INF-MAT5360 Mathematical optimization – Preparing for the written exam, 2010

General:

- This year there is a written exam in INF-MAT5360, while previously only oral exam has been used. Some guidelines on how to prepare are given next.
- The syllabus is clear. The lectures have focused on all the main topics in suitable detail, so the notes from the lectures will be very useful.
- The goal of the course is to understand several concepts and related theorems/results/algorithms: the exam therefore tests this understanding, and may use central techniques/arguments in proofs.

Here is a list of the main concepts/ideas/results/methods. The indication "(proof)" means that knowledge of proof could be useful.

Main results/concepts:

- *Convexity:*
  - Chapter 1: convex set and cone, LP problem, polyhedron, Prop.1.4.1, halfline, affine set, Prop.1.5.1, main results for convex functions (see summary, lecture notes from Chapter 1).
  - Chapter 2: convex/nonneg. combination, Prop.2.1.1 (proof), convex hull, Prop.2.2.1, polytope, LP and polytopes, affine independence, Prop.2.3.1, dimension, Prop.2.3.2, affine hull, simplex, some knowledge of topology for convex sets, Thm.2.5.1 (proof), Thm.2.5.2, Prop.2.5.4.
  - Chapter 3: distance function, nearest point (projection), Prop.3.1.1, Thm.3.1.2 (proof), hyperplane, Prop.3.2.1 (proof), separating hyperplane, supporting hyperplane, Thm.3.2.2, Thm.3.2.3, Cor.3.2.4, Thm.3.2.5, supporting halfspace, Thm.3.2.6.
  - Chapter 4: face, exposed face, Prop.4.1.1, extreme point and extreme halfline, recession cone, Prop.4.2.1, Thm.4.3.2 (proof main idea), Cor.4.3.3, Cor.4.3.4., Lemma 4.4.1 (proof), Thm.4.4.4, Cor.4.4.5, main idea in polyhedral combinatorics.

- *Combinatorial optimization:*
  - Shortest paths and trees (Schrijver notes): main concepts, Dijkstra Algorithm, shortest path with arbitrary lengths, existence of solutions Thm. (proof), minimum spanning trees, greedy forest, Theorem 1.11, Dijkstra-Prim method, Kruskal method.
  - Chapter 1: combinatorial optimization problem, convex hull, the maximum weight forest problem and the forest polytope, Theorem 1.2 (only the statement), formulation, relaxations, upper and lower bounds, quality of formulation, separation oracle, separation oracle for subtour elimination constraints, dynamic simplex method.
  - Chapter 2: Integer hulls and integral polyhedra, Prop.2.2, Cor.2.4, TU matrices, Thm.2.5., Cor.2.6, Prop.2.9, Prop.2.10 (proof), Prop.2.12 (proof).
  - Chapter 3: constructive heuristics, greedy algorithm, approximation guarantee, Christofides Algorithm, Thm.3.1. (proof), improvement heuristic, neighborhood function and local search, 2-exchange move and 2-opt for TSP, exponential neighborhood search, Sarvanov/Doroshko neighborhood.
  - Chapter 4: relaxations, Prop.4.1., divide and conquer, enumeration tree, pruning, Prop.4.3., branch-and-bound algorithm, cutting plane, valid inequality, Gomory cuts, TSP, comb inequalities.

Good luck!

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