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INF 3300, INF4300 Mathematical morphology

An introduction with applications

Lars Aurdal,

Norsk Regnesentral,

September 14th 2006

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Reading recommendations

1. Excellent book: *Morphological Image Analysis, Principles and Applications*, 2nd ed., by Pierre Soille, Springer Verlag, ISBN 3-540-42988-3.

The approach we choose

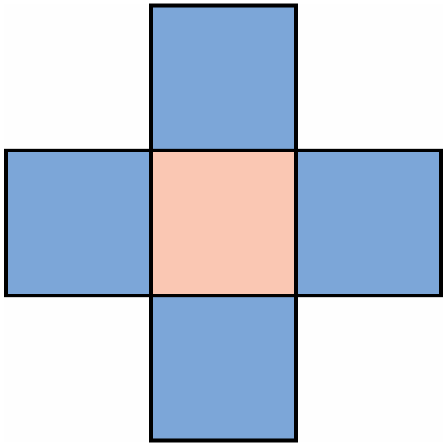
1. Mathematical morphology can be explained from very mathematical standpoint.
2. We will not choose that approach, rather we will focus on how these operators work from a visual point of view and how they can be implemented on a computer.
3. This avoids us a lot of gory mathematical detail.
4. If you want more details then consider INF5300.

The structuring element

1. The fundamental tool in mathematical morphology is the structuring element.
2. The structuring element is a (small) group of pixels. One of these pixels is designated as the origin of the structuring element.
3. The relation between this small group of pixels and the foreground pixels in the image to be analyzed is what we use to define the morphological operators.

The structuring element

1. Let’s consider the following structuring element:



2. This structuring element consists of 5 pixels organized in a cross, let’s consider the pink pixel in the center the origin of the structuring element.

Erosion

1. The first question we might ask is: “To where could I move the center of the structuring element in such a way that the entire structuring element overlaps foreground pixels”
2. Those pixels to where the structuring element can be moved like this are called the erosion of the original image.
3. Follow this link for an illustration.

Erosion

1. Matlab example.

Dilation

1. Another question we might ask is: “To where could I move the center of the structuring element in such a way that the structuring element overlaps foreground pixels in at least one position”.
2. Those pixels to where the structuring element can be moved like this are called the dilation of the original image.
3. Follow this link for an illustration.

Dilation

1. Matlab example.

Opening and closing

1. Remember that the basic operations of erosion and dilation can be combined.
2. Erosion followed by dilation is often referred to as an opening. It will remove noise and partly restore large objects.
3. The opposite, dilation followed by erosion is referred to as closing. In particular, this will close gaps on object boundaries as well as holes.

Opening and closing

1. Matlab example

Geodesic operators

1. One very important class of morphological operators are the so called geodesic operators.
2. They are based on simple erosions and dilations, but operate on two images, the input image and a mask.
3. The operator is applied to the input image, but the effect of the operator on the input image is limited by the mask.

Geodesic erosion

1. Consider two binary images, f and g.
2. We will consider f the input image and g the mask.
3. A geodesic erosion of f over the mask g is simply the following:
   a. Erode f by your structuring element.
   b. Perform a pixelwise maximum operation using the eroded f and the mask g as input.

Geodesic dilation

1. Consider two binary images, f and g.
2. We will consider f the input image and g the mask.
3. A geodesic dilation of f under the mask g is simply the following:
   a. Dilate f by your structuring element.
   b. Perform a pixelwise minimum operation using the dialted f and the mask g as input.

Geodesic dilation

1. Matlab example.

Geodesic operators

1. On “their own” the basic geodesic operators aren’t all that interesting.
2. They are however at the basis of the very important reconstruction operators.
3. The reconstruction operators are based on iterations of the geodesic operators **until idempotence**.
   a. An operator O is said to be idempotent if repeated applications has the same effect as a single application.

Morphological reconstruction

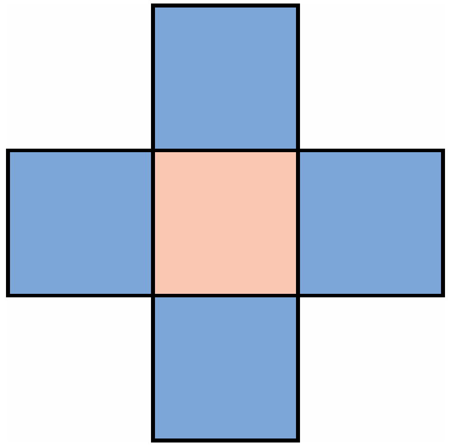
1. The morphological reconstruction (by dilation) of the input image f under the mask g consists in repeating a geodesic dilation of f under the mask g until idempotence (that is, until no further change).
2. Matlab example.

The extension to gray level images

1. The formal definition for morphological operations on gray level images can be quite confusing.
2. We will consider a slightly simplified definition, based on the assumption that the structuring element is “flat”.
3. In that case, the basic mathematical operations of erosion and dilation are reduced to local min and max operations over the structuring element as it is “moved” over the image.

Gray level erosion

1. Consider the previous structuring element:



2. This structuring element consists of 5 pixels organized in a cross, let’s consider the pink pixel in the center the origin of the structuring element.

Gray level erosion

1. Eroding a gray level image with this structuring element is very simple:
    a. Move the origin of the structuring element from pixel to pixel in the input image.
    b. Select the local **minimum** over the region defined by the structuring element.
    c. Write this minimum value to the same pixel position in the output image.

Gray level erosion

1. Remember: This is **nothing else** than a calculation of local minima values.
2. The **only** difference with what you would typically consider a local minimum calculation is that the **shape** of the region over which you calculate the minimum is defined by the shape of the structuring element.

Gray level erosion

1. Simple matlab example.

Gray level erosion

1. As we pointed out in the first lecture, structuring elements come in all sizes and shapes.
2. One very common type of structuring element is a line shaped structuring element.
3. What can they be used for?

Gray level erosion

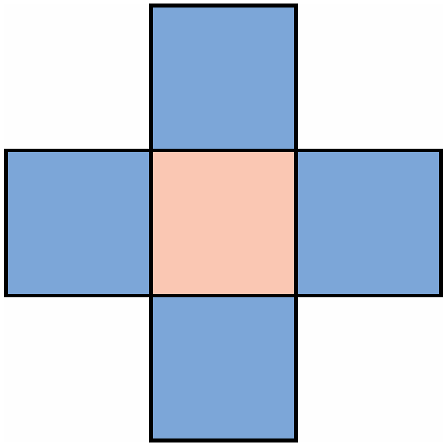
1. Consider the following image showing carbon fibers embedded in a special glue (epoxy).
2. Such carbon fiber matrices make very light but strong structures.

Gray level erosion

1. What if you wanted to answer a question like: "What is the distribution of fiber directions in this image?".
2. Matlab example.

Gray level dilation

1. Again, consider this structuring element:



Gray level dilation

1. Dilating a gray level image with this structuring element is also very simple:
   a. Move the origin of the structuring element from pixel to pixel in the input image.
   b. Select the local **maximum** over the region defined by the structuring element.
   c. Write this maximum value to the same pixel position in the output image.

Gray level dilation

1. Again: This is **nothing else** than a calculation of local maxima values.
2. The **only** difference with what you would typically consider a local maximum calculation is that the **shape** of the region over which you calculate the maximum is defined by the shape of the structuring element.

Gray level dilation

1. Simple matlab example.

Gray level opening and closing

1. The definition is completely parallel to that for the binary case:
   a. Gray level opening consists of a gray level erosion of the given image followed by a gray level dilation of the eroded image (using the same structuring element).
   b. In gray level closing we just invert the order of the basic operators.

Gray level opening and closing

1. Matlab example

Morphological gradients

1. A gray level dilation will (under certain conditions that we will not discuss here) produce an image with equal or higher values than in the input image.
2. This is easy to understand since the dilation is a local maximum.
3. Likewise, the erosion will (under the same conditions) produce an image with equal or smaller values.
4. This can be used for edge detection!

Morphological gradients

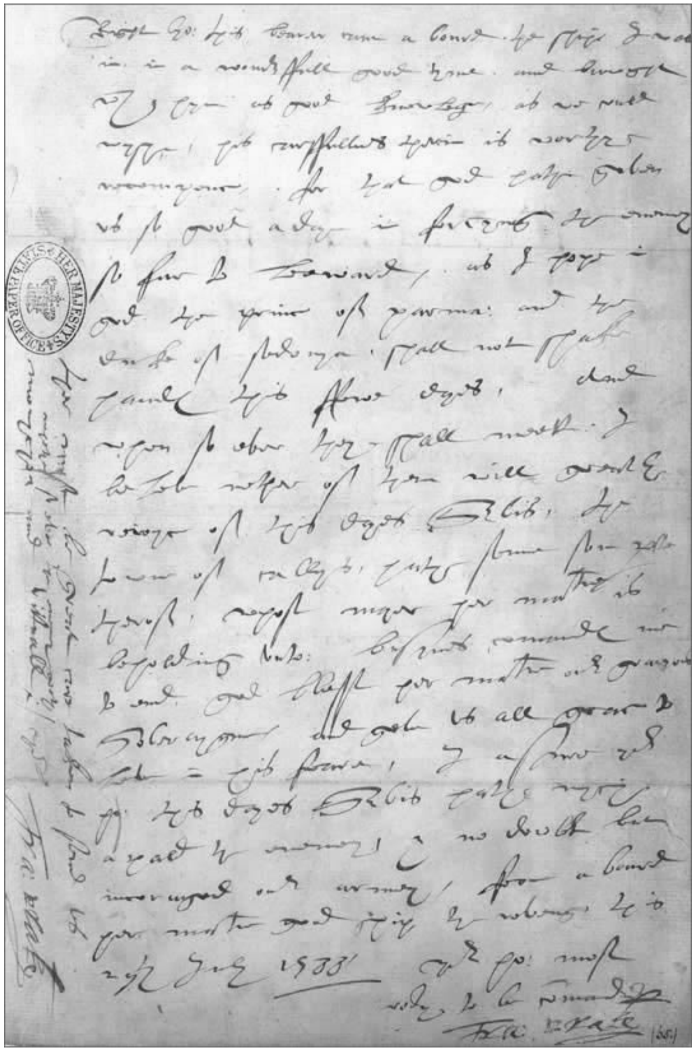
1. Matlab example.

The top hat transforms

1. The most frequently used of the top hat transforms is the so called white top hat transformation (WTH).
2. The WTH is primarily used in order to deal with images with uneven illuminations.
3. The WTH transform of an image is simply the image itself minus its opening!

The top hat transforms

Consider this image that we have already looked at earlier. We have already observed that there is no single, global threshold that will succeed in separating text from background. So far we have seen how this can be solved using local thresholding algorithms. But can anything be done in order to compensate for the uneven illumination.



The top hat transforms

1. Matlab example.