory, algorithms (Dijkstra, Bellman-Ford) with correctness and running time, applications

- Chapter 2
 - Definition of convexity, convex hull, polytopes and polyhedra, vertices, Farkas lemma, LP duality
 - Main results: Theorems 2.1, 2.2, 2.3, 2.4 (not proof), 2.5, 2.6 (not proof).
- Chapter 3
 - Matching, vertex cover, relations, (Gallai), Königs theorem
 - Max. cardinality matching (bipartite graph): theory and algorithm
 - Max. weight bipartite matching, theory and algorithm
 - Matching polytopes (bipartite and general graphs)
- Chapter 4
 - Flow theory, max-flow min-cut theorem, the max-flow algorithm
 - Hoffmans circulation theorem
 - Min. cost flow (extreme flow + algorithm)
- Chapter 5
 - Some knowledge of nonbipartite matching (example of algorithm + matching polytope)
- Chapter 8
 - ILP (integer LP), examples, connection to LP relaxation
 - TU (totally unimodular), concept and connections to integrality of polyhedra, consequence for ILP
 - TU matrices: examples, incidence matrices for graphs and directed graphs

Geir Dahl: *An introduction to convexity, polyhedral theory and combinatorial optimization*, 1997.

- Chapter 4
 - Graphs: walk, cycle, subgraph, bipartite graph, tree
 - Main results: Proposition 4.4 and 4.5
- Chapter 5
 - Forest polytope (Theorem 4.5)
- Chapter 6
 - Integer linear programming, integer hull, linear description
 - Valid inequalities: Chvatal-Gomory procedure, examples (knapsack, node packing)
 - Branch and bound algorithm, pruning
 - Cutting plane algorithm

Good luck! Truls Flatberg, Nov. 2007.